ALARMING OF EXCHANGE RATE CRISIS: A RISK MANAGEMENT APPROACH

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Abstract

Recently, with increasing volatility of foreign exchange rate, risk management becomes more and more important not only for multinational companies and individuals but also for central governments. This paper attempts to build an econometrics model so as to forecast and manage risks in foreign exchange market, especially during the eve of turbulent periods. By following McNeil and Frey's (2000) two stage approach called conditional EVT to estimate dynamic VaR commonly used in stock and insurance markets, we extend it by applying a more general asymmetric ARMA-GARCH model to analyze daily foreign exchange dollar-denominated trading data from four countries of different development levels across Asia and Europe for a period of more than 10 years from January 03, 2005 to May 29, 2015, which is certainly representative of global markets. Conventionally, different kinds of backtesting methods are implemented ultimately to evaluate how well the model behaves. Inspiringly, test results show that by taking several specific characteristics (including fat-tails, asymmetry and long-range dependence) of the foreign exchange market return data into consideration, the violation ratio of out-of-sample data can be forecasted very well for both fixed and flexible foreign exchange regimes. Moreover, all of the violations are evenly distributed along the whole period which indicates another favorable property of our model. Meanwhile, we find evidence of asymmetry volatility in all of the studied foreign exchange markets even though the magnitudes of the most of them are weak.

Keywords: Foreign Exchange Crisis, Value-at-Risk, Extreme Value Theory, Asymmetric GARCH, Generalized Pareto Distribution

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1. INTRODUCTION

Exchange rate crisis is one of the main types of financial crises and has caused devastating impacts on economies. Therefore, lots of empirical and theoretical studies attempt to investigate what kinds of factors can induce crises or more importantly, if there are any standard economic indicators to predict it? Famous papers addressing this issue include Frankel and Rose (1996) and Kaminsky et al (1998). However, Phornchanok and Roy (2013) found that almost all macroeconomic variables have poor predictive ability on extreme exchange rate changes despite of favorable in-sample results.

A second-best way is to investigate the trend of foreign exchange rate data itself since all of the relevant information is conveyed and summarized in the volatility of the exchange rate. Clearly, forecasting ability of this method will decrease drastically with the increase of the prediction length even though backtesting of the asymmetric VaR-GARCH based method in the paper shows that one-day ahead forecast is statistically convincing.

Generally, Value-at-risk (VaR) as a market risk measure is used for such analysis. It refers to the level of financial risk at a given confidence level within investment portfolio over a specific period, for example, days, months or even years. During the past decades, it is suggested by Basel Committee on Banking Supervision (BCBS) for calculating the market risk minimum capital requirement (MCR) which may concretely be contingent on the backtesting performance of banks' internal models (Ergen, 2015). Hence, accuracy of out-of-sample prediction gradually becomes a key topic both for industry managers and researchers. However, the most commonly used version of VaR requires distribution of the return rate of a certain asset be normal which is not realistic in foreign exchange markets.

During the past several years, several solutions such like historical simulation and GARCH-EVT were developed. However, Pritsker (2006) pointed out that historical simulation-based models are under-responsive to changes in conditional risk. Furio and Climent (2013) retrospect past methods and claim that the GARCH-EVT model behaves better than the GARCH models with Gaussian or student t distributed residuals in stock market. Meanwhile, some other researchers find that a significant characteristic in VaR prediction is the tail-thickness of the data, since remaining features like skewness and dynamic volatility can improve model performance only if the former is taken into consideration (Ergen, 2015). However, in a general scenario or in a specific market, modeling asymmetric components is more important than specifying error distribution for improving volatility forecasts of financial returns in the presence of fat-tails, leptokurtosis, skewness and leverage effects (see Liu and Hung, 2010).

In this paper, we will firstly relax assumption to allow for heavy tailed distributions which can characterize stylized facts properly. To accomplish that, famous extreme value theory (EVT) is employed to focus only on large price movements and their associated probabilities i.e. solely modeling tail behavior of the distribution rather than the entirety of the sample. Precisely, EVT theory calls for independent and identically distributed observations. Nevertheless, daily returns always exhibit volatility clustering or say dependence property. To address these issues, McNeil and Frey (2000) proposed a two-stage method: in the first stage, they employ a GARCH model to estimate the conditional volatility of a time series data and naturally obtain *i.id* residuals since these kinds of models can always be viewed as a process for determining a filter that transforms the given data into a white noise series. In the second stage, they utilize outcomes from the first procedure and import the *i.id* residuals into the EVT procedure. Note that the first stage filtering process not only captures the volatility clustering characteristic but also makes using EVT more reasonable. However, their model does not ponder on the phenomenon of asymmetric volatility, that is, unexpected bad news typically has a larger impact on future volatility than unexpected good news of similar magnitude which is strongly detected in equity markets while the evidence in exchange rate markets is mixed (see Wang and Yang (2009), Laopodis (2011)). One theory supporting asymmetry put forward by Pontines and Rajan (2011) claims that economies manage their currencies asymmetrically against a trade-weighted basket which rationalizes the relative exchange rate stability as well as the sustained reserve accumulation.

Based on the analysis above, in our paper, we continue to follow McNeil and Frey's two stage method but make the GARCH model in stage one to be more general (e.g. E-GARCH ,GJR-GARCH and PGARCH) to explain asymmetric volatility. Furthermore, as verified by abundant theoretical and empirical classical papers including Tversky and Kahnemans (1991), Barberis et al (1999) that people tend to be more sensitive to losses than gains, we will concentrate simply on left tail of the distributions. Or equivalently, since the long position of holding USD is studied in this case, when CNY, JPY or GBP are appreciated, the holder will suffer a loss. One can simply apply it to the upper tail by multiplying returns with minus one.

