EX ANTE EFFICIENCY OF STRUCTURED BARGAINING PROCEDURES UNDER COORDINATION FAILURE AMONG CREDITORS

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Abstract

We analyze the ex ante efficiency of structured bargaining procedures, especially the absolute priority rule (APR) violations and the revocation of preferential payments (PP). We show that when creditors receive clear signals about firms, the debtor is more likely to choose a risky action if APR violations are adopted. On the other hand, when a noisy signal is transmitted to creditors, a high liquidation value may induce coordination failure among creditors. Because this also induces moral hazard on the part of the debtor, adopting APR violations may be a useful way of improving ex ante efficiency. Finally, if there is complementarity between a firm's assets, the revocation of PP can mitigate the coordination problem and thus increase ex ante efficiency**.

Keywords: Coordination Failure, Absolute Priority Rule, Preferential Payments, Global Game

JEL Classification Codes: G33, K40, D74

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1 Introduction

It is widely recognized that bankruptcy law plays an important role in protecting creditors, inducing an efficient allocation of firms' assets and disciplining the behavior of firms' managers (Aghion et al. (1992)).[38] Although bankruptcy laws not only differ across countries but also change over time, their broad aim is twofold (Hart (1995)).[39] The first goal is to distribute firms' assets efficiently among stakeholders or to determine the best allocation of firms' assets ex post. The second goal is to provide efficient incentives to stakeholders ex ante by designing the distribution and the allocation of firms' rights held by stakeholders ex post. In this sense, the design of bankruptcy law should consider the interaction between ex ante and ex post efficiency by resolving two types of conflict: conflict between a debtor and creditors, and conflicts among creditors (von Thadden et al. (2010)).

In a structured bargaining procedure such as the Civil Rehabilitation Act in Japan or Chapter 11 in the United States, once the procedure is commenced, an automatic stay is declared, and creditors do not extract any gains until a reorganization plan has been implemented. Then, a firm's manager or equity holders can extract absolute priority rule (APR) violations and retain some of the firm's value. Therefore, considering the conflict between a debtor and creditors corresponds to identifying the role of APR violations. On the other hand, a conflict among creditors becomes more acute as the debtor becomes unprofitable. That is, each creditor wants to be repaid its own money or to seize some of the firm's assets at the expense of the other creditors' stakes. Preferential payments (PP) are payments or transfers of firms' assets to creditors who have some advantage over other creditors. However, in a structured bargaining procedure, if the trustees find that any creditor has received such repayment before the bankruptcy, they are given the right to void the transactions in order to make a more equitable distribution to all creditors or to make the reorganization plan of the firm more reliable. In this case, considering the conflict among creditors corresponds to identifying the role of the revocation of PP. Therefore, the main goals of this paper are to show how the two types of conflict affect each other and to determine the role of structured bargaining procedures, particularly APR violations

[38] It is also important to investigate why bankruptcy laws exist. To consider this, see Longhofer and Peters (1999).
[39] Acharya et al. (2011) analyze how differences in bankruptcy codes across countries affect firms' capital structures.

and the revocation of PP.

To evaluate the role of violating APR, much research has been done. However, the usefulness of APR violations is yet to be determined. Some researchers insist that violating APR ex post has a positive effect on ex ante efficiency. Bebchuk and Picker (1993) and Berkovitch et al. (1997), (1998) show that APR violations encourage the firm's managers to increase investment in firm-specific human capital ex ante. Gertner and Scharfstein (1991) and Eberhart and Senbet (1993) show that APR violations deter the firm's managers from taking more risky actions when the firm is in financial distress. Moreover, Berkovitch and Israel (1998) show that APR violations facilitate the transfer of information to creditors for reorganization. On the other hand, Longhofer (1997) and Bebchuk (2002) conclude that APR violations have a negative effect on ex ante efficiency. Longhofer (1997) shows that APR violations not only exacerbate credit rationing problems but also make default more likely to occur. Bebchuk (2002) shows that APR violations encourage firm managers to take risky actions ex ante.

On the other hand, in considering how the other type of conflict (i.e., a conflict among creditors) ex post affects the ex ante financial contract, we face a theoretical difficulty. That is, when incorporating a coordination problem among creditors into a debtor--creditors relationship, one faces the problem of multiple equilibria. With multiple equilibria, because it is impossible to determine which equilibrium is achieved, one cannot determine how the coordination problem affects the firm manager's incentives and the ex ante financial contract, and therefore, the role of the revocation of PP is indeterminate. However, recent progress in the literature on equilibrium selection, especially the notion of the global game, enables us to analyze these issues. This is because, when one introduces incomplete information and strategic complementarities among players into the model, one can obtain a unique equilibrium solution. There has been much recent research on how coordination failure among creditors affects the ex ante financial contract, which is based on the concept of the global game (Morris and Shin (2004), Hubert and Schafer (2002) and Kasahara (2009)).[40]

These researchers, however, do not derive ex ante and ex post efficiency by considering the two types of problems jointly; one is an agency problem between a debtor and creditors, and the other is a coordination problem among creditors. Thus, in this paper, using the concept of the global game, we consider how a structured bargaining procedure affects ex ante and ex post efficiency when both problems exist. We set up a model in which a firm's manager (debtor) and two creditors interact strategically.[41] Creditors, observing a signal about the firm's project in the interim period, decide whether to roll over their loans or to withdraw their money. After creditors have made these decisions, a debtor chooses whether to allocate its assets to a safe project or to a risky project. The firm's final return depends on which project the debtor chooses, what signals are received in the interim period, and how many creditors roll over their loans.

In this framework, we derive the following conclusions. First, when creditors receive clear signals about the firm, the amount of repayment creditors demand ex ante decreases as the liquidation value of the firm increases. Second, when creditors receive a noisy signal, the repayment decreases until the liquidation value reaches the threshold level, after which it increases. The first conclusion indicates that if APR violations are adopted, creditors require a greater amount of repayment ex ante. This, in turn, exacerbates ex ante inefficiency, because a debtor is more likely to choose a risky project. This result is consistent with the results of Longhofer (1997) and Bebchuk (2002). On the other hand, the second conclusion indicates that when signals are noisy, the amount of repayment increases as the liquidation value approaches its upper bound. The intuition behind this can be explained as follows. When the liquidation value is high, creditors are more likely to withdraw their loans. However, the less informative is the signal, the more severe is the coordination problem for creditors. To compensate for this, creditors require increased repayments ex ante. Because this also induces moral hazard for the debtor, adopting APR violations may be useful for improving ex ante efficiency when creditors receive noisy signals about the debtor. Third, we show that when there is complementarity between the physical assets of the firm, adopting the revocation of PP provides creditors with more incentive to cooperate with each other. Therefore, adopting the revocation of PP increases ex ante efficiency.

