# COST OF CAPITAL ADJUSTED FOR GOVERNANCE RISK THROUGH A MULTIPLICATIVE MODEL OF EXPECTED RETURNS

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**Abstract**

This paper sets forth another contribution to the long standing debate over cost of capital, firstly by introducing a multiplicative model that translates the inner structure of the weighted average cost of capital rate and, secondly, adjusting such rate for governance risk. The conventional wisdom states that the cost of capital may be figured out by means of a weighted average of debt and capital. But this is a linear approximation only, which may bring about miscalculations, whereas the multiplicative model not only takes account of that linear approximation but also the joint outcome of expected costs of debt and stock, and their proportions in the capital structure. And finally, we factor into the cost of capital expression a rate of governance risk.

**Keywords**: cost of capital; governance risk; weighted average cost of capital; governance index; multiplicative model of returns

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## Introduction

Regarding its meaning, cost of capital has three prevailing alternatives of usage. Although its importance and relevance seems widely established, the concept itself and how to work it out have faced a widespread concern from critics. Let us handle usage and criticisms separately.

***a) As a criterion for financial decisions.***

It's the current benchmark, clearly depicted in a well-known textbook on Investment Valuation[1]:

*[…] is the cost of the different components of financing used by the firm, weighted by their market value proportion. […] Since a firm can raise its money from three sources – equity, debt, and preferred stock – the cost of capital is defined by the weighted average of each of these costs.*

***As standard for investment decisions that furnishes a minimal rate of return on proposed new investments.***

This was the approach taken by Ezra Solomon (1955) in his seminal paper which attempted to measure any company's cost of capital:

*Its function is to provide a correct and objective criterion by which management can determine whether it should or should not accept available proposals involving the expenditure of capital. Because of this function, this concept has also been called the "minimun required rate of earnings" or the "cut-off" rate for capital expenditure.*

To put it in other words, if this required rate for a new investment project is higher than the cost of capital, firm value will increase; otherwise, it will lose value[2].

On this line of analysis, Ross et al. (1995) argued that being the cost of capital the minimum required return on a new investment, it can be translated like "*the opportunity cost associated with the firm's capital investment.*"

[1] Damodaran (2002), see the References section for further details.

[2] There are particular settings for which this statement becomes fuzzy and requires further qualifications. Apreda (2009, forthcoming) will deal with this issue in the context of investment decisions.

Therefore, cost of capital becomes a "hurdle rate" in the following sense:

i) for an investment project in the firm's line of business such a rate would grant that the basic business risk of the new asset will be the same as the one of already existing assets;

ii) valuation of an investment project from a different risk class would demand a cost of capital metrics that takes into account the proper line of business.

***b) As a link between investment decisions and financing decisions.***

This was the viewpoint firstly brought to light by Modigliani and Miller (1958), grounded on portfolio theory, complete markets, and perfect arbitrage. Albeit constrained by utterly restrictive assumptions, it has provided academics and practitioners with plenty of potential for further research so far[3].

On these grounds and focusing on a portfolio management approach, Myers (1995) stated that the cost of capital is the opportunity cost "*borne by investors who can put their money into securities with the same risk as the proposed project"*.

***c) Criticism against the current cost of capital usage***

However, these contributions have come under an impressive array of disapproval nurtured by scholars and practitioners for whom both the concept of cost of capital and the weighted average method of calculation employed so far, suffer from variegated shortcomings. For instance, Haley and Schall (1976) pointed out that:

*As our understanding of more realistic and complex situations increases, the concept of cost of capital becomes either irrelevant, misleading or both […] However, even in textbooks to the extent that it is used as a decision criterion it should be confined to the investment decision. The cost of capital concept offers no advantage in research and, in the long run, the term cost of capital might best be abandoned.*

Another critical remark was raised by Reilly and Wecker (1973) who highlighted that within the weighted average cost of capital (WACC) paradigm