The contribution of our research lies in integration of different kinds of asymmetric GARCH models to GPD based EVT theory to form a more general model for forecasting risks of foreign exchange markets. Indeed, we find evidence of the leverage effect in most of the market even though the magnitude is weak but at least it rationalizes the legitimacy of using asymmetric GARCH model in foreign exchange market. In the end, conventional backtesting is employed to evaluate the validity and performance of the generalized method. To our satisfactory, all of the statistics pass the test.

The rest of the paper is organized as follows. Section 2 briefly illustrates the principle of how different asymmetric GARCH models can be nested with conditional EVT to estimate 99% of VaR. Section 3 presents the empirical counterpart such as descriptive features of the foreign exchange data and formal assessment of the model accuracy is the key subject afterwards. Section 4 concludes the paper, and indicates future study focuses.

2. THEORETICAL FRAMEWORK OF ASYMMETRIC GARCH BASED EVT MODEL

The first subsection describes how EVT can deal with tail related issues i.e. rare events and its advantage over other estimation methods in estimating VaR. In the second subsection, we briefly explore the background of asymmetric GARCH model as well as how it can be integrated into subsection 1.

2.1. POT-EVT

Most statistical methods are concerned with center values of a statistical distribution, and rarely pay attention to extreme values which is a key issue in the risk management area. However, EVT provides a creative way which attempts to construct the best possible tail estimator of the tail area so as to model rare events. In particular, the first challenge facing researchers is what kind of numbers can be defined as rare events. Generally, there are two various criteria to select the so-called extreme values: block maxima model (BMM) and peaks-over-threshold (POT) model. Basically, the former model intends to choose the maximum value in different periods while the latter method considers all large observations which exceed a high threshold. Clearly, the latter method uses the data more efficiently in terms of financial data due to its volatility clustering property (see figure 1). That is, a large number of relevant data are dropped by block maxima model since extreme events follow one another during the foreign exchange crisis period. Therefore, POT-EVT method is applied in this paper to identify extreme values and will be discussed in more details.

If we denote X to be a random variable with unknown underlying distribution F , our aim in this analysis is to obtain the tail estimator of F . Before doing that, one needs to introduce the concept of conditional excess distributions. Formally, the distribution of excesses over a threshold u has the following function form:

$$F_u(y) = Pr(X - u \le y \mid X > u), 0 \le y \le x_0 - u \quad (1)$$

Where x_0 is the right endpoint of *F*. Intuitively, $F_u(y)$ measures the probability that a loss exceeds the threshold u by at most an amount *y*, given that *X* is deemed to be an extreme number. Furthermore, we formulate a relationship between the conditional probability and the underlying population through Bayesian formula: for $X>u$,

$$F_u(y) = \frac{Pr(X - u \le y, X > u)}{Pr(X > u)} = \frac{F(y+u) - F(u)}{1 - F(u)} \quad (2)$$

Or after simple manipulation:

$$F(x) = [1 - F(u)]F_u(y) + F(u) \quad (3)$$

Note that at the juncture, as long as we can get both an estimator of $F(u)$ and $F_u(y)$, we will naturally get a tail estimator of the underlying population from (3) which is the motivation of EVT. Since $(n\text{-}k)/n$ is a sample analogue estimator of $F(u)$ where n, k counts the number of whole sample and observations above the threshold u respectively. Afterwards, it remains to formulate an estimator of $F_u(y)$ which is guaranteed by the Pickands-Balkema-de Haan theorem for large enough threshold u (see Balkema and de Haan (1974); Pickands (1975)). To make the statement much clearer, we denote the theorem formally:

For sufficiently large u, a class of underlying distributions $F_u(y)$ can be approximated by the generalized Pareto distribution (GPD):

$$sup|F_u(y) - G_{\xi\varphi}(y)| \xrightarrow{u - x_0} 0 \quad (4)$$

$$where\, G_{\xi\varphi}(y) = \begin{cases} 1-(1+\dfrac{\xi y}{\Psi})^{-1/\xi}, & if\ \xi \neq 0 \\ 1-e^{-y/\xi}, & if\ \xi = 0 \end{cases} \quad (5)$$

Where ξ is the shape parameter, while φ the scale parameter. Note that the GPD is heavy-tailed when $\xi > 0$, hence it is a desired model to characterize excess distribution over a high threshold in foreign exchange markets.

Lastly, by plugging the two estimators into (3), we arrive at the tail estimator of F:

$$F(x) = [1 - F(u)]G_{\xi\varphi}(y) + F(u) = 1 - \frac{k}{n}[1 + \xi\frac{(x-u)}{\varphi}]^{\frac{-1}{\xi}} \quad (6)$$

It is noteworthy that this estimator is only for estimation of the tails $X>u$ as that of formula (3).

Once we get the estimate from the sample data by applying maximum likelihood, we can immediately measure the level of risk given probability q by inverting formula (6):

$$x_q = u + \frac{\varphi}{\xi}[(\frac{q}{k/n})^{-\xi} - 1] \quad (7)$$

Which is exactly the estimator of VaR. Clearly, the only thing we need to do is to assign two values u and q for the system.

2.2. Asymmetric GARCH

As mentioned in section 1, EVT-VaR can only be applied to i.id observations, while the foreign exchange data always exhibits property of dependence especially clustering volatility. Hence, McNeil and Frey (2000) advise to use GARCH model in the first stage to filter the raw data to be i.id residuals which can then be used in the second stage in 2.1. Additionally, in order to accommodate the possibility of asymmetry or say leverage effect, we will replace the basic symmetric GARCH model with a family of asymmetric GARCH models.

2.2.1. EGARCH (p, q) model

Nelson (1991) proposed the exponential GARCH model to account for asymmetric effect:

$$\log h_t = \omega + \sum_{i=1}^{p}\alpha_i \frac{|\varepsilon_{t-i}| + \gamma_i\varepsilon_{t-i}}{\sqrt{h_{t-i}}} + \sum_{j=1}^{q}\beta_j \log h_{t-j} \quad (8)$$

Clearly, the parameter γ_i delivers a leverage effect: if the value is negative, "bad news" will affect the volatility of the future exchange rate more.

