AGGREGATION OF UNDERWRITING RISKS IN INSURANCE INDUSTRY OF IRAN USING VINE COPULA

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Abstract

In this paper, the underwriting risks of the insurance industry of Iran were aggregated using various vine copula classes and historical data of loss ratios which corresponds to each business line. The estimated economic capital (EC) for the entire insurance industry considerably varies across different risk measures and vine copula models. In addition, less than the risk-based capital (RBC) charge assessed based on the standard model of RN69 and amounted to 96,943,391 million of Iran Rials. Therefore, it was concluded that using the Vine copula method and allowing symmetry and tail dependence for pairs of business lines' risks in the risk aggregation process leads to overestimation of the RBC risk charge, as compared to the estimated results of simple and linear aggregation methods of such standard model. Furthermore, the choice of dependency structure and risk measures have a paramount effect on the aggregate economic capital. Highlights: Estimated aggregated economic capital varies across different risk measures and vine copula models; Selecting the appropriate copula model is an important consideration in risk aggregation process; Using the Vine copula method in the risk aggregation leads to overestimation of the RBC risk charge; The estimated economic capital is less than RBC risk charge calculated under standard model of RN69.

Keywords: Dependency Structure, Risks Aggregation, Underwriting Risks, Vine Copulas, Insurance Industry, Iran

JEL codes: G22, C4

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1. Introduction

Insurance companies are faced with a multitude of risks in their underwriting business and investment activities. Regardless of identifying, assessing and classifying the risks in insurance firms and finding the factors affecting the risks in each business line of an insurance firm, modeling the dependency structures of risks is crucial to risk management and most notable for the risk aggregation process. The purpose of risk aggregation is to calculate the aggregated economic capital[9]. In addition, the process of risk aggregation involves risk measurement and aggregation.

There are different methods of risk aggregation. The traditional method is the linear risk aggregation[10]. The new progressive method is based on the Copula models. In linear risk aggregation, the only data required is the estimate of each risks' economic capital and the correlation between pairs of risks[11]. The Copula methods of aggregation depend on the joint distribution of all individual risks. Copula is defined as a function that joins univariate distribution to a multivariate distribution function, denoted by C in the following equation for random variables $(X_1, \cdots, X_d)$ (Joe, 1997; Nelsen, 2006):

[9] There are several different interpretations of the concept of economic capital (EC) in the finance literature. In the solvency view or the regulatory-type view EC is defined as the sufficient surplus to support the solvency at a given level of risk tolerance, over a specified time horizon. For more details, see Mueller and Siberon (2004).

[10] The simple summation and variance-covariance matrixapproaches are known as the linear risk aggregation.

[11] Note that the simple summation method is a particular case of the variance-covariance matrix approach when all correlation coefficients set to 100%. This approach does not allow for diversification and is known to yield usually very conservative economic capital.

$$\Pr(X_1 \leq x_1, \cdots, X_d \leq x_d) = \mathrm{C}(\Pr(X_1 \leq x_1), \cdots, \Pr(X_d \leq x_d)) \quad (1)$$

Based on this definition, it is known that estimation of the joint distribution of risks is not independent of the specified marginal distributions and implemented copula models. Thus, selecting the appropriate copula model is an important consideration in risk aggregation. Linear risk aggregation is simple and convenient to use, but this method can lead to overestimating the aggregated economic capital[12]. For this reason, the recent use of Copula models as an alternative approach in risk aggregation is expanding in financial studies.