This paper represents the first attempt to determine the role of structured bargaining procedures, particularly the roles of APR violations and the revocation of PP in incorporating the two types of conflicts among stakeholders. Moreover, these conclusions provide important implications for the optimal design of bankruptcy law, particularly in relation to small and medium-sized enterprises (SMEs), because creditors receive less clear information about SMEs than about larger firms.

In the next section, we describe the model. In

[40] Bolton and Scharfstein (1996) show that multiple creditors can deter the debtor's moral hazard (i.e., strategic default). Although they address the conflict between a debtor and multiple creditors, they do not consider the coordination problem among creditors, because they analyze *ex post* bargaining problems among creditors.

[41] The following arguments can be applied when there are more than two creditors. For more details, see Moris and Shin (2003).

Section 3, we consider the problem of the manager's project choice. In Section 4, we derive the equilibrium strategies of the manager and creditors. In Section 5, we investigate the comparative statics and derive the effects of APR violations. Section 6 concludes the paper.

2 The model

There are three periods, $t = 0,1,2$, and two types of agent, a firm (a manager) and two creditors. Everyone is risk neutral, and the riskless interest rate is zero. The firm has a fixed-scale technology that requires two different physical assets. The purchase of each asset costs one unit. A firm has no wealth, and each creditor lends one unit to the firm at $t = 0$. Creditors are assumed to face a competitive financial market.

At $t = 1$, the prospect of the project is realized as the signal x. Ex ante, x is a random variable with a uniform distribution $f(x)$ on $X = [\underline{X}, \overline{X}]$. Knowing x, each creditor decides whether to roll over or to withdraw the loan.[42] If both creditors withdraw, the project is terminated and the creditors receive the liquidation value of the asset, L $(0 < L < 1)$.[43] If at least one creditor rolls over, the project is continued. If the project is continued, it is subject to the manager's decision of project choice. That is, the firm's assets can be used to implement the safe project or the risky project. At $t = 2$, the firm's final output is realized. If the project succeeds, each creditor demands a repayment of R. If the project generates enough cash to meet the repayments of both creditors, the manager receives the residual output. If the final output does not meet the total repayments of the creditors, the firm goes bankrupt at $t = 2$, and the creditors divide the output equally and the liquidation values of the two assets are assumed to be zero.[44]

The payoff of the project depends on the decisions of the manager and creditors; that is, whether the manager chooses the safe project or the risky project and how many creditors roll over their loans. Consider the case in which both creditors roll over their loans. When the manager chooses the safe project, the firm's output at $t = 2$ is x for sure (with probability 1). When the manager chooses the risky project, the firm's output at $t = 2$ is $x' + Z$ with probability 1/2 and $x' - Z$ with probability 1/2, where $x' = x - \delta$, $\delta > 0$ and $Z > 0$. Note that $x > x'$, so that choosing the safe project is always more efficient than choosing the risky project. In addition, we suppose that $Z > \delta$, so that $x' + \delta > x$, which means that a successful risky project generates more output than does the safe project.

On the other hand, consider the case in which only one creditor rolls over its loan. The creditor withdrawing its loan receives the liquidation value L at *t=1*. When the manager chooses the safe project, the firm's output at *t=2* is θx for sure, where $0 < \theta < 1/2$. In this context, θ represents the degree of complementarity between two physical assets. When the manager chooses the risky project, the firm's output at *t=2* is $\theta x' + Z$ with probability 1/2 and $\theta x' - Z$ with probability 1/2.

In the later arguments, the ex ante efficiency is evaluated by the expected NPV of the project. In this model, the level of *R* is determined at *t=0* and it affects the manager's incentive to take a risky action at *t=1*. Therefore, the level of *R* plays an important role to evaluate the ex ante efficiency. On the other hand, the ex post efficiency is evaluated by the expected NPV of the project given the realized signals. Since the inefficienct liquidation caused by the creditors' withdrawal reduces the return of the project and affects the manager's incentive to take a risky action, the coordination problem between creditors affects the ex post efficiency.[45]

3 Project choice by the manager

In this section, we consider the manager's incentive. As implied above, it is efficient for the safe project to be selected for every value of *x*. Because the condition for the safe project being chosen depends on the number of creditors who roll over their loans up to $t = 2$, we first consider the case in which both creditors roll over their loans. Then, we consider the case in which only one creditor rolls over.

3.1 The case in which both creditors roll over

In this subsection, we consider the case in which both creditors roll over and derive the condition under which the manager chooses the safe project. Note that the final output of the safe project is given by x if the signal x is revealed at *t=1*. Thus, let M_S be the manager's payoff at *t=2* when the manager chooses the safe project. Because the firm will go bankrupt if $x < 2R$, M_S can be written as:

[42] In this paper, in order to focus on the inefficiency caused by coordination failure, we describe the model as a noncooperative game and rule out the possibility of renegotiation among stakeholders.

[43] The liquidation values of both assets are assumed to be the same.

[44] In this sense, the financial contract between a debtor and creditors is a debt contract with an option to withdraw at *t=1*. This framework is also used by Morris and Shin (2004). In addition, the main results of our paper hold if we allow the firm to borrow from new creditors but make switching lenders costly.

[45] Of course, since the coordination problem between creditors affects the level of *R*, it also affects the ex ante efficiency.

$$M_s = \begin{cases} x - 2R \; if \; 2R \leq x \leq \bar{X} \\ 0 \;\; if \; \underline{X} \leq x \leq 2R. \end{cases}$$

Next, consider the case in which the manager chooses the risky project. As in the previous case, let M_R be the manager's payoff at *t=2* when the manager chooses the risky project. Then, M_R can be written as follows:

$$M_r = \begin{cases} x' \quad 2R & if \quad 2R + \delta + Z \leq x \leq \bar{X} \\ \dfrac{x' + Z \quad 2R}{2} & if \quad 2R + \delta \quad Z \leq x < 2R + \delta + Z \\ 0 & if \quad \underline{X} \leq x < 2R + \delta \quad Z. \end{cases}$$

Note that if $2R + \delta - Z \leq x < 2R + \delta + Z$, the manager receives the output only when the project succeeds at *t=2*.

Thus, the manager chooses the safe project if and only if $M_S \geq M_R$. That is, the manager chooses the safe project at *t=1* when the signal *x* satisfies:

$$x - 2R \geq \frac{x - \delta + Z - 2R}{2},$$

$$x \geq 2R - \delta + Z \equiv \bar{x}. \qquad (1)$$

By the definition of $\bar{x}$, we can derive the following lemma.

Lemma 1. Consider the case in which both creditors roll over. The manager is more likely to choose the risky project the higher is R, the higher is Z and the lower is δ.

Lemma 1 implies that, under limited liability, the risky project is more likely to offer a higher expected return the higher is the amount of repayment *R*, the more final output fluctuates (i.e., the higher is *Z*) and the lower is the efficiency loss of the risky project δ.