2.2.2. GJR-GARCH (p, q) model

Another asymmetric GARCH model introduced by Glosten, Jagannathan, and Runkle (1993) is the GJR-GARCH model which allows for different impacts of lagged positive and negative innovations:

$$h_t = \omega + \sum_{i=1}^{p}(\alpha_i + \gamma_i D_{t-i}(\varepsilon_{t-i} < 0))\varepsilon_{t-i}^2 + \sum_{j=1}^{q}\beta_j \log h_{t-j} \quad (9)$$

Wwhere $D_{t\text{-}i}$ denotes an indicator function. The leverage effect is captured if $\gamma_i > 0$.

2.2.3. PGRACH (p, q) model

The last asymmetric GARCH model named power GARCH was proposed by Ding and Granger (1996) and is shown below:

$$\sigma_t^{\delta} = \omega + \sum_{i=1}^{p}\alpha_i(|\varepsilon_{t-i}| - \gamma_i\varepsilon_{t-i})^{\delta} + \sum_{j=1}^{q}\beta_j\sigma_{t-j}^{\delta} \quad (10)$$

Note that when $\delta>0$ and $\gamma_i>0$, past "bad news" will have larger impact on current volatility than past "good news".

2.3. Two-stage EVT-VaR

In a more general context, GARCH-type disturbances could also appear on the right-hand side of ARMA regression models, that is, the conditional mean can be an ARMA process:

$$r_t = a_0 + \sum_{i=1}^{p}a_i r_{t-i} + \sum_{j=1}^{q}b_j\varepsilon_{t-j} + \varepsilon_t = \mu_t + \varepsilon_t \quad (11)$$

Where r_t denotes daily returns or losses, $\mu_t = a_0 + \sum_{i=1}^{p}a_i r_{t-i} + \sum_{j=1}^{q}b_j\varepsilon_{t-j}$ is the conditional mean, $\varepsilon_t = \sqrt{h_t}z_t$ and h_t follows the asymmetric GARCH model discussed in section 2.2.

Referring to the estimation details, even though under regularity conditions, quasi-maximum likelihood estimator (QMLE) satisfies adequately the large sample properties of both consistency and asymptotic normality, if an alternative parametric distribution can be reasonably assumed, maximum likelihood (ML) may outperform QMLE in terms of efficiency. As shown in both table 1 and figure 2, the filtered residuals display specific property of heavy-tailed, which indicates that the innovations ε_t in (11) follow general error distribution (GED) rather than standard normal distribution. Correspondingly, the log-likelihood function of a zero-mean random variable ε_t with conditional variance h_t will be:

$$l(\varepsilon_t \mid \theta, v) = \log(v) - \frac{1}{2}\left|\frac{\varepsilon_t/\sqrt{h_t}}{\lambda}\right|^{v} - (1+\frac{1}{v})\log(2) - \log(\Gamma(\frac{1}{v})) - \log(\lambda) - \frac{1}{2}\log(h_t) \tag{12}$$

Where $\lambda = [\frac{\Gamma(1/v)}{2^{2/v}\Gamma(3/v)}]^{-1/2}$ and V is a positive parameter controlling the thickness of the tail.

In case v=2, the density is equal to the $N(0,h_t)$ density and the distribution becomes leptokurtic if v<2. As usual, θ contains contains the parameters which are of interest such as $a_i, b_j, \alpha_i, \beta_j$. Clearly, allowing for an ARMA part considerably extends the range of applications, but it also entails serious technical difficulties since we have to estimate both ARMA and GARCH models simultaneously rather than choosing to fit an ARMA model first and then fit a GARCH model on the ARMA residuals (see Engle (1982)).

After obtaining estimate of the asymmetric ARMA-GARCH parameters, both 1-step ahead conditional mean and conditional variance forecast can be formulated recursively through (8)-(11).

To make things clear, it is necessary to briefly summarize at this conjuncture. In stage one, we estimate and fit different asymmetric ARMA-GARCH models to return series. In stage two, we get the filtered i.id but tail-thickness residuals from stage one and apply EVT theory to characterize the lower tail of the distribution of standardized residuals according to equation (7) and therefore obtain VaR of the residuals. Eventually, by using formula (11), we arrive at a VaR estimate of the original return rate directly. For example, the 1-day VaR is:

$$VaR(r)_q^{t+1} = \mu_{t-1} + \sqrt{h_{t+1}}VaR(Z)_q \tag{13}$$

Where μ_{t+1}, h_{t+1} are the one-day ahead forecasts of the conditional mean and variance which can be calculated by minimizing the mean square errors (MSE) as usual. Apparently, if two or more day ahead forecast is needed, it is requires to compute conditional variances as well as the conditional mean recursively according to their different asymmetric GARCH models.

3. EMPIRICAL ANALYSIS

In section 2, we discussed theoretical framework of the paper, in the following, we focus on real data to do the corresponding empirical study of section 2.