Tang and Valdez (2009) compared the copula method with linear risk aggregation approaches, thereby, documenting how the latter may overestimate total risk. Shi and Frees (2011) used some copula models instead of the chain-ladder method to determine the loss reserves. Savelli and Clemente (2011) also applied the Hierarchical structural method in risk aggregation and quantified the capital required for premium risk for multi-line non-life insurance companies. Czado et al. (2012) used mixed Gaussian copula approach to model the dependency between the number of claims and its corresponding average claim size of car insurance policies in Germany. The study revealed a significant small positive dependency between the average claim size and the number of claims. Guegan and Jouad (2012) in their research also applied different pair-copula models for aggregating market risks[13]. Diers et al. (2012) used the Bernstein copula for calibrating the claims data on storm, flood, and water damage insurance in Germany. Wang (2013) considered both multi-year (temporal) dependencies and dependencies among lines of business in the multi-year, and multi-line reinsurance contract. In his study, copulas were used to show that assumption of temporal independence in modeling the distribution of the underlying loss variables, potentially led to a significant under-estimation of the risk embedded in the reinsurance product. Brechmann (2013) modeled the Systemic and Operational Risk using a hierarchical approach called the "hierarchical Kendall copula". Brechmann and Czado (2013) in their study used Vine copulas to analyze the Euro Stoxx 50 index, as it is a major market indicator for the Eurozone and highlighted the use and effectiveness of vine copulas in financial risk management. Belkacem (2014) modelled the dependence of two lines of business; Auto Damage and Auto Liability, using the Archimedean copulas and their survival copulas to evaluate the solvency capital required (SCR) for a Tunisian insurance company. The aggregate, SCR of the internal model proposed by Solvency II is lower than the assessed SCR using the copula model and suggested retaining the internal model. Yoshiba (2015) utilized several parametric copulas for enterprise risk management and risk aggregations in the banking industry, especially for the market and credit portfolios.

As earlier mentioned, the empirical studies utilizing the copula method in risk aggregation are numerous. This research focused on the portfolio of underwriting risks and estimated the corresponding economical capital requirement for the entire insurance industry of Iran, using various types of Vine copula models.

In the classification of IAA[14], underwriting risk is one of the main categories of risks that insurance companies face in their insurance business activities[15]. Further, this category of risks could have a significant effect on the activities of business investment insured (Zou et al., 2012) and on the insurance cycles (Jakovčević and Žaja, 2014). Therefore, managing this category of insurers' risks is fundamentally important for insurance companies.

Using the vine copula provides us with the model complex dependency patterns by benefiting from the rich variety of bivariate copulas as building blocks of vine copula. However, there are some serious weaknesses of copula particularly, concerning the recognition of extreme events, such as natural disasters and terrorist attacks. Nevertheless, the modeling dependencies with copulas would incur significant cost for smaller companies (Nguyen and Molinari, 2011).

As mentioned earlier, modeling is the dependent structure of underwriting the risk of insurance industry in which Vine copula models has several advantages. Specially, R-vine copula models the complex dependency of larger numbers of dimensions of risk. Hence, it allows estimation of the total economical capital for the entire insurance industry more accurately than other parametric copulas such as the elliptical and Archimedean copula models, which are even more accurate than the linear risk aggregation method.

The Central Insurance of Iran as the sole regulatory authority proposed the Regulation No. 69 (RN69) on insurers to assess the solvency capital required to determine the solvency margin ratio (SMR) since February 15th, 2012. The structure of this regulation is based on the linear risk aggregation method (Safari, 2014). Thus, the results of this research would be useful to future regulatory decisions.

There are some rational explanations for using the entire insurance industry as the data sample for this research. Some of the insurance companies are active in a few lines of business, while others

[12] In the linear risk aggregation approach it is implicitly assumed that the aggregated risk has the same quantiles of the individual risks. However, this assumption is satisfied only if the aggregated risk and individual risks come from the same elliptic density family (See Rosenberg and Schuermann [2004]).

[13] Market risk is the risk that the value of an investment will decrease due to moves in market factors. The five main market sub- risks considered in the Guegan and Jouad,s study are Equity, Interest rates, Spread, Foreign Exchange and Implied Volatility.

[14] International Actuarial Association

[15] Investment risks are another one main categories of risks in the IAA classification. And Zou et al [2012]) examined the association between underwriting risks and investment risks.

established previous years ago began some of their insurance business activities early. The aim of this research is to model the dependency structure of all lines of insurance business activities and estimate the economic capital for the whole underwriting risks.

To achieve this end, the historical data of loss ratios corresponding to each business line was used to model the dependency structure between different business lines of the insurance industry with different classes of vine copula. The Economic capital for the entire Iranian insurance industry was calculated using 1000 simulated data of loss ratio based on the specified vine copula models and the risk measures value at risk (VaR) and expected shortfall (ES). Furthermore, total risk-based capital (RBC) of the industry is based on the RN69 model assessed to explore the advantages of using the vine copula model instead of the RN69 method which represented the diversification effect. To estimate the parameters of vine copula model and simulate the aggregated loss ratios, statistical computing software R 3.1.2 (which is freely accessible from an online website) was used.