3.2 The case in which only one creditor rolls over

Next, consider the case in which only one creditor rolls over. Let $M_S^{'}$ and $M_R^{'}$ be the manager's payoffs at *t=2* when the manager chooses the safe and risky projects, respectively. Note that, in this case, the manager must repay *R* to one creditor at *t=2* when the project succeeds. Then, similar to the previous cases, $M_S^{'}$ and $M_R^{'}$ can be written as follows:

$$M_s^{'} = \begin{cases} \theta x - R & if \quad R/\theta \leq x \leq \bar{X} \\ 0 & if \quad \underline{X} \leq x < R/\theta, \end{cases}$$

$$M_r^{'} = \begin{cases} \theta x' - R & if \quad \dfrac{R + \delta + Z}{\theta} \leq x \leq \bar{X} \\ \dfrac{\theta x' + Z - R}{2} & if \quad \dfrac{R - Z}{\theta} + \delta \leq x < \dfrac{R + Z}{\theta} + \delta \\ 0 & if \quad \underline{X} \leq x < \dfrac{R - Z}{\theta} + \delta. \end{cases}$$

Therefore, $M_S^{'} \geq M_R^{'}$ holds if and only if the following inequality is satisfied:

$$\theta x - R \geq \frac{\theta x' + Z - R}{2},$$

$$x \geq \frac{R + Z}{\theta} - \delta \equiv \bar{x}'. \qquad (2)$$

In other words, the manager chooses the safe project if the manager observes $x \geq \bar{x}'$. By the definition of $\bar{x}'$, we can derive the following lemma.

Lemma 2. Consider the case in which only one creditor rolls over. The manager is more likely to choose the risky project the higher is *R*, the higher is *Z*, the lower is δ, and the lower is θ.

The intuition behind Lemma 2 is very similar to that behind Lemma 1. Moreover, Lemma 2 implies that the manager has a greater incentive to choose the risky project the greater is the degree of asset complementarity (i.e., the smaller is θ). In addition, note that, because $\bar{x} < \bar{x}'$, the manager is more likely to choose the risky action when only one creditor rolls over.[46]

4 The creditors' decision

In this section, we investigate how a coordination problem among creditors affects the moral hazard problem of the manager. First, we consider the benchmark case of perfect information, under which creditors observe the signal *x* publicly. In this case, we show that there exist multiple Nash equilibria. Next, we investigate the case of imperfect information, under which each creditor receives a private signal t_i about the prospect of the project. In this case, we use the concept of the global game to show that multiple equilibria do not arise.

4.1 Perfect information

First, we consider the case in which the signal *x* is publicly revealed. Because the payoff of each creditor

[46] $\bar{x} < \bar{x}'$ if θ is small enough to satisfy $\theta \leq (R+Z)/(2R+Z)$. Because we have assumed that $\theta \leq 1/2$, this inequality is satisfied.

depends on the other creditor's decision, we consider three cases: both creditors withdraw; both roll over; and one rolls over while the other withdraws.

When both creditors withdraw their loans, the project is terminated. Then, each creditor receives the liquidation value of the asset *L*.

Next, consider the case in which both creditors roll over. As we showed in the previous section, the payoffs of the creditors depend on the value of *x*. If $x < \bar{x}$, the manager chooses the risky project. Let π_R be the payoff of creditor $i(i=1,2)$. When *x* is large enough for the manager to repay both creditors, each creditor retains *R* even if the project fails (i.e., $x' - Z > 2R$). However, given that $\bar{x} = 2R - \delta + Z$, it follows that $\bar{x} < 2R + \delta + Z$. Then, π_R can be written as follows:

$$\pi_R = \begin{cases} \dfrac{x' - Z + 2R}{4} & if \quad 2R + \delta - Z < x \le \bar{x} \\ \dfrac{x'}{2} & if \quad \underline{X} \le x \le 2R + \delta - Z. \end{cases}$$

Note that when *x* is too small for the manager to repay both creditors even if the project succeeds (i.e., $x' + Z < 2R$), the creditors divide the final outputs in both states, and thus, each creditor's expected payoff is $x'/2$.

If $x \ge \bar{x}$, the manager chooses the safe project. Let π_S be the payoff of creditor *i*. Then π_S can be written as follows:[47]

$$\pi_S = R \quad if \quad \bar{x} \le x \le \overline{X}.$$

In addition, relative to the case in which both creditors withdraw, each creditor is better off by rolling over the loan if the signal *x* satisfies the following inequality:

$$\frac{x - \delta - Z + 2R}{4} > L,$$

$$x > 4L - 2R + \delta + Z \equiv x_0. \tag{3}$$

That is, if the creditors observe that the value of *x* is larger than x_0, they are willing to roll over their loans.

Next, consider the case in which one of the creditors rolls over while the other withdraws. The payoff of the creditor who withdraws the loan is the liquidation value of the asset *L*. The payoff of the creditor who rolls over the loan can be derived as follows. If $x < \bar{x}'$, the manager chooses the risky project. Let π_R' be the expected payoff of the creditor who rolls over. Then, π_R' can be written as follows:[48]

$$\pi_R' = \begin{cases} \dfrac{\theta x' - Z + R}{2} & if \quad \dfrac{R - Z}{\theta} + \delta < x \le \bar{x}' \\ \theta x' & if \quad \underline{X} \le x \le \dfrac{2R - Z}{\theta} + \delta. \end{cases}$$

If $x \ge \bar{x}'$, the manager chooses the safe project. Then, the expected payoff of the creditor who rolls over, π_S', can be written as follows:[49]

$$\pi_S' = R \quad if \quad \bar{x}' \le x \le \overline{X}.$$

The creditor is better off by rolling over the loan than by liquidating it if the following inequality is satisfied:

$$\frac{\theta x' - Z + R}{2} > L,$$

$$x > \frac{2L - R + Z}{\theta} + \delta \equiv x_1. \tag{4}$$

From (3) and (4), x_1 is larger than x_0 if:

$$\frac{2L - R + Z}{\theta} + \delta > 4L - 2R + \delta + Z,$$

$$\theta < \frac{2L - R + Z}{4L - 2R + Z}. \tag{5}$$

Having already assumed that $\theta < 1/2$, (5) is satisfied.

The payoffs of the creditors are illustrated in the figure below[50].

To summarize this discussion, when the signal *x* represents perfect information for creditors, we can derive the following proposition.

Proposition 1. Consider the case in which a signal *x* represents perfect information for creditors (see Appendix 1). Then, we have the following:

- if $x < x_0$, the equilibrium strategies for creditors are (withdraw, withdraw);
- if $x_0 \le x < x_1$, the equilibrium strategies for creditors are (withdraw, withdraw) and (rollover, rollover);
- if $x_1 \le x$, the equilibrium strategies for creditors are (rollover, rollover).

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[47] From the definition of $\overline{X}$ and the assumption $Z > \delta$, it follows that $\bar{x} > 2R$.

[48] As in the previous case, $\bar{x}' < (R + Z)/\theta + \delta$.