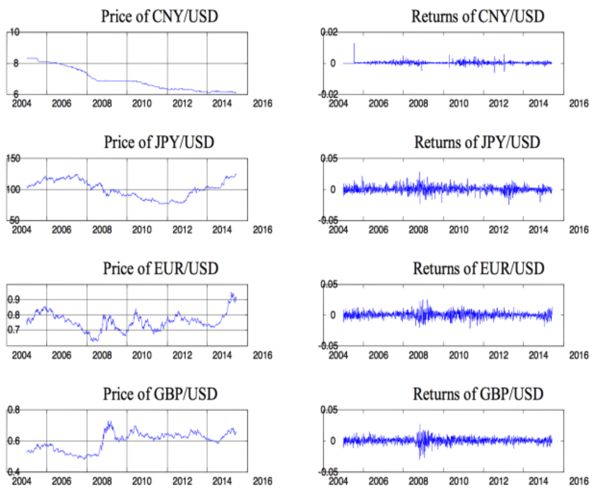
3.1. Descriptive statistics of the data

At the beginning, we do some graphical analysis as well as list several basic characteristics to illustrate the necessity of using asymmetric GARCH based two-stage EVT method and GPD to fit the heavy-tailed nature of the data. Precisely, since the observations should be approximately i.id for modeling tails of a distribution with a GPD and EVT needs to be fat-tailed distributes, Jarque-Bera and Ljung-Box Q statistics are reported. Data used in this paper is extracted from the Quandl website and includes four countries' daily exchange rates: Chinese Yuan/Dollar (CNY/USD), Euro/Dollar (EUR/USD), Japanese Yen/Dollar (JPY/USD), and Pound/Dollar (GBP/USD) from January 03, 2005 to May 29, 2015 including around 3710 observations. This time span covers the period of the global financial crisis of 2008-2010, the collapse of the Russian ruble beginning in the second half of 2014 and sharp depreciation of the Yen, Pound and Euro. Therefore, the examination of the effectiveness of VaR prediction will be believable. We define $r_{j,t}$ to be price changes in market of country j on day t for a long position i.e. holding USD. Note that each return is multiplied by 100 without altering intrinsic of the problem:

$$r_{j,t} = 100 * \log(P_{j,t}/P_{j,t-1}). \tag{14}$$

Situations of the four markets are depicted in figure 1:

Figure 1. Price and return of the four different foreign exchange markets



A rough comparison of left and right panels in figure 1 reveals that evidence of asymmetry is insufficient. For instance, volatility during downward periods and upward periods in the market of JPY/USD are almost the same, which is also true in the market of EUR/USD from 2010 to 2012. Hence, checking asymmetry formally becomes necessary in section 3.2. Differently, volatility clustering is clearly detected in the right panel of figure 1.

Moreover, table 1 presents some descriptive statistics of daily returns. As we can see in the first row, sample skewness except EUR/USD is quite different from 0 indicating asymmetry of the return distribution. High kurtosis in the second row shows leptokurtic i.e. tail thickness, both of the features suggesting non-normality of the return distribution which is formally identified by a Jarque-Bera normality test in the third row. The much thicker tail phenomenon of China (with kurtosis 42.40) than those of its developed counterparts (with kurtosis less than 10) attracts us, which may reflect its susceptibility to extreme shocks (Kim, 2015).

Detailed inspection shows some rationality of this result: Firstly, China reformed its peg regime in July 21, 2005 when under pressure from its major trading partners especially the United States, it moved into a supply-demand based managed peg system and began to allow the RMB to gradually appreciate over the next three years. Until June 2013, the RMB appreciated more than forty percent on a real (inflation-adjusted) basis against the US dollars. This event can also be identified on the right panel of figure 1. Secondly, many papers show that emerging countries such as those in Asian and Latin American countries tend to share fat tailed in their foreign exchange market compared with their developed counterparts due to their vulnerability to both internal and external shocks as well as poor management ability. (Frigyes Ferdinand Heinz and Desislava Rusinova, 2015;Kim, 2015).Thirdly, from figure 1 we can clearly find that volatility of the developed group are much higher than that of China, therefore, the same magnitude of a certain event may be extreme for China but not for the other three in a relative sense such as the global economic crisis in 2008.

Like most financial return series, data in foreign exchange markets also exhibits some degree of autocorrelation and volatility clustering referring to values of Q (16) and Q2(16). For the sake of deliberation, specific properties such as non-normality and the nature of tail-thickness can also be directly confirmed through the Q-Q plot in figure 2.

In summary, the descriptive statistics motivate us to search for new methods (i.e. asymmetric EVT-GARCH based VaR) to replace the typical models which rely on normality and serial independence assumptions.

Table 1. Diagonal statistics of the raw foreign exchange returns

	CNY/USD	JPY/USD	EUR/USD	GBP/USD
Skewness	2.12	0.12	-0.04	-0.47
Kurtosis	42.40	6.37	5.90	8.42
Jarque-Bera	177702	1295	955	3429
	(0.00)	(0.00)	(0.00)	(0.00)
$Q(16)$	438.38	791.45	765.50	952.34
	(0.00)	(0.00)	(0.00)	(0.00)
$Q^2(16)$	269.96	782.20	1028.64	2515.43
	(0.00)	(0.00)	(0.00)	(0.00)

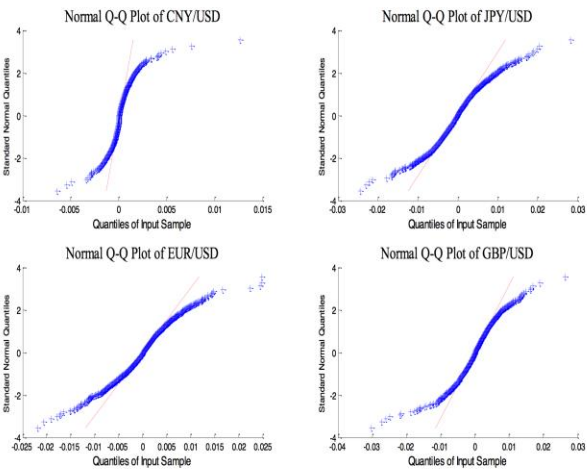
Note: kurtosis for a normal distribution is 3

In the following, we will divide the whole sample into two parts. The first part (January 03, 2005 to May 29, 2014) is used for model construction while the second part (from June 02, 2014 to May 29, 2015) for prediction accuracy evaluation in part 3.4.