*[there is] a mathematical error of using weight average cost of capital to represent the true cost of capital. […] It is not possible to express such cost of capital as an algebraic combination of the coefficients of the financing polynomials for the specific sources of capital. Use of the weighted average cost of capital may lead to the establishment of an erroneous investment cut-off point and/or a nonoptimal capital structure.*

A truly debatable issue has been put forth on the grounds of the so-called "circularity" problem. For instance, Mohanty (2006) defines circularity as taking place when

*while valuing a company using the Discounted Cash Flow approach, we need to know the cost of capital to value the company, and we need to know the value of the company (in particular the market debt-to-equity ratio) to find the cost of capital.*

The referred author proposed a solution of the problem, by means of an iterative algorithm that ultimately finds out the actual value of equity to be used in the cost of capital assessment. Following another track of research, Velez-Pareja and Tham (2001) support a solution based on market value corrections, period after period.

Finally, a deeper analysis on the limitations of WACC paradigm and the convenience of shifting towards an institutional-behavioral paradigm has been advocated by Dempsey (1996).

Starting out from the mainstream discussion, this paper intends to make two contributions:

a) To frame the notion of cost of capital within the context of a multiplicative model of returns, instead of the usual one which is only a linear approximation of the latter.

b) To adjust the cost of capital for governance risk.

So as to accomplish our goals, in section 1 we provide an overview of the conventional wisdom of cost of capital. It will be stressed that the current procedure to assess cost of capital lies on a linear approximation only.

In section 2, the unconventional wisdom is unfolded, showing the linkage between the linear approximation and a multiplicative model for expected returns.

It is for section 3 to introduce governance risk, stemming from a former contribution of ours that sets up a new governance index and a rate of governance risk (Apreda, 2007a). Last of all, section 3.1 maps out an adjustment to the cost of capital for governance risk.

## 1. The Conventional Wisdom about Cost of Capital

Let us assume that we start our analysis with certain firm endowed by the following capital structure:

- **Debt (simple bonds or bank loans)**: the company has different kinds of debt that can be deployed in vectorial notation as follows:

$$[\,\mathbf{D}\,] = [\,\mathbf{D}_1 ; \mathbf{D}_2 ; \mathbf{D}_3 ; \ldots\ldots\ldots ; \mathbf{D}_N\,]$$

such that the monetary value of debt is a weighted average of debt components:

[3] On this account, see section 1.1.

$$D = x_1 . D_1 + x_2 . D_2 + x_3 . D_3 + \ldots\ldots\ldots + x_N . D_N$$

provided that

$$x_1 + x_2 + x_3 + \ldots\ldots\ldots\ldots + x_N = 1$$

where

$$x_g = D_g \ / \sum D_h \quad ; \quad h : 1, 2, 3, \ldots\ldots , N$$

- **Stock (ordinary shares)**: the company might have issued shares in different dates; perhaps with distinctive voting features in each case.

$$[\, S \,] = [\, S_1 ; S_2 ; S_3 ; \ \ldots\ldots\ldots \ ; S_M \,]$$

such that the monetary value from the portfolio of equity varieties is a weighted average of its components:

$$S = y_1 . S_1 + y_2 . S_2 + y_3 . S_3 + \ldots\ldots\ldots + y_M . S_M$$

provided that

$$y_1 + y_2 + y_3 + \ldots\ldots\ldots\ldots + y_M = 1$$

where

$$y_i = S_i \ / \sum S_j \quad ; \quad j : 1, 2, 3, \ldots\ldots , M$$

- **Financial Hybrids (mainly preferred stock, convertible preferred stock, bonds with warrants, and convertible bonds)**

$$[\, FH \,] = [\, FH_1 ; FH_2 ; FH_3 ; \ \ldots\ldots\ldots \ ; FH_O \,]$$

such that the monetary value from the portfolio of financial hybrids is a weighted average of its components:

$$FH = z_1 . FH_1 + z_2 . FH_2 + z_3 . FH_3 + \ldots\ldots + z_O . FH_O$$

provided that

$$z_1 + z_2 + z_3 + \ldots\ldots\ldots\ldots + z_O = 1$$

where

$$z_k = FH_k \ / \sum D_l \quad ; \quad l : 1, 2, 3, \ldots\ldots , O$$

The conventional wisdom states that the rate **k**, the cost of capital for such company, can be worked out as a weighted average of the expected return of each component in the capital structure[4]:

$$k = x_D R_D + y_S R_S + z_{FH} R_{FH}$$

such that

$$x_D + y_S + z_{FH} = 1$$

where

$$x_D = D/(D+S+FH); \ y_S = S/(D+S+FH); \ z_{FH} = FH/(D+S+FH);$$

As for the expected returns from the three main components of the capital structure, we have to assess them the following way:

$$R_D = x_1 . R(D_1) + x_2 . R(D_2) + \ldots\ldots + x_N . R(D_N)$$

$$R_S = y_1 . R(S_1) + y_2 . R(S_2) + \ldots\ldots + y_M . R(S_M)$$

$$R_{FH} = z_1 . R(FH_1) + z_2 . R(FH_2) + \ldots\ldots + z_O . R(FH_O)$$

It is from the firm's valuation side that the expected rate of return from debt, $R_D$, stands for the after-tax cost of debt. In point of fact,

$$R_D = (\, 1 - \text{tax rate} \,) \ . \ r_D$$

where $r_D$ denotes the nominal average rate of return from the portfolio **D**. Such expression derives from the next one:

$$R_D = (\, 1 - \text{tax rate} \,) \ . \ r_D$$

$$= x_1 . (\, 1 - \text{tax rate} \,) . r(D_1) + x_2 . (\, 1 - \text{tax rate} \,) . r(D_2) +$$

$$\ldots\ldots + x_N . (\, 1 - \text{tax rate} \,) . r(D_N)$$

A similar procedure would hold if the firm has some financial hybrid that qualifies for a tax shield, as it is the case with convertible bonds.

### 1.1 The Portfolio Approach to cost of Capital

Since Markowitz's innovative method for managing portfolios[5], there have been distinctive developments far beyond the founding issue. Therefore, organizations were regarded as dual portfolios (the first one given by their assets, the second consisting in their liabilities and equity). On this line of research, value enhancement meant that

[4] For valuation purposes, at date **t**, the rate **k** should be referred as the "expected cost of capital", because the rates of return for stock, debt and financial hybrids are expected values. It is only for ease of notation we do not use as from now the expected value operator **E[ . ]**.

[5] Markowitz (1952, 1959)

the rate of return from the former should be higher than the return from the latter portfolio.

Another constructive framework of analysis was employed by Modigliani and Miller through a series of consequential papers, most remarkably the one published in 1958[6]. On their own viewpoint, the company has a portfolio that consists of its own securities

$$\mathbf{P} \;=\; \textbf{Company's Portfolio of Securities}$$

defined as the following vector of proportions:

$$\mathbf{P} = \{\, x_D \,;\, y_S \,;\, z_{FH} \,\}$$

such that $x_D + y_S + z_{FH} = 1$

Applying the well-known expression for the expected return from any portfolio, in Markowitz's sense:

$$\mathbf{R\,(\,P\,)} = x_D\,\mathbf{R}_D + y_S\,\mathbf{R}_S + z_{FH}\,\mathbf{R}_{FH}$$

For all intents and purposes, the rationale behind the conventional wisdom would lie on the following identity:

$$\mathbf{R\,(\,P\,)} = \mathbf{k}$$

## 2. The Unconventional Wisdom about Cost of Capital

We now settle down to another perspective that consists in factoring the cost of capital into debt, stock and financial hybrids returns, through a multiplicative model[7].

$$1 + \mathbf{K} = <1 + x_D\,\mathbf{R}_D>.<1 + y_S\,\mathbf{R}_S>.<1 + z_{FH}\,\mathbf{R}_{FH}>$$