2. Analytical methods

The concept of copula was introduced by Sklar (1959), (in his seminal paper for the first time. Copula captures the dependence among n-variables, irrespective of their marginal distributions and utilizes the copula for risk aggregation. Modeling the multivariate distribution function of a set of risks $\{X_1, \cdots, X_n\}$ is necessary for the purpose of risk aggregation. Based on the first theorem of Sklarwe, one can construct the multivariate cumulative distribution function (cdf) F(X) from the univariate cumulative distribution function $F_1, \cdots, F_n$ and an n-variate arbitrarily chosen copula C. That is:

$$F(X) = C(F_1(x_1), \cdots, F_n(x_n)) \quad (2)$$

Consequently, the joint distribution of risks $X_1, \cdots, X_n$ are not independent to the implemented copula(C) and specified margins $F_1(x_1), \cdots, F_n(x_n)$. This means that selecting the appropriate copula model is of crucial importance in the process of risk aggregation and estimating the total economic capital. For this purpose, Brechmann (2013) in his study suggested that four (4) key characteristics; heterogeneous pairwise dependence, tail dependence, interpretable parameters and computational tractability should take into account, the selection of the ideal copula model practically. For the purpose of risks aggregation, various copulas have been used in the empirical research. The most famous parametric copula models are Elliptical copulas such as Gaussian copulas, t-Student copulas, and Archimedean copulas model such as Gumbel, Frank, Clayton and Joe copula models, Hierarchical Archimedean Copula (HAC) models and Vine copula models. Table 1 shows some famous parametric bi-variate copulas and their rank correlation.

Elliptical copulas are unable to model financial asymmetries. They are generally applied to symmetric distributions (Patton, 2009) and can be easily extended to a higher dimension. Although, Archimedean copulas are not satisfactory in the description of multi-variate dependence in dimensions higher than two (Joe, 1996). This class of copulas is symmetric with respect to the permutation of their random uniform variable and therefore suffers from a very limited dependence structure[16]. Although, the hierarchical Archimedean copulas (HAC) prevail over restriction of exchangeability in the Archimedean copulas class, there are some drawbacks to the HAC. It is presupposed that in the HAC, the within-group dependence should be homogeneous and between-group dependence should be lower than the within-group dependence (Savu and Trede, 2010)[17].

In this research, Vine copula was used as an efficient technique for describing and analyzing the multivariate dependence as it is widely used in risk aggregation since the last decade.

2.1 Vine copula

Multivariate Vine copulas[18] or ''pair of copula constructions''[19] are constructed by sequentially applying bi-variate copulas to build up a higher dimension copula. Therefore with a vine copula, it is possible to model complex dependency patterns by benefiting from the rich variety of bi-variate copulas as building blocks (Joe, 1996).

Considering the random set of variables $X = \{X_1, \cdots, X_n\}$ with joint probability distribution function (pdf) $f(x_1, \cdots, x_n)$ and marginal function pdf's $f_i(x_i)$ for $i = 1, 2, \ldots, n$. Then, the joint distribution function of these random variables can be decomposed into terms of conditional distributions and the marginal function pdf's to be:

$$f(x_1, \cdots, x_n) = \left[\prod_{t=2}^{n} f(x_t | x_1, \cdots, x_{t-1})\right] \cdot f_1(x_1) \quad (3)$$

By Sklar's theorem in Equation (3) the component $f(x_t | x_1, \cdots, x_{t-1})$ can be expressed as[20]:

[16] Because in the Archimedean copulas assumed that the multi-variate dependency structure depends on a single parameter of its generator function.

[17] In addition, to fully describe the complexity of the observed dependence, it could be necessary to use hierarchical copulas with more complexities due to identification of the kind and the sort of the structure.

[18] Vine copulas initially proposed by Joe 1996

[19] The pair copula constructions (PCCs) was introduced by Aas et al. (2009).