[49] As in the previous case, $\bar{x}' > R/\theta$.

[50]

	rollover	withdraw
rollover	π_R, π_R	π_R', L
withdraw	L, π_R'	L, L

Proposition 1 shows that if $x_0 \leq x < x_1$, a coordination problem arises between creditors. That is, if creditor *i* believes that creditor *j* will roll over (withdraw) the loan, creditor *i* may be better off by rolling over (withdrawing) its loan. With multiple equilibria, we cannot predict which equilibrium will occur or the likelihood of each equilibrium. Therefore, if we pursue our analysis based on these multiple equilibria by, for example, determining the equilibrium repayment or by considering the effect of the coordination problem on the manager's moral hazard, we encounter the problem of subjective probabilities.

To overcome this problem, in the next subsection, we incorporate imperfect information about the state of the project, X, and derive a unique equilibrium by using the concept of the global game.

4.2 Imperfect information

In this section, we modify the model by assuming that each creditor $i(i=1,2)$ receives a different signal t_i about the prospect of the project *x*, with a small amount of noise. That is, eachcreditor *i* obtains a signal $t_i = x + \varepsilon_i$, where ε_i is a small error term that is uniformly distributed over $[-\varepsilon, \varepsilon]$. In addition, for $i \neq j$, ε_i and ε_j are independent. The density function of t_i, when the true state is *x*, is denoted by $g(t_i \mid x)$.

The strategy of each creditor is to determine which action (withdrawal or rollover) to choose for each private signal t_i. The equilibrium strategies for both creditors are given by a profile of strategies such that each creditor maximizes its expected payoff conditional on the information available, given that the other creditor is following the strategies in the profile.

First, we define $t_0 = x_0 - \varepsilon$ and $t_1 = x_1 + \varepsilon$. Then, if the private signal $t_i < t_0$ is realized, both creditors are certain that the true states satisfy $x < x_0$. Thus, from Proposition 1, withdrawal is the dominant strategy in this region. Similarly, if $t_i \geq t_1$, then $x \geq x_1$, in which case, rolling over the loans is the dominant strategy in this region. To explore the equilibrium strategy in the interval $[t_0, t_1)$, suppose that the creditors adopt a ``switching strategy" in which they roll over their loans when t_i is larger than some threshold $\hat{t} \in [t_0, t_1)$ and withdraw when t_i is below the threshold. Let us denote the threshold for creditor *i* as $\hat{t}_i$. Then, let $p(t_j \mid t_i) = h(t_j - t_i)$ be the density function of creditor *j*'s private signal, t_j, conditional on creditor *i* observing the private signal t_i. Then, $p(t_j \mid t_i)$ can be written as follows:

$$p(t_j \mid t_i) = \begin{cases} \dfrac{-t_j + t_i + 2\varepsilon}{4\varepsilon^2} & if \quad t_j > t_i \\ \dfrac{t_j - t_i + 2\varepsilon}{4\varepsilon^2} & if \quad t_j < t_i. \end{cases}$$

If creditor *i* withdraws, the payoff is the liquidation value of the asset *L*. Let $E\pi(t_i, \hat{t})$ be the expected payoff of creditor *i* when it receives the signal t_i and rolls over given that creditor *j* adopts a switching strategy with a threshold of $\hat{t}$. We derive $E\pi(t_i, \hat{t})$ based on the following arguments. First, let *H* be the cumulative distribution function of t_j when creditor *i* receives the private signal t_i. Then, the probability that creditor i rolls over and creditor *j* withdraws is $H(\hat{t} - t_i)$. Note that, by definition, $t_1 < \bar{x}'$ and $t_1 < \bar{x}$. Therefore, after only one creditor rolling over its loan, the manager chooses the risky project in $t \in [t_0, t_1)$. Because *x* is uniformly distributed on $[t_i - \varepsilon, t_i + \varepsilon]$, the expected payoff of creditor *i*, when only creditor *i* rolls over its loan, $E[\pi'_R \mid t_i]$, is as follows:[51]

$$E[\pi'_R \mid t_i] = \begin{cases} \dfrac{\theta(t_i - \delta) - Z + R}{2} & if \quad \dfrac{R - Z}{\theta} + \delta < t_i \leq t_1 \\ \theta(t_i - \delta) \quad if & t_0 \leq t_i < \dfrac{R - Z}{\theta} + \delta. \end{cases}$$

Similarly, the probability that both creditors roll over their loans is $1 - H(\hat{t} - t_i)$. Then, the expected payoff of creditor *i* when both creditors roll over, $E[\pi_R \mid t_i]$, is as follows:[52]

$$E[\pi_R \mid t_i] = \begin{cases} \dfrac{t_i - \delta - Z + 2R}{4} & if \quad 2R - Z + \delta < t_i \leq t \\ \dfrac{t_i - \delta}{2} & if \quad t_0 \leq t_i < 2R - Z + \delta. \end{cases}$$

Thus, $E\pi(t_i, \hat{t})$ can be written as:

$$E\pi(t_i, \hat{t}) = H(\hat{t} - t_i) E[\pi'_R \mid t_i] + \\ + (1 - H(\hat{t} - t_i)) E[\pi_R \mid t_i]. \tag{6}$$

Using these relations, we can derive the following unique equilibrium for this game under imperfect information.

Proposition 2 (see Appendix 2). There exists a unique equilibrium in which both creditors follow switching strategies with a threshold of:

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[51] If $t_0 > (R - Z)/\theta + \delta$, $E[\pi'_R \mid t_i] = \dfrac{\theta(t_i - \delta) - Z + R}{2}$.

[52] If $t_0 > 2R - Z + \delta$, $E[\pi_R \mid t_i] = \dfrac{t_i \quad \delta \quad Z + 2R}{4}$.

$$\hat{t} = \begin{cases} \dfrac{4L}{1+2\theta} + \delta & if \quad 0 \le L \le R - Z \\ \dfrac{4L+Z-R}{1+\theta} + \delta & if \quad R - Z < L \le R - \dfrac{Z}{2} \\ \dfrac{8L+3Z-4R}{1+2\theta} + \delta & if \quad R - \dfrac{Z}{2} < L \le \min\{1, R - \dfrac{\delta}{2}\}. \end{cases}$$

Proposition 2 implies that, unlike in the complete information case, there exists a unique Nash equilibrium. Clearly, from Proposition 2, creditors are more likely to withdraw their money the higher are L,Z and δ, and the lower are R and θ.