3.2. Model Estimation

As illustrated previously, we need to firstly get filtered i.id standardized residuals through specifying the asymmetric GARCH model. Table 2 below lists the most proper fitted models of the four different markets mainly based on the criteria of AIC and FPE. Note that, since our interest here centers on prediction ability but not structural analysis, other criteria such as HQ and SC may be inferior in this scenario.

Figure 2. Normality test by applying Q-Q plot



Importantly, values of γ_i how that most of the leverage parameters are significant from 0, even though the magnitude is weak. Consequently, asymmetry does exist in the four foreign exchange markets. A dominating explanation for the weakness lies on the two-sided nature of exchange rates, namely, a positive return shock to one currency is a negative shock for the other position. This can also be validated by coexistence of both positive and negative values of γ_i in table 2. Unsurprisingly, value of γ_i for CNY/USD is much smaller than those of other currencies reflecting China's managed exchange rate regime i.e. hybrid of fixed and floating.

Table 2. Model specification and parameter estimate

Parameters	*CNY/USD*	*JPY/USD*	*EUR/USD*	*GBP/USD*
		Con.mean		
a_0	0.0056	-0.0046	-0.0180	-0.0012
	(0.0702)			(0.0913)
a_1	0.095	0.154	0.133	0.195
b_1	0.55	0.665	0.652	0.550
		Con.variance		
W	-0.771	0.004	0.008	0.001
		(0.0861)		(0.0994)
a_1	0.588	0.319	0.063	0.076
γ_1	0.006	-0.072	0.027	-0.022
β_1	0.85	0.953	0.791	0.934
δ			0.485	

Note that the fitted models are ARMA(1,1)-EGARCH(1,1), ARMA(1,1)-EGARCH(1,1), ARMA(1,1)-PGARCH(1,1) and ARMA(1,1)-GJRGARCH(1,1) respectively. p values are shown in parentheses only when not significant at the 5% level.

There exist two popular theories to explain why the asymmetry γ varies among different countries just as shown in Table2 that asymmetry effect being much smaller for Yuan/USD compared with others. Firstly, given the super economic and trade volumes between the US and China, Yuan and USD act as base currencies to each other. For example, most of the trade enterprises use either US dollars or Chinese

Yuan to measure their benefits and losses while for case of GBP, more companies may calculate their profits in US dollars. Hence, a large volatility of GBP/USD will cause the sale of GBP-denominated assets, which leads to the devaluation of the pound. But for US and China, a volatility of Yuan/USD will make Chinese to sell USD-denominated assets and Americans to sell EUR-denominated assets and therefore the asymmetry between China and the US is weaker. Secondly, even though the Chinese government allowed the Yuan to move to an unknown basket of currencies peg, it only requires the value of the Yuan to fluctuate 0.3 % per day over the previous day closing price which can be verified in figure 1.

Once the relevant parameters constituting the conditional mean $\overline{\mu_t}$ and conditional variance $\overline{h_t}$ are estimated, we can immediately apply (8)-(11) to calculate the standardized residuals:

$$z_t = (r_t - \overline{\mu_t})/\sqrt{\overline{h_t}} \tag{15}$$

Likewise, autocorrelation and normality diagnosis of standardized residuals are presented in Table 3. As expected, bad property like serial correlation or volatility clustering disappears through examining $Q(16)$ and $Q^2(16)$. However, non-normality or tail-thickness of the data still exists from indicators of skewness, kurtosis or Jarque-Bera. In consideration of the two features of standardized errors, POT-GPD based EVT in stage 2 is reasonably motivated.

Table 3. Diagonal statistics of the standardized errors

	CNY/USD	*JPY/USD*	*EUR/USD*	*GBP/USD*
Skewness	-0.053	0.132	0.331	-0.053
Kurtosis	14.92	5.26	8.00	8.95
Jarque-Bera	13928	507	2497	3468
	(0.00)	(0.00)	(0.00)	(0.00)
$Q(16)$	23.79	15.39	18.84	5.99
	(0.09)	(0.50)	(0.28)	(0.91)
$Q^2(16)$	1.2613	25.47	24.64	17.30
	(1.00)	(0.07)	(0.14)	(0.32)

Note: p-values are shown in the parentheses

3.3. POT-based EVT

The implementation of POT involves the following steps: select the and then fit the GPD function to the exceedances over u and finally compute point estimates as well as interval estimates of VaR.

3.3.1. Selection of threshold u

As discussed in Pickands-Balkema-de Haan theorem, the shape parameter and scale parameter estimators are functions of the selected threshold u which will be determined in this subsection. Intuitively, choice of u is a not a matter of science but a matter of art in balancing trade-off between being unbiased and small variance of the estimators: on the one hand, if u is high enough, then, Pickands-Balkema-de Haan theorem mentioned above is well satisfied, we can therefore apply GPD to fit the exceedances. However, as u gets larger and larger, there will be fewer observations left for the estimation of the parameters of the tail distribution function which will definitely increase the variance of the estimator.

Nevertheless, the issue of determining the fraction of data belonging to the tail is discussed by Danielsson et al. (2001). A subjective and popular tool is to plot the mean excess against u and choose threshold u^* which has the smallest mean excess value. Intuitively, a larger mean excess value parallels to larger bias of the GPD estimator. Particularly, the mean excess function (MEF) for the GPD with parameter $\xi < 1$ is:

$$e(u) = E(X - u \mid X > u) = \frac{\psi + \xi u}{1 - \xi}, \psi + \xi u > 0 \tag{16}$$

And a sample analogue estimator of the MEF is given by

$$e(u) = \frac{\sum_{i=k}^{n}(X_i - u)}{n - k + 1}, k = min\{i \mid X_i > u\} \tag{17}$$

As tabulated in Table 4, exceedances of each market account for around 10% of the whole sample. Hence, the tail sample is enough according to McNeil and Frey (2000).