The right side of this equation can be broken up into the following components:

$$1 + \mathbf{K} = 1 + x_D\,\mathbf{R}_D + y_S\,\mathbf{R}_S + z_{FH}\,\mathbf{R}_{FH} +$$

$$+\; x_D\, y_S\,\mathbf{R}_D\mathbf{R}_S + x_D\, z_{FH}\,\mathbf{R}_D\mathbf{R}_{FH} +$$

$$+\; y_S\, z_{SFH}\,\mathbf{R}_S\mathbf{R}_{FH} + x_D\, y_S\, z_{FH}\,\mathbf{R}_D\mathbf{R}_S\,\mathbf{R}_{FH}$$

or, equivalently,

$$\mathbf{K} = x_D\,\mathbf{R}_D + y_S\,\mathbf{R}_S + z_{FH}\,\mathbf{R}_{FH} +$$

$$+\; x_D\, y_S\,\mathbf{R}_D\mathbf{R}_S + x_D\, z_{FH}\,\mathbf{R}_D\mathbf{R}_{FH} +$$

$$+\; y_S\, z_{SFH}\,\mathbf{R}_S\mathbf{R}_{FH} + x_D\, y_S\, z_{FH}\,\mathbf{R}_D\mathbf{R}_S\,\mathbf{R}_{FH}$$

Lastly,

$$\mathbf{K} = \mathbf{k} + x_D\, y_S\,\mathbf{R}_D\mathbf{R}_S + x_D\, z_{FH}\,\mathbf{R}_D\mathbf{R}_{FH} +$$

$$+\; y_S\, z_{SFH}\,\mathbf{R}_S\mathbf{R}_{FH} + x_D\, y_S\, z_{FH}\,\mathbf{R}_D\mathbf{R}_S\,\mathbf{R}_{FH}$$

Hence, the cost of capital stemming from the multiplicative model contains a linear approximation that amounts to the cost of capital according to the conventional wisdom. However, a non-linear component is also embedded in the multiplicative model and the bridge between both the linear and non-linear components may become significant and non-rejectable eventually, measured by the expression:

$$\mathbf{K} - \mathbf{k} = x_D\, y_S\,\mathbf{R}_D\mathbf{R}_S + x_D\, z_{FH}\,\mathbf{R}_D\mathbf{R}_{FH} +$$

$$+\; y_S\, z_{SFH}\,\mathbf{R}_S\mathbf{R}_{FH} + x_D\, y_S\, z_{FH}\,\mathbf{R}_D\mathbf{R}_S\,\mathbf{R}_{FH}$$

By far, this sort of approach lends a coherence and unity to our subject matter that the linear perspective lacks eventually.

### 2.1 The Linkage between K and k (the Metrics of Substitution)

We wonder to what extent it is advisable for the analyst to substitute the linear approximation of cost of capital

$$\mathbf{k} = x_D\,\mathbf{R}_D + y_S\,\mathbf{R}_S + z_{FH}\,\mathbf{R}_{FH}$$

for the multiplicative interpretation of cost of capital

$$\mathbf{K} = \mathbf{k} + x_D\, y_S\,\mathbf{R}_D\mathbf{R}_S + x_D\, z_{FH}\,\mathbf{R}_D\mathbf{R}_{FH} +$$

$$+\; y_S\, z_{SFH}\,\mathbf{R}_S\mathbf{R}_{FH} + x_D\, y_S\, z_{FH}\,\mathbf{R}_D\mathbf{R}_S\,\mathbf{R}_{FH}$$

The rationale behind the substitution of **K** for **k** in the context of applications, should be tracked down into whether next condition is fulfilled or not, eventually:

$$\left|\,\mathbf{K} - \mathbf{k}\,\right| < 10^{-\alpha}$$

For the sake of illustration, we now move on to **Table 1**, where we deal with a company which offers, at valuation date, 8 % of return from debt (net of tax) and 11% on the standing stock[8]. We figure out

[6] See the References section.
[7] More background and foundations in Apreda (2006) who introduced, for the first time, multiplicative models in the context of residual information sets, differential rates, and transactional algebras.
[8] These rates are current values for many developing countries. In fact, rates are usually much higher.