Using the result of Proposition 2, let us evaluate ex post efficiency when there is incomplete information. First-best ex post efficiency is attained if both creditors adopt a switching strategy with a threshold of $\hat{t}^*$ that satisfies $E\pi_R(\hat{t}^*) = L$. That is:

$$\hat{t}^* = \begin{cases} 2L + \delta & if \quad 0 \le L \le R - \dfrac{Z}{2} \\ 4L + \delta + Z - 2R & if \quad R - \dfrac{Z}{2} < L \le \min\{1, R - \dfrac{\delta}{2}\}. \end{cases}$$

Note that $\hat{t} - \hat{t}^*$ represents the extent of inefficient liquidation that occurs because of coordination failure among creditors. Because:

$$\hat{t} - \hat{t}^* = \begin{cases} \dfrac{2(1-2\theta)(2L-R) + 2(1-\theta)Z}{1+2\theta} & if \quad R - \dfrac{Z}{2} < L \le \min\{1, R - \dfrac{\delta}{2}\} \\ \dfrac{2(1-\theta)L + Z - R}{1+\theta} & if \quad R - Z < L < R - \dfrac{Z}{2} \\ \dfrac{2(1-2\theta)L}{1+2\theta} & if \quad 0 \le L \le R - Z, \end{cases}$$

coordination failure is more severe the larger are L and Z, and the smaller are R and θ. These results have important implications for the roles of APR violations and the revocation of PP. Under APR violations, creditors get less, and thus, asset liquidation values are lower. This means that in this model, L decreases. Therefore, we argue that APR violations mitigate the coordination problem among creditors and reduce ex post inefficient liquidation. In addition, under the revocation of PP, a transfer of assets can be voided, and a manager can reuse the assets. Therefore, the revocation of PP also mitigates the coordination problem and reduces ex post inefficient liquidation.

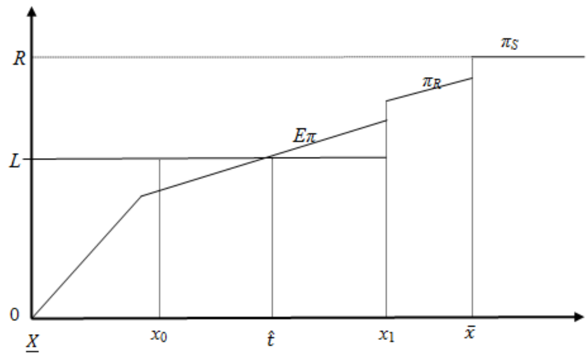
We are now in a position to consider how R is determined at $t=0$. Let us define $\hat{x}_0 \equiv \hat{t} - \varepsilon$ and $\hat{x}_1 \equiv \hat{t} + \varepsilon$. As Figure 4 shows, both creditors withdraw their money for sure when $\underline{X} \le x < \hat{x}_0$ and roll over their loans for sure when $\hat{x}_1 < x \le \overline{X}$. When $\hat{x}_0 \le x < \hat{x}_1$, creditors adopt the switching strategy described in Proposition 2. Because we assume that creditors face a competitive financial market, R is determined such that the following equality is satisfied:

$$1 = \int_{\underline{X}}^{\hat{x}_0} Lf(x)dx + \int_{\hat{x}_0}^{\hat{x}_1} \left(\int_{\hat{t}-\epsilon}^{x} E\pi(t_i,\hat{t})g(t_i)dt_i + \int_{x}^{\hat{t}+\epsilon} Lg(t_i)dt_i \right) f(x)dx + \int_{\hat{x}_1}^{\overline{x}} \pi_R f(x)dx + \int_{\overline{x}}^{\overline{X}} \pi_S f(x)dx \tag{7}$$

As is discussed in Lemma 1 and 2, since the level of R affects the manager's incentive to take a risky action, the ex ante efficiency can be evaluated by what level R is determined.

In the next section, we consider the comparative statics and discuss how APR violations and the revocation of PP affect ex ante efficiency and when they should be adopted.

Figure 1. The Expected payoff of a creditor under imperfect information



5 The roles of APR violations and the revocation of PP

So far, we have considered two problems that arise among stakeholders. One is an agency problem between creditors and a manager (shareholders), and the other is a coordination problem among creditors. Consideration of how to solve the former problem enables us to determine the role of APR violations. Consideration of how to solve the latter problem enables us to determine the role of the revocation of PP. In this section, we analyze these problems by investigating the comparative static properties of the model.

First, we consider the role of APR violations. When APR violations are adopted, the manager can obtain some value even if the firm's value is not large enough to repay both creditors. To formulate this situation, we assume that if APR violations are adopted, the amount each creditor can obtain when both creditors withdraw their money becomes $(1-\alpha)L$, where $0<\alpha<1$. Then, we consider how the amount of repayment (R) changes if the degree of APR violations (a) changes. Under APR violations, the manager obtains $2\alpha L$ when both creditors withdraw. However, as (1) and (2) show, this change in payoffs does not directly affect the condition under which a manager has an incentive to take a risky action. Therefore, to consider whether adopting APR violations generates ex ante inefficiency, we need only investigate the comparative static effect of a change in a on R. In relation to this comparative static effect, from (7), we can derive the following proposition.

Proposition 3. When ε is small or when L is small, $\partial R/\partial a$ is positive for all $\alpha\in[0,1]$. However, when ε is large or when L is large, there exists a threshold, $\hat{\alpha}$, such that $\partial R/\partial a$ is negative for $\alpha\in[0,\hat{\alpha}]$ and $\partial R/\partial a$ is positive for $\alpha\in[\hat{\alpha},1]$ (see Appendix 3).

Figure 2. The effect of APRv whenεis small

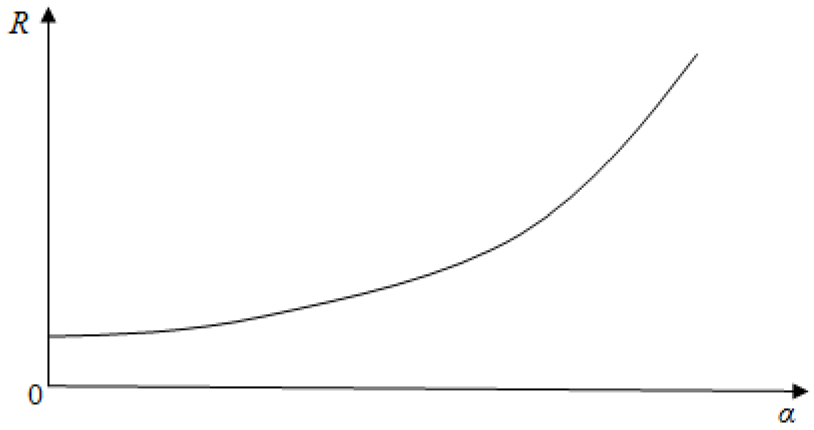
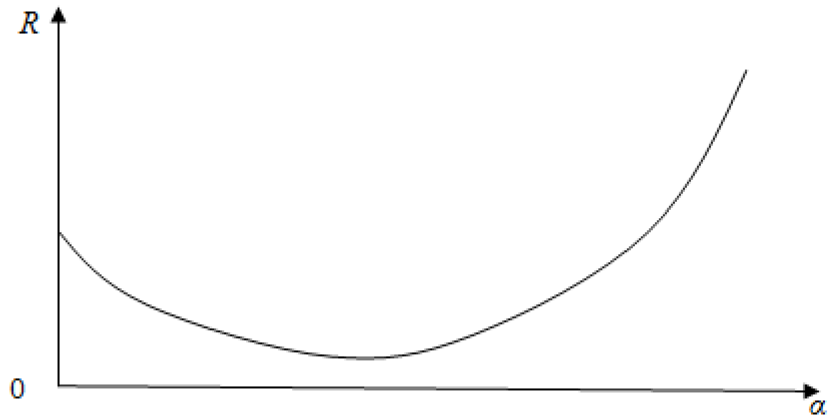