3.3.2. MLE of GPD parameters

As guaranteed by Pickands-Balkema-de Haan theorem, extreme values should approximately follow GPD. That is, for a sample $y = \{y_1, \ldots, y_n\}$ where $y_i = Z_i - u^*$ and Z_i is the standardized residual, the log-likelihood function $log(\xi, \psi \mid y)$ for the GPD is the logarithm of the joint density of the n observations:

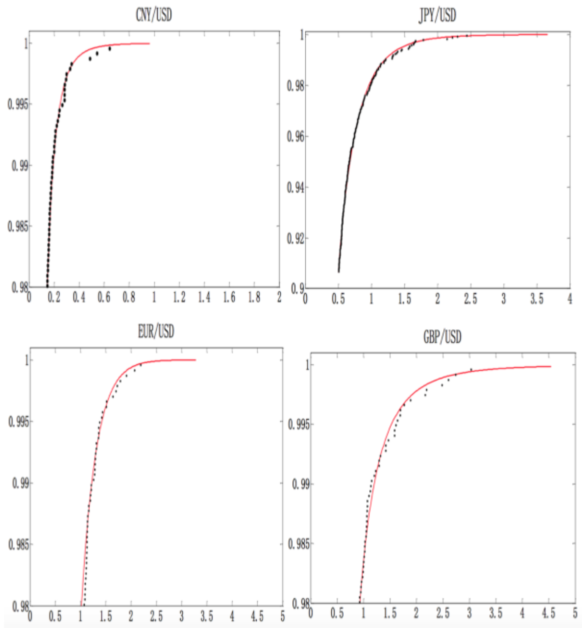
$$\log(\xi, \psi \mid y) = \begin{cases} -n\log(\psi) - (\frac{1}{\xi} + 1)\sum_{i=1}^{n}\log(1 + \frac{\xi}{\psi} y_i) & \text{if} \xi \neq 0 \\ -n\log(\psi) - \frac{1}{\psi}\sum_{i=1}^{n} y_i & \text{if} \xi = 0 \end{cases} \tag{18}$$

On a regular basis, ξ, ψ are estimated by maximizing the log-likelihood function as shown in table 4. Correspondingly, figure 3 plots how well the estimated GPD could fit exceedances of the lower tails. Note that in table 4, positive values of ξ indicate tail-thickness of the residuals and therefore the plausibility of using EVT.

Table 4. Estimated parameters of GPD distribution

		CNY/USD	*JPY/USD*	*EUR/USD*	*GBP/USD*
EVT threshold	u	0.05	0.5	0.49	0.45
% of exceedance	k/n	11.6	9.4	11.3	10.6
Shape parameter	ξ	0.19	0.12	0.215	0.26
Scale parameter	ψ	0.04	0.04	0.287	0.22

Figure 3. GPD fitted to the left tail exceedances above their corresponding threshold



Given the estimated GPD, high quantiles VaR can now be directly calculated from formula (7) by substituting ξ, ψ with their estimates.

3.3.3. VaR forecasting

At this point, we just estimated VaR of the standardized errors. In order to obtain VaR estimate of the daily returns, it remains to be seen the links between the two values by utilizing equation (13) derived from asymmetric ARMA-GARCH model. Results are reported in table 5 and table 6, e.g. in market of JPY/USD and CNY/USD, with probability 0.01, loss of a long position (holding USD) will exceed the 1.202% , 0.194% respectively.

To do comparison, we would also like to report another commonly used risk measure named expected shortfall (ES) proposed by Artzner et al (1999). Formally, if we define L as the level of losses, ES is denoted as $E(L \mid L > VaR)$. It is easy to see that ES measures the tail conditional expectation which is exactly the average loss exceeding VaR. Similarly, by replacing $VaR(Z)_q$ in equation (13) with $E(L \mid L < VaR(Z)_q)$, we arrive at the formula of ES for daily returns:

$$VaR(r)_q^{t+1} = \mu_{t+1} + \sqrt{h_{t+1}}\, E(Z \mid Z < VaR(Z)_q) \qquad (19)$$

Furthermore, confidence intervals could provide additional information regarding the accuracy of the point estimates, single confidence intervals are considered by using a bootstrap method or log-likelihood based method as that in Gilli (2006). Given the fact that bootstrap method does not depend on asymptotic theory, it may be more reliable under small sample circumstance.

As we can see from the last row of table 5, the corresponding expected loss in CNY/USD market is 2.85% .That is, on average, loss in situations where drops exceed 0.194% is 0.285%.

Table 5. Point estimates, maximum likelihood (ML) and bootstrap (BC) confidence intervals at 95% confidence level for the CNY/USD market

	Lower bound		*Point estimate*	*Upper bound*	
	BC	*ML*	*ML*	*ML*	*BC*
	Left tail				
$VaR_{0.01}$	0.178	0.175	0.194	0.221	0.218
$ES_{0.01}$	0.247	0.243	0.285	0.368	0.351

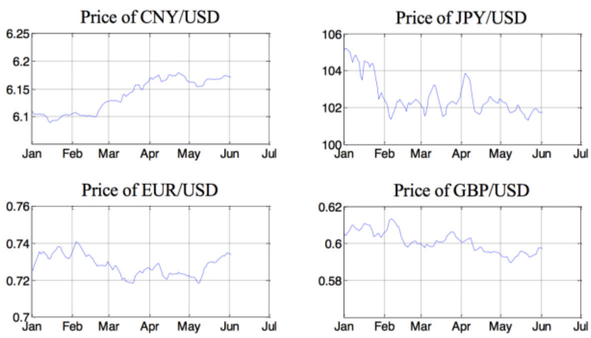
For simplicity, only point estimates of the markets remained are listed below:

Table 6. Point estimates for the remained three markets

	JPY/USD	*EUR/USD*	*GBP/USD*
$VaR_{0.01}$	1.202	1.221	1.201
$ES_{0.01}$	1.614	1.658	1.789

Interestingly, risks of the latter three countries are quite distinct from those of China: they are much more risk exposed. This result is also logical without intuition: since the in-sample date end on May 29, 2014, detailed price trends from January 01, 2014 to June 2,2014 are reported below: as we can see in figure 4 , CNY was depreciating most of the time during first half of 2014 while others showed the opposite which increases the risks of holding USD against JPY, EUR or GBP and therefore leading higher VaR forecasting values.