**K** and **k** under five different sets of weights for debt and stock. It can be witnessed that the gap

$$|\mathbf{K} - \mathbf{k}|$$

is relevant. In all cases the discrepancy keeps over $\mathbf{10^{-3}}$, which does not make reliable the linear approximation.

**Table 1**

| $x_D$ | $y_S$ | $R_D$ | $R_S$ | K | k | K - k |
|---|---|---|---|---|---|---|
| 0.30 | 0.70 | 0.08 | 0.11 | 0.1028 | 0.1010 | 0.0018 |
| 0.40 | 0.60 | 0.08 | 0.11 | 0.1001 | 0.0980 | 0.0021 |
| 0.50 | 0.50 | 0.08 | 0.11 | 0.0972 | 0.0950 | 0.0022 |
| 0.60 | 0.40 | 0.08 | 0.11 | 0.0941 | 0.0920 | 0.0021 |
| 0.70 | 0.30 | 0.08 | 0.11 | 0.0908 | 0.0890 | 0.0018 |

## 3. Governance Risk

In a recently published paper (Apreda, 2007a), I introduced a new weighted average governance index out of which a measure of governance risk can be derived[9].

The governance index, at date **t** and for certain company "**c**", arises from the expression [10]

$$\mathbf{G(C; T)} =$$

$$= \mathbf{w(1).\ G(c, 1, t) + w(2).\ G(c, 2, t) + \ldots\ldots + w(Q).\ G(c, Q, t)}$$

It is for the rate of change worked out from this index to gauge good or bad governance performance, throughout the horizon **[t; T]**:

$$\mathbf{1 + r_c\,(governance) = G(c; T) \ / \ G(c; t)}$$

Taking advantage of the rate of change of this governance index, we set forth a measure of governance risk, by solving:

$$\mathbf{< 1 + r_c\,(governance) > . < 1 - \Delta\ govrisk > = 1}$$

to get at last,

$$\mathbf{\Delta\ govrisk = r_c\,(governance) \ / < 1 + r_c\,(governance) >}$$

Whenever the company improves its governance, from date t to date **T**, it holds that

$$\mathbf{r_c\,(governance) > 0}$$

whereas if governance performance lessens, the rate becomes negative. By the same token, good governance makes

$$\mathbf{\Delta\ govrisk > 0}$$

and the final outcome is a decrease of the adjustment for governance risk measured up by

$$\mathbf{< 1 - \Delta\ govrisk >}$$

whereas bad governance leads to the opposite outcome:

$$\mathbf{\Delta\ govrisk < 0}$$

and, therefore,

$$\mathbf{1 - \Delta\ govrisk > 1}$$

[9] Gompers et al. (2001) provided with a qualitative index intended to measure the compliance with a set of provisions in the foundational charters of listed companies in the United States. Our index goes beyond those provisions and takes into account a set of governance variables not necessarily contained in the charters. Besides, it applies also to closed companies, not listed, as it seems the rule in developing countries.

[10] Further details about the index components can be found in the Appendix at the end of this paper.

### 3.1 Cost of Capital Adjusted for Governance Risk

The adjustment for governance risk has two alternative courses of action: either we embed it into the linear approximation or we deal with the multiplicative model outright.