Figure 3. The effect of APRv whenεis large



The first part of Proposition 3 implies that when creditors receive clear signals about the firm, the amount of repayment, *R*, increases if APR violations are adopted so that creditors get less when they withdraw their money. Because the risky project is more likely to be chosen the larger is *R*, ex ante efficiency is reduced if APR violations are adopted. This result is consistent with the results of Longhofer (1997) and Bebchuk (2002). However, the second part of Proposition 3 indicates that when signals are noisy, high liquidation values may generate large repayments that induce the debtor to take risky actions. The intuition behind this can be explained as follows. When liquidation values are high, creditors are more likely to withdraw their loans. However, when signals are less informative, coordination problems among creditors are more severe. To compensate for this, creditors require higher amounts of repayment ex ante, which, in turn, induces risky action by the debtor. Therefore, contrary to the case in which ε is small, adopting APR violations may be a useful way of improving ex ante efficiency when either L or ε is large (i.e., when there is severe coordination failure among creditors).

Proposition 3 suggests that adopting APR violations is beneficial for the reorganization of SMEs. This is because, generally speaking, creditors observe SMEs less clearly than they observe large firms, and thus, coordination among the creditors of SMEs may be more difficult.[53] In addition, according to our model, even a firm that is economically viable may become financially distressed and go bankrupt because of coordination failure among creditors. Although the source of this inefficiency is an interim shortage of liquidity for the firm, Proposition 3 suggests that APR violations can mitigate this shortage because creditors have more incentive to roll over their loans.

Next, we consider the role of the revocation of PP. To consider this, we modify the payoff of the model when one creditor withdraws but the other rolls over, as follows.[54] The creditor who withdraws at *t=1* gets $(1-\beta)L$, where β $(0<\beta<1)$ represents the degree of revocation of PP. Under the revocation of PP, although a manager can use two physical assets, one of them is partially liquidated, of which only the fraction β can be used. Then, the manager generates $\tilde{\theta}(\beta)x$ for sure when choosing the safe project and, when choosing the risky project, generates $\tilde{\theta}(\beta)x' + Z$ with probability 1/2, and generates $\tilde{\theta}(\beta)x' - Z$ with probability 1/2, and $\tilde{\theta}(0)=\theta$, $\tilde{\theta}(1)=1$, $\theta' > 0$ and $\theta'' > 0$.

Under these modified settings, let $\tilde{M}_S$ and $\tilde{M}_R$ denote the manager's expected payoffs when choosing the safe and risky projects, respectively.

$$\tilde{M}_S = \begin{cases} \tilde{\theta}(\beta)x - R & if \quad \dfrac{R}{\tilde{\theta}(\beta)} < x \le \overline{X} \\ 0 \quad if & \underline{X} \le x \le \dfrac{R}{\tilde{\theta}(\beta)} \end{cases}$$

$$\tilde{M}_R = \begin{cases} \tilde{\theta}(\beta)x' - R \quad if & \dfrac{R+Z}{\tilde{\theta}(\beta)} + \delta < x \le \overline{X} \\ \dfrac{\tilde{\theta}(\beta)x' + Z - R}{2} \quad if & \dfrac{R-Z}{\tilde{\theta}(\beta)} + \delta < x \le \dfrac{R+Z}{\tilde{\theta}(\beta)} + \delta \\ 0 \quad if & \underline{X} \le x \le \dfrac{R-Z}{\tilde{\theta}(\beta)} + \delta \end{cases}$$

Therefore, the manager chooses the safe project if the following inequality is satisfied:

$$x \ge \frac{R+Z}{\tilde{\theta}(\beta)} + \delta \equiv \overline{x}'(\beta).$$

Note that $\overline{x}'$ decreases when β increases. Thus, adopting the revocation of PP can mitigate the risk-taking behavior of the manager ex post.

Next, consider the creditor's expected payoff when only one creditor rolls over under imperfect information. Then, the expected payoff of creditor *i*, $E[\tilde{\pi}_R' \mid t_i]$, is given by:[55]

$$E[\tilde{\pi}_R' \mid t_i] = \begin{cases} \tilde{\theta}(\beta)(t_i - \delta) \quad if & t_0 \le t_i < \dfrac{R-Z}{\tilde{\theta}(\beta)} + \delta \\ \dfrac{\tilde{\theta}(\beta)(t_i - \delta) - Z + R}{2} \quad if & \dfrac{R-Z}{\tilde{\theta}(\beta)} + \delta \le t_i \le t_1. \end{cases}$$

By the definition of $\tilde{\theta}(\beta)$, $E[\tilde{\pi}_R' \mid t_i] \ge E[\pi_R' \mid t_i]$ always holds. This means that the creditor who rolls over can get more when PP is revoked than when it is not. Then, given that creditor *j* adopts a switching strategy with a threshold of $\tilde{t}$, the expected payoff of creditor *i* who receives the signal t_i and chooses to roll over the loan is:

$$\tilde{E}\pi(t_i, \tilde{t}) = H(\tilde{t} - t_i)E[\tilde{\pi}_R' \mid t_i] + \\ + (1 - H(\tilde{t} - t_i))E[\pi_R \mid t_i].$$

Using these relations, we can derive the following proposition.

Proposition 4. When there is complementarity between the firm's assets, adopting the revocation of PP can increase ex ante efficiency (see Appendix 4).

Compared with Proposition 3, Proposition 4 indicates that the revocation of PP always increases ex ante efficiency. This is because a creditor is more likely to roll over its loan under the revocation of PP, and this mitigates the problem of coordination failure.

[53] Of course, the number of creditors also affects whether coordination failure arises. However, in order to focus on the effect of information asymmetry between firms and creditors, in this paper, we assume that the number of creditors is determined exogenously. For an analysis of this problem, see Bolton and Scharfstein (1996).

[54] To focus on the role of the revocation of PP, in the following arguments, we suppose that APR violations are not adopted.

[55] As is the case for $E[\pi_R' \mid t_i]$, the manager chooses the risky project in $t \in [t_0, t_1)$.