[20] For bivariate X_1 and X_2 that is: $f(x_1, x_1) = c_{1,2}\,(F_1(x_1), F_2(x_2)) \cdot f_1(x_1) \cdot f_2(x_2)$

$$f(x_t|x_1,\cdots,x_{t-1}) = c_{1,t|2,\cdots,t-1} \times f(x_t|x_2,\cdots,x_{t-1}) = \left[\prod_{s=1}^{t-2} c_{s,t|s+1,\cdots,t-1}\right] \times c_{(t-1),t} \times f_t(x_t) \tag{4}$$

Where $c_{s,t|s+1,\cdots,t-1}$ denotes the conditional density function of bivariate copula with uniformly distributed marginal u_s and u_t correspond to variables X_s and X_t respectively. Using four (4) and $s = i$, $t = i + j$ it follows that:

$$f(x_1,\cdots,x_n) = \left[\prod_{j=1}^{n-1}\prod_{i=1}^{n-j} c_{i,(i+j)|(i+1),\dots,(i+j-1)}\right] \cdot \left[\prod^{nk=1} f_k(x_k)\right] \tag{5}$$

Decomposition of Equation (5) is called pair copula decomposition (PCC). Bedford and Cooke (2001, 2002) called such pair copula constructions as the regular vine copulas since the dependency structure can be visualized as resembling a grapevine. In addition, Kurowicka and Cooke (2007) classified vine copulas into three sub-classes; R-vine, D-vine and C-vine copulas. There are $\binom{n}{2} \times (n-2)! \times 2^{\binom{n-2}{2}}$ number of possible ways to decompose the n-dimensional density functions Equation (6) to the components pair copula constructions and marginal distributions. The number of C- vines and D-vines are equal to $n!/2$ and the reminders decomposition refers to R-vines (Morales-Nápoles et al., 2010; Aas et al., 2009).

Table 1. Some famous parametric bi-variate copulas and their rank correlation

Name	Parameter range	Kendall's τ	Tail dependence (lower, upper)
Gaussian	$\rho \in (-1,1)$	$\frac{2}{\pi}\arcsin(\rho)$	$(0,0)$
t-student	$\rho \in (-1,1), v > 2$	$\frac{2}{\pi}\arcsin(\rho)$	$2t_{v+1}\left(-\sqrt{v+1}\sqrt{\frac{1-\rho}{1+\rho}}\right)$
Clayton	$\theta > 0$	$\frac{\theta}{\theta+2}$	$\left(2^{\frac{-1}{\theta}},0\right)$
Gumbel	$\theta \geq 1$	$1-\frac{1}{\theta}$	$\left(0,2-2^{\frac{1}{\theta}}\right)$
Frank	$\theta \in \mathbb{R}\{0\}$	$1-\frac{4}{\theta}+4\frac{D_1(\theta)}{\theta}$	$(0,0)$
Joe	$\theta > 1$	$1+\frac{4}{\theta^2}\int_0^1 t\log(t)(1-t)^{2(1-\theta)/\theta}.dt$	$\left(0,2-2^{\frac{1}{\theta}}\right)$
BB1	$\theta > 0, \delta \geq 1$	$1-\frac{2}{\delta(\theta+2)}$	$\left(2^{\frac{-1}{\theta\delta}},2-2^{\frac{1}{\delta}}\right)$
BB6	$\theta \geq 1, \delta \geq 1$	$1+\frac{4}{\theta\delta}\int_0^1\{-\log\left(1-(1-t)^{\theta}\right)\times(1-t)\left(1-(1-t)^{-\theta}\right)\}.dt$	$\left(0,2-2^{\frac{1}{\theta\delta}}\right)$
BB7	$\theta \geq 1, \delta > 0$	$1+\frac{4}{\theta\delta}\int_0^1\{-\left(1-(1-t)^{\theta}\right)^{\delta+1}\times\frac{\left(1-(1-t)^{\theta}\right)^{-\delta}-1}{(1-t)^{\theta-1}}\}.dt$	$\left(2^{\frac{-1}{\delta}},2-2^{\frac{1}{\theta}}\right)$
BB8	$\theta \geq 1, \delta \in (0,1]$	$1+\frac{4}{\theta\delta}\int_0^1\{-\log\left(\frac{(1-\delta t)^{\theta}-1}{(1-\delta)^{\theta}-1}\right)\times(1-t\delta)\left(1-(1-t\delta)^{-\theta}\right)\}.dt$	$(0,0)$