***Conventional approach***

In keeping with the linear expression for the cost of capital, the approximation would be given by[11]

$$\mathbf{k}_{+\,gov} = \mathbf{x}_D \mathbf{R}_D + \mathbf{y}_S \mathbf{R}_S + \mathbf{z}_{FH} \mathbf{R}_{FH} - \Delta\ \mathbf{govrisk}$$

***Unconventional approach***

In contrast with the former approach, the framing of governance risk into the multiplicative model proceeds from

$$1 + \mathbf{K}_{+\,gov} = \langle 1 + \mathbf{x}_D \mathbf{R}_D \rangle . \langle 1 + \mathbf{y}_S \mathbf{R}_S \rangle .$$

$$. \langle 1 + \mathbf{z}_{FH} \mathbf{R}_{FH} \rangle . \langle 1 - \Delta\ \mathbf{govrisk} \rangle$$

Bear in mind that if

$$\Delta\ \mathbf{GOVRISK} < 0$$

then $\mathbf{K}_{+\,gov}$ becomes larger since governance worsens, adding up to the overall risk premium in cost of capital.

Again, the suitability of both models follows from the gap between **K** and **k**.

In **Table 2**, we profit from **Table 1** by substituting $\mathbf{K}_{+\,gov}$ and $\mathbf{k}_{+\,gov}$ for **K** and **k**. The gap

$$|\ \mathbf{K}_{+\,gov} - \mathbf{k}_{+\,gov}\ |$$

is less than $10^{-3}$ in four out of five sets of weight.

**Table 2**

| $x_D$ | $y_S$ | $R_D$ | $R_S$ | $\Delta$ govrisk | $K_{+\,gov}$ | $k_{+\,gov}$ | $K_{+\,gov} - k_{+\,gov}$ |
|---|---|---|---|---|---|---|---|
| 0.30 | 0.70 | 0.08 | 0.11 | 0.01 | 0.0918 | 0.0910 | 0.0008 |
| 0.40 | 0.60 | 0.08 | 0.11 | 0.01 | 0.0891 | 0.0880 | 0.0011 |
| 0.50 | 0.50 | 0.08 | 0.11 | 0.01 | 0.0862 | 0.0850 | 0.0012 |
| 0.60 | 0.40 | 0.08 | 0.11 | 0.01 | 0.0832 | 0.0820 | 0.0012 |
| 0.70 | 0.30 | 0.08 | 0.11 | 0.01 | 0.0799 | 0.0790 | 0.0009 |

[11] When adjusting for governance risk we denote cost of capital as $\mathbf{K}_{+\,gov}$ and $\mathbf{k}_{+\,gov}$.

## Conclusions

Summing up, this paper raises the issue of to what extent the conventional usage conveys reliable information or distorts the value we expect from any fair assessment of cost of capital.

To avoid a faulty linear approximation to the cost of capital, the multiplicative model turns out to be more functional and also wide-ranging to the needs of the analyst.

Finally, governance risk is a subject matter that should not go on unnoticed any longer. We have brought forth its inclusion both in the conventional approach as well the multiplicative framework so as to get a more down-to-earth measure of cost of capital.

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## APPENDIX

The subsequent vector comprises a chosen list of explanatory variables for governance, at date **t**,

$$\mathbf{G} = [\, G(1), G(2), \ldots\ldots\ldots\ldots, G(Q) \,]$$

A weighting system, at date t, will arise out of the vector

$$\mathbf{W} = [\, w(1), w(2), w(3), \ldots\ldots\ldots\ldots, w(Q) \,]$$

The index should be defined, at date **t** out of a universe of available companies, also framed as a vector

$$\Gamma = [\, c_1 ; c_2 ; c_3 ; \ldots\ldots ; c_V \,]$$

and to compute its value at date **t**, for certain company **c** belonging to **Γ**, we avail ourselves of the scalar product of vectors **G** and **W**:

$$G(c; t) =$$

$$= [G(c; 1; t), G(c; 2; t), \ldots , G(c; Q; t)] \,.\, [w(1), w(2), \ldots , w(Q)]$$

that is to say, the index springs up from the dated expression:

$$G(c; t) =$$

$$= w(1) \,.\, G(c; 1; t) + w(2) \,.\, G(c; 2; t) + \ldots + w(Q) \,.\, G(c; Q; t)$$

or, equivalently[12],

$$G(c; t) = \sum w(i) \,.\, G(c; i; t) \quad ; \quad i: 1, 2, 3, \ldots\ldots , Q$$

and we are going to make explicit each governance variable by means of a recursive relationship[13]:

$$G(c; i; t) = G(c; i; t-1) + \varepsilon(c; i; t-1; t) \tag{App.1}$$

(App.1) provides the dynamical setting from which the index evolves as time passes by.