6 Conclusion

In this paper, we analyzed how a structured bargaining procedure such as the Civil Rehabilitation Act in Japan or Chapter 11 in the United States affects ex ante and ex post efficiency when there are two types of conflict: a conflict among creditors and a conflict between a debtor and creditors. We focused on the role of absolute priority rule (APR) violations and the revocation of preferential payments (PP). First, we showed that when creditors receive clear signals from a firm, the amount of repayment that creditors demand ex ante decreases as the liquidation value of the firm increases. Second, if noisy signals are transmitted to creditors, repayment falls until the liquidation value reaches a threshold level, after which repayment increases. The first result suggests that if APR violations are adopted, creditors require greater amounts of repayment ex ante, which, in turn, induces the debtor to undertake risky actions. This result is consistent with those of Longhofer (1997) and Bebchuk (2002). The second result suggests that when the signal is less accurate, coordination problems become more severe for creditors, and thus, adopting APR violations may be useful for improving ex ante efficiency. Third, when there is complementarity between the firm's assets, the revocation of PP can mitigate the coordination problem and thus increase ex ante efficiency. These results have important implications for the optimal design of bankruptcy laws.

To simplify our arguments, we assumed a simple standard debt contract. However, it would be interesting to generalize the framework used in this paper to consider the optimal contract between a debtor and creditors and to evaluate the role of APR violations and the revocation of PP. This is left for future research.

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Appendix 1

Proof of Proposition 1

Note that $x_0 < x_1$ holds. Then, if $x < x_0$, from (3), (4) and $x_0 < x_1$, it follows that $L > \pi_R$, $L > \pi_R^{'}$. Therefore, the best response for both creditors is to withdraw.

If $x_0 \leq x < x_1$, from (3), (4) and $x_0 < x_1$, it follows that $L < \pi_R$, $L > \pi_R^{'}$. Therefore, in this case, there exist multiple Nash equilibria: (rollover, rollover), (withdraw, withdraw).

If $x_1 \leq x$, then $L < \pi_R$, $L < \pi_R^{'}$. Therefore, rollover is the dominant strategy for both creditors.

Appendix 2

Proof of Proposition 2

First, from (6), we can derive the following two lemmas.[56]

Lemma 3. $E\pi(t_i,\hat{t})$ is a continuous function: $[t_0,t_1]\times[t_0,t_1] \to \mathrm{R}$ and is strictly increasing in t_i and weakly decreasing in $\hat{t}$.

Lemma 4. $E\pi(\hat{t},\hat{t})$ is strictly increasing in $\hat{t}$ and $E\pi(\hat{t},\hat{t}) = L$ has a unique solution.

Creditor *i* rolls over the loan if $E\pi(t_i,\hat{t}) \geq L$ and withdraws if $E\pi(\hat{t},\hat{t}) < L$. From Lemma 3, we can define $b(\hat{t})$, which satisfies $E\pi(b(\hat{t}),\hat{t}) = L$ for every $\hat{t}$ uniquely. Then, from Lemma 3, $E\pi(t_i,\hat{t}) \geq L$ holds if $t_i \geq b(\hat{t})$ and $E\pi(t_i,\hat{t}) < L$ holds if $t_i < b(\hat{t})$. Therefore, when creditor *j* adopts a switching strategy with a threshold of $\hat{t}$, creditor *i*'s best response is to follow a switching strategy with a threshold of $b(\hat{t})$. In particular, when $\hat{t} = b(\hat{t})$, neither player has an incentive to deviate from the strategy. In this case, $\hat{t}$ satisfies the following:

$$L = E\pi(\hat{t},\hat{t}),$$

$$\Leftrightarrow \hat{t} = \begin{cases} \dfrac{4L}{1+2\theta} + \delta & if \quad 0 \leq L \leq R - Z \\ \dfrac{4L + Z - R}{1+\theta} + \delta & if \quad R \quad Z < L \leq R - \dfrac{Z}{2} \\ \dfrac{8L + 3Z - 4R}{1+2\theta} + \delta & if \quad R - \dfrac{Z}{2} < L \leq \min\{1, R - \dfrac{\delta}{2}\}. \end{cases}$$

Next, we show that the switching strategy with a threshold of $\hat{t}$ is the unique equilibrium for both creditors. Recall that the strategy of creditor *i* involves choosing between rollover and withdrawal for each private signal t_i. Let $\sigma_i (i = 1,2)$ be the strategy profile for creditor *i*. Then, if (σ_1,σ_2) is the equilibrium, the following relations should be satisfied:

$$\sigma_i(t_i) = \begin{cases} r & if \quad t_i > b^{n-1}(t_1) \\ w & if \quad t_i < b^{n-1}(t_0), \end{cases}$$

where *r* denotes rollover and *w* denotes withdrawal.

$$b^n(k) = \begin{cases} k & (n = 0) \\ b(b^{n-1}(k)) & (n \geq 1) \end{cases}.$$

We prove this relation by induction.

First, consider the case in which *n=1*. In this case, $t_i > b^0(t_1) = t_1$, so creditor *i*'s best response is to roll over. In addition, $t_i < b^0(t_0) = t_0$, so creditor *i*'s best response is to withdraw. Therefore, the above relation is satisfied when *n=1*.

Next, suppose that the above relation is satisfied when *n=k*. Given that creditor *j* rolls over if $t_j > b^{k-1}(t_1)$:

[56] This proof is adapted from Morris and Shin (2003) and Ui (2009).

$$Pr(\sigma_j = r \mid t_i) \geq Pr(s[b^{k\text{-}1}(t_1)]_j = r \mid t_i), \tag{8}$$

where $Pr(s[b^{k\text{-}1}(t_1)]_j = r \mid t_i)$ denotes the probability conditional on t_i that creditor j chooses to roll over when adopting a switching strategy with a threshold of $b^{k\text{-}1}(t_1)$.

Let $\overline{E\pi_R}$ be the expected payoff of creditor i when this creditor observes a private signal t_i and rolls over the loan. Then, $\overline{E\pi_R}$ can be written as follows:

$$\overline{E\pi_R} \equiv (1\text{-}Pr(\sigma_j = r) \mid t_i))E[\pi_R^{'} \mid t_i] + Pr(\sigma_j = r \mid t_i)E[\pi_R \mid t_i].$$

In addition, let $E\pi(t_i, b^{k\text{-}1}(t_1))$ be the expected payoff of creditor i when the creditor observes t_i and the opponent adopts the switching strategy with a threshold of $b^{k\text{-}1}(t_1)$. Then, $E\pi(t_i, b^{k\text{-}1}(t_1))$ can be written as follows:

$$E\pi(t_i, b^{k\text{-}1}(t_1)) \equiv \{1\text{-}Pr(s(b^{k\text{-}1}(t_1))_j = r \mid t_i)\}E[\pi_R^{'} \mid t_i] + Pr(s(b^{k\text{-}1}(t_1))_j = r \mid t_i)E[\pi_R \mid t_i].$$

Then, from (8) and $E[\pi_R \mid t_i] \geq E[\pi_R^{'} \mid t_i]$, $\overline{E\pi_R} \geq E\pi(t_i, b^{k\text{-}1}(t_1))$ is satisfied.