Source: Allen et al. (2014)

An n-dimensional vine tree structure is a sequence of n-1 trees. Tree j has n+1-j nodes and n-j edges and Edges in tree j become nodes in tree j+1. The density of a regular vine distribution is defined by the product of pair copula densities over the $n(n-1)/2$ edges identified by the regular vine tree structure and the product of the marginal densities. Canonical vine distributions are regular vine distribution for which each tree has a unique node connected to $n - j$ edges. D-vine distributions are regular vine distributions for which no node in any tree is connected to more than two edges. It should, however, be noted that C- and D-vine copulas are most appropriate if their structure is explicitly motivated by the data. In particular, C-vine copulas may be used if there is a set of pivotal variables such as stock indices and D-vine copulas are particularly attractive to model variables with temporal order (Jaworski et al., 2013).

The full specification of a vine model requires the choice of a vine tree structure, the copula families for each pair copula term and their corresponding parameters[21]. Morales-Nápoles et al. (2010) and Dissmann et al. (2013) specified the R-Vine copula in matrix notation. One matrix contains the R-vine tree structure, one the copula families utilized and two matrices corresponding to parameter values.

To estimate parameters of the R-vine distribution ($f(x_1,\cdots,x_n)$ in Equation 6) for a given R-vine tree structure and bivariate copula families for each pair copula term, an i.i.d. sample from the R-vine distribution is required. Given this multivariate

[21] To specify the D-vine tree structures, the order of the variables in the first tree has to be chosen and for the C-vine the root nodes for each tree need to be determined.

sample, there is the need to estimate marginal and copula parameters. The marginal parameters are either estimated using parametric (IFM[22]) or non-parametric (MPL[23]) approach. To estimate the parameters of pair copulas with known tree structure, there are several methods available. The first is the sequential estimation method suggested by Aas et al. (2009) and further studied in detail by HobakHaff (2012). Maximum likelihood was also discussed in the research of Aas et al. (2009) and Bayesian estimation for D-vine copula in the study of Min and Czado (2010). For general R-vine copula Gruber and Czado(2013) gave a Bayesian estimation approach which estimated the tree structure, pair copulae and their parameters starting with Tree 1 to Tree d-1 (Czado et al., 2013). In this paper, parameters of vine copulas through maximum likelihood estimation method were estimated using Package 'VineCopula' in R.

3. Results and discussion

3.1 Data

Loss ratio (LR) is defined as the ratio of the gross incurred claims to earned premium. This index obviously represents the magnitude of the underwriting loss from each business line (Adams and Buckle, 2003; Tang and Valdez, 2009). Here, the loss ratio as a proxy of underwriting risk was used. However, use of loss ratio as a measure of underwriting risk has been widely criticized. Zou et al. (2012) emphasized that loss ratio did not capture the underwriting expenses and therefore, expense ratio was added to loss ratio for measuring underwriting risk. Jakovčević and Žaja (2014) argued that change in reserves for claims and the costs of loss liquidation are not taken into account in the loss ratio index.

The historical data of earned premium and incurred claims in each business line of the insurance industry of Iran are accessible from the website of the Central Insurance of Iran (http://www.centinsur.ir/). An annual sample of these data was used to calculate the historical aggregate loss ratio of the insurance industry from 1975-2013, according to Equation (6) (Tang and Valdez, 2009). :

$$LR_t = \frac{\sum_{i=1}^{n} IC_{i,t}}{\sum_{i=1}^{n} EP_{i,t}} = \sum_{i=1}^{n} LR_{i,t} * \frac{EP_{i,t}}{\sum_{i=1}^{n} EP_{i,t}} = \sum_{i=1}^{n} LR_{i,t} * w_{i,t} \quad (6)$$

Where $LR_{i,t}$ is loss ratio of line i during period t, and there are n lines of business in total; and $IC_{i,t}$ and $EP_{i,t}$ denote the incurred claims and earned premium from line i during period t respectively; and $w_{i,t} = \frac{EP_{i,t}}{\sum_{i=1}^{n} EP_{i,t}}$ represents the weight of line i in period t by earned premium.