It's worth noticing the inner structure of the second term on the right side of the expression above:

$$\varepsilon(c; j; t-1; t) = \begin{cases} \textbf{+ 1 (compliance}^{14}\textbf{ level)} \text{ if there is material evidence that the underlying variable has moved for the better over the valuation period.} \\ \textbf{0 (neutral level)} \text{ if there is no conclusive evidence that any material change has taken place.} \\ \textbf{– 1 (non compliance level)} \text{ if there is material evidence that the underlying variable has moved for the worse over the valuation period.} \end{cases}$$

Summing up, (App.1) defines each governance variable inductively. In other words, (App.1) conveys the idea of an accumulative process that holds for every company **c**. As time goes by, the process rewards compliance and punishes non-compliance, period after period.

At this juncture, we have to render account of our choice of governance variables. They are sorted out in ***Exhibit 1***[15] under the headings of six broad categories, namely Board of Directors, Owners, Governance Architecture, Management, Creditors, Gatekeepers and Regulators.

It goes without saying that, in actual practice, the analyst or econometrician laboring over this index may shorten the list of variables on the grounds of tractability, relevance, research costs, or statistical fitness.

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[12] When writing down **G(c; t)**, we mean the value of the index at date **t** for company **c,** whereas **G(c; j; t)** stands for the value of the governance variable **G(j)** at date **t**, for company **c.**

[13] We assume that the variable "date at **t**" belongs to a denumerable set that stands for an index set. More background on recursive or inductive definitions can be found in Bloch (2000).

[14] Compliance risk and compliance functions are newcomers in the governance parlance, since their introduction by the Bank of Basel like guidelines for financial institutions worldwide. The first extension of both notions to non-financial organizations was provided by Apreda (2007b).

[15] Further background on the semantics of the variables included in **Exhibit 1** can be found in Apreda (2007c, 2005, 2003)

**Exhibit 1.** Governance Variables

| | |
|---|---|
| **Board of Directors**<br><br>Independent Directors<br>CEO and Chair as separate functions<br>Control and fiduciary duties<br>Audit Committee<br>Staggering appointments<br>Compliance risk committee<br>Compensation packages committee<br>Self-dealing issues | **Management**<br><br>Control and decision rights<br>Tight-budget constraints<br>Rent-seeking avoidance mechanisms<br>Compensation packages<br>Severance payments<br>Anti-takeover provisions<br>Compliance risk function |
| **Owners**<br><br>One share, one vote<br>Differential voting rights<br>Pyramids and cross-holdings structures<br>Minority protection rights<br>Tunneling<br>Capital structure | **Creditors**<br><br>Control rights<br>Protective covenants in bonds and bank's loans<br>Financial hybrids and capital structure<br>Banks' influence on Boards<br>Sinking funds provisions in bonds and bank's loans |
| **Governance architecture**<br><br>Founding Charter<br>Governance Statute<br>Codes of Good Practices<br>Reorganization provisions<br>Design of accountability mechanisms<br>Transparency procedures<br>Private or public placements of securities | **Gatekeepers and regulators**<br><br>Federal or state incorporation rules<br>Design of open or closed organizations<br>Auditor independence<br>Credit risk ratings<br>Compliance risk<br>Corporate or Private Companies Laws |