Figure 4. Detailed price trends from January 01, 2014 to June 2, 2014



3.4. Backtesting

Note that checking model accuracy is not equivalent to evaluating forecast ability is another thing, even though the two targets overlap to some extent. Actually, in terms of practical application, managers prefer to care more about the latter more than the former. Hence, in this subsection, we test how our model will behave in reality by adapting out-of-sample from June 02, 2014 to May 29, 2015. A method of rolling the data is employed and illustrated below:

Figure 5. Illustration of rolling data

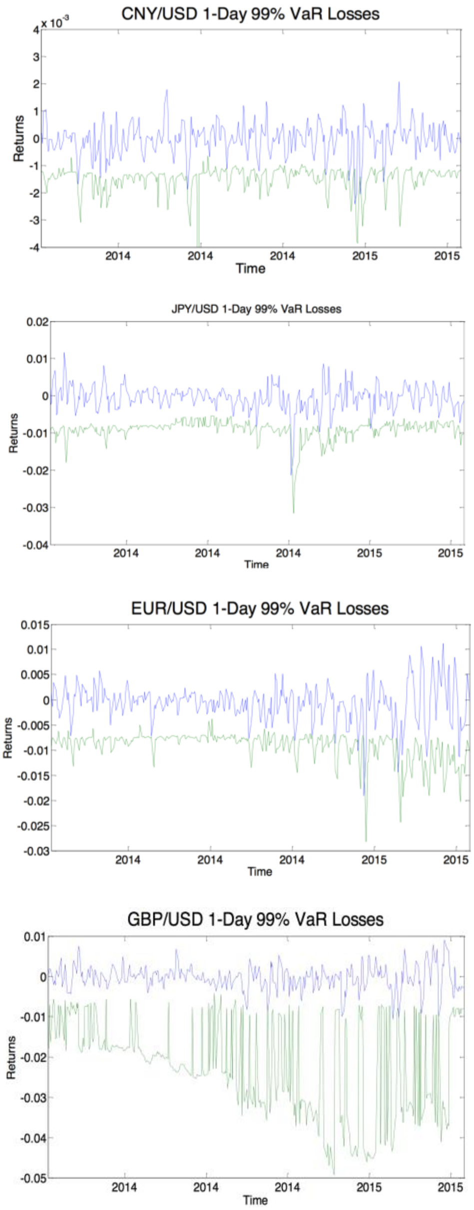
1 ---------------- m ----------- T   Step 1

---- k ----------------- m + k ------- T   Step k

Where m is the number of the in-sample data. However, reselecting model order and re-estimating model parameters (including multiple different ARMA-GARCH parameters as well as threshold and parameters of GPD) every time we roll the data is quite demanding. Luckily, as suggested by McNeil and Frey (2000), by applying the same model specification and meanwhile setting uniform threshold as 10 percentile of the data, we can still check prediction accuracy without losing too much credibility. Namely, corresponding specifications like ARMA(1,1)-EGARCH(1,1) , ARMA(1,1)-EGARCH(1,1), ARMA(1,1)-PGARCH(1,1) and ARMA(1,1)-GJRGARCH(1,1) for CNY/USD, JPY/USD, EUR/USD and GBP/USD will be accepted throughout. Afterwards, asymmetric ARMA-GARCH based 1-day ahead VaR forecast is depicted in figure 5.

In the following, backtesting is introduced step by step: from intuitive value comparison to statistical tests and lastly we develop it to be based on various forms of loss functions given that magnitude of exceedance helps to view forecasting ability differently.

Figure 6. $VaR(r)_{0.99}$ forecast and returns of the out-of-sample period (June 02, 2014 to May 29, 2015)



3.4.1. Informal violence ratio inspection

In order to deepen the understanding of figure 6 and accordingly have a basic cognition about the prediction effectiveness, the concept of violence ratio (VR) is introduced. Briefly, violation happens when the real returns beyond its corresponding forecast value and violation ratio or the failure ratio is the ratio of violations to number of out-of-sample (Jorion, 2001). If the constructed model is accurate, VR would be coincident with the chosen significance level i.e. 1% in our case by a statistical view. Or equivalently, expected violations are around 3 to 4 among the 365 days forecast period.

As we can see from both figure 6 and table 7, most of the markets are empirically in line with theory even though some of the returns hit exactly the VaR bound. It is apparent that a much higher violation ratio implies the risk is not properly hedged while a much lower VR does not signify a better management even though risk is controlled very well, since the holder of the asset suffers an opportunity cost of the interest rate (Gencay et al.2003).

Table 7. Failures of prediction at significance level 1% and 0.5% (long position)

	CNY/USD	*JPY/USD*	*EUR/USD*	*GBP/USD*
1 %Violations	4.38	3.29	4.01	3.28
0.5 % Violations	1.92	1.68	1.87	1.71

However, VR is only a rough measurement, in order to statistically determine whether empirical and theoretical VRs are significantly different with each other, we adopt the standard test put forward by Christoffersen (1998) as well as Kupiec (1995).