On the other hand, when creditor j withdraws, which occurs if $t_j < b^{k\text{-}1}(t_0)$:

$$Pr(\sigma_j = r \mid t_i) \leq Pr(s[b^{k\text{-}1}(t_1)]_j = r \mid t_i) \tag{9}$$

is satisfied. Then, $E\pi(t_i, b^{k\text{-}1}(t_0))$ can be written as follows:

$$E\pi(t_i, b^{k\text{-}1}(t_0)) \equiv \{1\text{-}Pr(s(b^{k\text{-}1}(t_0))_j = r \mid t_i)\}E[\pi_R^{'} \mid t_i] + Pr(s[b^{k\text{-}1}(t_0)]_j = r \mid t_i)E[\pi_R \mid t_i].$$

Then, from (9), $\overline{E\pi_R} \leq E\pi(t_i, b^{k\text{-}1}(t_0))$ is satisfied. Therefore, from Lemmas 3 and 4, if $t_i < b(b^{k\text{-}1}(t_0)) = b^k(t_0)$ is satisfied, creditor i chooses to withdraw its loan.

In summary:

$$E\pi(t_i, b^{k\text{-}1}(t_1)) \leq \overline{E\pi_R} \leq E\pi(t_i, b^{k\text{-}1}(t_0))$$

is satisfied. From the first inequality and Lemmas 3 and 4, it follows that if $t_i \geq b(b^{k\text{-}1}(t_1)) = b^k(t_1)$, then creditor i chooses to roll over the loan when $E\pi_R(t_i, b^{k\text{-}1}(t_1)) \geq L$. Similarly, from the second inequality, if $t_i < b(b^{k\text{-}1}(t_0)) = b^k(t_0)$, then creditor i chooses to withdraw the loan when $E\pi_R(t_i, b^{k\text{-}1}(t_0)) < L$. Thus, we have proved that the above relation is satisfied when $n=k+1$.

Now, we show that $\lim_{n\to\infty}\{b^n(t_0)\} = \lim_{n\to\infty}\{b^n(t_1)\} = \hat{t}$. First, if $t > \hat{t}$, then $E\pi_R(t,t) > E\pi(\hat{t},\hat{t}) = L$, so $b(t) < t$ is satisfied. In addition, from Lemma 3, if $t > \hat{t}$, then $E\pi_R(\hat{t}, t) < L$. In summary, if $t > \hat{t}$, then $\hat{t} \leq b(t) < t$.

Similarly, if $t < \hat{t}$, then $E\pi(t,t) < E\pi(\hat{t},\hat{t}) = L$, so $b(t) > t$ is satisfied. In addition, from Lemma 3, if $t < \hat{t}$, then $E\pi_R(\hat{t}, t) > L$. Because $E\pi_R(b(t), t) = L$, it follows that $b(t) < \hat{t}$. Therefore, if $t < \hat{t}$, then $t < b(t) \leq \hat{t}$.

Because the sequence $b^n(t_1)$ is decreasing if $t > \hat{t}$ and the sequence $b^n(t_0)$ is increasing if $t < \hat{t}$, it follows that $\lim_{n\to\infty}\{b^n(t_1)\} = \lim_{n\to\infty}\{b^n(t_0)\} = \hat{t}$, and thus the equilibrium is unique.

Appendix 3

Proof of Proposition 3

Note that x and t_i are uniformly distributed. Then, under APR violations, (7) can be rewritten as follows:

$$X = (\hat{x}_0 - \underline{X})(1-\alpha)L + (\bar{x} - \hat{x}_1)\pi_R + (\bar{X} - \bar{x})\pi_S + \varepsilon(E\pi_R + L). \tag{10}$$

In addition, from our definitions, we can derive the following equations:

$$\partial\pi_S/\partial a = \partial R/\partial a,$$

$$\partial\pi_R/\partial a = \tfrac{1}{2}\partial R/da,$$

$$\partial E\pi_R/\partial a = -h(\hat{t} - \bar{t})(\partial\hat{t}/\partial a)\left\{E\left[\pi'_R \,|\, \bar{t}\right]\right\}/\partial a,$$

$$\partial\bar{x}/\partial a = 2(\partial R/\partial a),$$

where $\bar{t} = (\hat{x}_0 + \hat{x}_1)/2$.

By using these relations and totally differentiating (10) with respect to a, we obtain:

$$\frac{\partial R}{\partial a} = \frac{(\hat{t} - \underline{X})L + (\pi_R + W - (1-a)L)(\partial\hat{t}/\partial a)}{\bar{X} - \bar{x} - 2(\pi_S - \pi_R) + \dfrac{\bar{x} - \hat{t} - \varepsilon}{2}},$$

where $W \equiv \varepsilon h(\hat{t} - \bar{t})(E[\pi_R \,|\, \bar{t}] - E[\pi'_R \,|\, \bar{t}])$. Note that W is increasing in L and ε. In addition, $\hat{t} - \underline{X} > 0$, and the denominator is positive. Thus, given that $\partial\hat{t}/\partial a < 0$ and ε is large, there exists a threshold $\hat{\alpha}$ such that $\partial R/\partial a < 0$ when a is large and $\partial R/\partial a > 0$ when a is small.

Appendix 4

Proof of Proposition 4

The equilibrium switching strategy is the one with a threshold of $\tilde{t}$, where:

$$\tilde{t} = \begin{cases} \dfrac{4L}{1+2\tilde{\theta}(\beta)} + \delta & \text{if} \quad 0 \le L \le R - Z \\ \dfrac{4L + Z - R}{1+\tilde{\theta}(\beta)} + \delta & \text{if} \quad R - Z < L \le R - Z/2 \\ \dfrac{8L + 3Z - 4R}{1+2\tilde{\theta}(\beta)} + \delta & \text{if} \quad R - Z/2 < L \le min\{1, R - Z/2\}. \end{cases}$$

Given that $\tilde{\theta}(\beta) \ge \theta$ for all $\beta \in [0,1]$, it clearly follows that $\tilde{t} < \hat{t}$.

Next, we define $\tilde{x}_0 \equiv \tilde{t} - \varepsilon$ and $\tilde{x}_0 \equiv \tilde{t} + \varepsilon$. Then, as shown in Appendix 4, (7) can be rewritten as follows:

$$X = (\tilde{x}_0 - \underline{X})L + (\bar{x} - \tilde{x}_1)\pi_R + (\bar{X} - \bar{x})\pi_S + \varepsilon(E\tilde{\pi}_R + (1-\beta)L). \tag{11}$$

Given that $\tilde{x}_0 < \hat{x}_0$, $\tilde{x}_1 < \hat{x}_1$ and $\tilde{E}\pi(t_i, \tilde{t}) \ge E\pi(t_i, \hat{t})$, from (7), R decreases when the revocation of PP is adopted.