Starting from a universe of **V** available companies, conveyed by the vector

$$\Gamma = [\ c_1 ; c_2 ; c_3 ; \ \ldots\ldots\ c_V ]$$

and taking into account the vector of **Q** governance variables

$$\mathbf{G} = [\ G(1), G(2), \ldots \ldots \ldots \ldots, G(Q)\ ]$$

we can define a sample space matching our purposes as the cartesian product

$$\mathbf{G} \times \Gamma = \{ \ ( G(i) ; c_j ) \ \mid \ i : 1, 2, \ldots, Q \ ; \ j : 1, 2, \ldots, V \ \}$$

Afterwards, we define a boolean-valuation function, **Bool**, from the cartesian **G** × **Γ** on the set

$$\{ \ ( a_i^{\ j} )_{Q \times V} \ \mid \ i : 1, 2, \ldots, Q \ ; \ j : 1, 2, \ldots, V \}$$

of all real matrix of **L** files by **S** columns, in the following way:

$$\mathbf{Bool} : \ \mathbf{G} \times \Gamma \ \rightarrow \ ( a_i^{\ j} )_{Q \times V}$$

such that

$$\mathbf{Bool}\, [\ ( G(i) ; c_j )\ ] \ = \ ( \delta_i^{\ j} )_{Q \times V}$$

where [16]

[16] Such a matrix is boolean, and its coefficientes become Kronecker's deltas.

$$\mathbf{a}_{\mathbf{i}}^{\mathbf{j}} = \boldsymbol{\delta}_{\mathbf{i}}^{\mathbf{j}} = \begin{cases} 1 & \text{if company } \mathbf{j} \text{ is responsive to variable } \mathbf{i} \\ 0 & \text{if company } \mathbf{j} \text{ is non-responsive to variable } \mathbf{i} \end{cases}$$

Hence, from the sample space stems a matrix of coefficients, whose files stand for governance variables, and columns for companies, as shown below.

$$(\boldsymbol{\delta}_{\mathbf{i}}^{\mathbf{j}})_{\mathbf{Q}\times\mathbf{V}} = \begin{pmatrix} \delta_1^1 & \delta_1^2 & \delta_1^3 & \dots\dots\dots \delta_1^V \\ \delta_2^1 & \delta_2^2 & \delta_2^3 & \dots\dots\dots \delta_2^V \\ \delta_3^1 & \delta_3^2 & \delta_3^3 & \dots\dots\dots \delta_3^V \\ \dots\dots\dots\dots\dots & & & \dots\dots\dots\dots \\ \dots\dots\dots\dots.. & & & \dots\dots\dots\dots \\ \delta_Q^1 & \delta_Q^2 & \delta_Q^3 & \dots\dots\dots \delta_Q^V \end{pmatrix}$$

Being responsive for the company $\mathbf{c}_{\mathbf{j}}$ to the variable **i**, means at least three things:

a) the variable becomes related to the company's governance in a distinctive way;
b) we can ascertain whether the company is performing well or badly, regarding that variable;
c) if the company $\mathbf{c}_{\mathbf{j}}$ is unrelated to certain variable **i**, then there is no responsiveness and $\boldsymbol{\delta}_{\mathbf{i}}^{\mathbf{j}}$ is zero.

We are going to take advantage of this matrix to set up the weighting system, by means of the cardinal number for the following finite set[17]:

$$\# \{\textbf{File ( h )}\} = \# \{ \boldsymbol{\delta}_{\mathbf{h}}^{\mathbf{j}} = \mathbf{1} \;;\; \mathbf{j: 1, 2, \dots , V} \}$$

that is to say, we count the number of non-zero elements in such file.

Lastly, we compute each weight, for any governance variable **h**, by solving

$$\mathbf{w(i)} = \# \{ \textbf{File ( i )} \} \;/\; \sum \# \{ \textbf{File ( h )} \;;\; \mathbf{h: 1, 2, \dots , Q} \}$$

[17] For ease of notation, we follow the widely used symbol # {**A**}, that stands for "the cardinal number of the set **A**", where **A** is a finite set. Bloch (2000) enlarges upon this subject matter by means of a basic and readable framework of analysis.