3.4.2. Unconditional coverage

In principle, to carry out every statistical test, we have to build a statistic generated from the simple value of VR. As we know, under the assumption that model is accurate, each realized return outcome produces a VaR violation with probability α which is exactly the significance level. Hence, the number of violations can be viewed as a Bernoulli experiment which can be approximated with a normal distribution. We assume length of the out-of-sample is T and number of violations during this period is S, then

$$\frac{S-\alpha T}{\sqrt{\alpha(1-\alpha)T}} \xrightarrow{d} N(0,1) \tag{20}$$

Maximum likelihood estimator of VR can be written as $\overline{VR}_{MLE} = S/T$, and the likelihood ratio statistic is denoted as:

$$LR_{uc} = -2(\ln L(\alpha, S) - \ln L(\overline{VR}_{MLE}, S)) \tag{21}$$

Where $L(\alpha, S) = \alpha^{S}(1-\alpha)^{T-S}$. It can be shown that under the null hypothesis (i.e. α and VR are statistically the same) and regularity conditions, the LR statistic follows a chi-square distribution with one degree of freedom. In view of thesimilarities, we notice the other two commonly used unconditional coverage statistics: POF-test (proportion of failures) and TUFF-test (time until first failure) also follow a χ^2 distribution.

3.4.3. Conditional coverage

The best known test called conditional coverage was proposed by Christoffersen (1998). It not only pays attention to VR but also sheds new light on clustered exceptions i.e. the null hypothesis implies that a violation today should not depend on whether or not a violation occurred on the previous day. Clearly, the joint test is more valid, since in reality large losses occurring in clustering periods are more likely to lead to disastrous events than individual exceptions taking place evenly during the whole period. For ease of exposition, we omit the details of an independence test and only conclude that the LR statistic also follows a chi square distribution with one degree of freedom.

Formally, the joint statistic is as follows:

$$LR_{cc} = LR_{uc} + LR_{ind} \tag{22}$$

It is worth noting, however, even when our model passes the joint test it may still be rejected according to the single independence test or unconditional coverage test. Hence, table 8 reports all of the test results.

Table 8. Backtesting Value-at-Risk

	CNY/USD	JPY/USD	EUR/USD	GBP/USD
TUFF	1.1524	0.4552	1.1524	0.5592
	(0.283)	(0.500)	(0.283)	(0.455)
Uncond.	0.4522	0.1245	0.0329	0.1245
	(0.501)	(0.724)	(0.856)	(0.724)
Indep.	0.0000	0.0000	0.0000	0.0000
	(1.000)	(1.000)	(1.000)	(1.000)
Cond.	0.4522	0.1245	0.0329	0.1245
	(0.798)	(0.940)	(0.984)	(0.940)

Note: p-values are in the parentheses, and all of the statistics are insignificant at level of 5%

As we can see from table 8, almost all independence statistic are statistically equal to 0, which indicates non-existence of violation clustering and it can also be verified intuitively by figure 5. In this case, $LR_{cc} = LR_{ind}$. In summary, the statistical analysis (including both joint and separate tests) shows evidence in favor of the null hypothesis: 1. Realized VR is insignificantly different from significance levels; 2. Violations are independent from each other.

Consequently, our asymmetric EVT-GARCH based VaR model can be well applied to foreign exchange risk management. In fact, the strong prediction ability is unreasonable. GARCH part of the model can quickly capture the volatility change of the market and then adopt the new information to make adjusted forecast for forthcoming days can lead to the beautiful non-existence of violation clustering.

3.4.4. Loss function based model evaluation

As mentioned previously, the magnitude of the violations may be more important in practice. Let r_t denote the vector of the realized volatility and f_t the vector of forecasted VaR. In the following we list several commonly used loss functions vary from mean square error (MSE), mean absolute deviation (MAD) to mean logarithm of absolute error (MLAE) in order to reflect specific concerns about the same method:

$$MSE = (1/T)\sum_{t=1}^{T}(f_t - r_t)^2 \tag{23}$$

$$MAD = (1/T)\sum_{t=1}^{T}|f_t - r_t| \tag{24}$$

$$MLAE = (1/T)\sum_{t=1}^{T}\log|f_t - r_t| \tag{25}$$

Table 9. Volatility Loss Functions

	CNY/USD	JPY/USD	EUR/USD	GBP/USD
MSE	0.0008	0.0054	0.0057	0.0342
MAD	0.0015	0.0039	-5.5677	0.0097
MLAE	-7.3111	-5.5862	0.0040	-4.8088

Similarly, the model also performs very well in terms of loss functions.

4. CONCLUSION

This paper introduces a more general asymmetric VaR-GARCH based tool to forecast risks of four different kinds of currencies as that of McNeil and Frey (2000). In step one, an asymmetric ARMA-GARCH model is used to foreign exchange market returns and residuals are obtained. In the second step, the extracted *i.id* residuals are modeled using the GPD.

As expected, by taking certain data properties such as asymmetry, heteroscedasticity, non-normality and tail-thickness into consideration, the model successfully passes all of the conventional backtesting. Therefore, it can be reasonably used in practice as an objective early warning system especially before a currency catastrophe which can be referred to make instant financial and economic policies[2].

Also, based on the findings of asymmetric volatility in foreign exchange markets and those in other mentioned papers, we are able to draw a conclusion that the property of asymmetry not only depends on the specific time period observed and market used. Furthermore, under the assumption that if central banks' intervention effect dominates the base-currency effect, we can roughly say that Chinese government may not implement currency manipulation to keep Yuan being undervalued. This is because many papers have shown that manipulations generate higher volatility (Galati et al 2005; Frenkel et al. 2005), their intervention on one side of the foreign exchange market but not the other will lead larger asymmetric. On the other hand, if base-currency effect dominates the central banks' intervention effect since the two giants share similar economic size and condition, we still cannot

[2] Even though the model can only perform well with short term forecasts, it is better than doing nothing to avert a crisis especially on the eve of such an event.

confidently accuse China of manipulating its currency since we know nothing about the magnitude of base-currency effect.

In the future, more attention should be paid towards evaluating the effects of an even more general model by considering vector-GARCH cases, so that we can include more economic variables according to economic theory (e.g. real interest rate) since it essentially integrates the two different methods discussed in section 1 together.

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