Table 2 presents the status of the insurance industry of Iran in the year 2013. There are sixteen (16) different lines of business in total and the corresponding underwriting risks are denoted by X1, X2… X16. As can be seen in Table 2, loss ratio varies across business lines from 0.0514 for line "Aircraft insurance" to 1.5286 for line "Credit insurance". Premium income of line "Third-party insurance and excess" and line "Premium treatment" and line "Credit insurance" were less than the corresponding claim payments. Furthermore, the line "Third-party" and line "Premium treatment" have the highest percentage of premium income earned by the industry (about 40 and 23%, respectively). However, the sum of the premium income is greater than the sum of the claim payment in the industry and thus, the aggregated loss ratio for the entire industry is less than one that is about 89.6%.

3.2. Modelling the dependency structure

Here, it is essential to estimate the marginal distributions of each business line's loss ratio before modelling the dependency structures. The maximum pseudo likelihood (MPL) approach introduced by Genest et al. (1995) was hereby used non-parametrically to estimate the marginal distributions of each risk $\{X1, X2, \ldots, X16\}$. The kernel density of underwriting risks corresponds to each business line of the insurance industry plotted in Figure 6 as shown in the appendix. From Figure 6, it is evident that the distributional behaviour of each business line differs from one another, owing to the variety of the risks that covered each business line of the insurance industry. In addition, the scatter plots of the pair's risk data (Figure 6) in the appendix gave a clear demonstration of simple dependency between pair's business lines intuitively.

The results of modeling the dependency structure of lines in the insurance industry of Iran through the vine copula approach are shown in Tables 3, 4 and 5. These tables as well as the Figures 1, 2 and 3 enable comparison of the dependency structure of business lines under different class of Vine copula models. Table 3 gives the estimated results of modeling dependency between different lines of business with regular vine (R-Vines) copulas, canonical vine (C-Vines) and drawable vines (D-Vines) copulas, respectively (Tables 4 and 5). It is noticeable that these tables provide only the first fifteen trees of specified dependency structure. The estimated results correspond with other trees of dependency structures which are not reported here. Tables 3 to 5 consist of selected pair copula families, corresponding parameters and pairwise dependence measure such as Kendall's τ [24] with respect to AIC criteria.

[22]inference functions for margin(IFM)
[23]maximum pseudo likelihood (MPL)
[24]There are other pairwise dependence measure such as the empirical tail dependence coefficient (λ)

Table 2. Business lines of the insurance industry and their loss ratio in 2013

Line of Business	Risk name	Premiums Income	Market Share	Claim Payment	Loss Ratio
Third-party insurance and excess	X1	51,745,766.2	0.4028	53,523,386.1	1.0344
Premium treatment	X2	30,500,850.9	0.2374	35,801,123.0	1.1738
The body of the car insurance	X3	10,287,407.1	0.0801	6,612,351.3	0.6428
Liability insurance	X4	8,323,430.5	0.0648	6,834,766.5	0.8211
Driver accident insurance	X5	6,085,592.3	0.0474	2,781,556.9	0.4571
Fire insurance	X6	5,880,291.9	0.0458	2,457,451.1	0.4179
Life insurance	X7	4,639,439.2	0.0361	3,322,801.3	0.7162
Oil and energy insurance	X8	2,047,876.5	0.0159	675,153.8	0.3297
Engineering insurance	X9	1,984,672.4	0.0154	980,214.7	0.4939
Accident insurance	X10	1,898,603.9	0.0148	656,572.3	0.3458
Ship insurance	X11	1,726,179.4	0.0134	683,090.9	0.3957
Cargo insurance	X12	1,598,648.8	0.0124	369,907.8	0.2314
Aircraft insurance	X13	1,349,454.1	0.0105	69,414.7	0.0514
Credit insurance	X14	202,878.2	0.0016	310,116.6	1.5286
Money insurance	X15	103,186.8	0.0008	20,014.7	0.1940
Other types of insurance	X16	88,066.8	0.0007	25,425.6	0.2887
Total insurance industry		128,462,344.7	1	83,027,498.2	0.8962

Note: numbers in columns 3 and 5 of the table are given in units of million Rials of Iran.

Table 3. Estimation results of dependency structure of risks with R-Vine copulas

Risks names	Bivariate copulas	Parameter(s)	Kendall's τ	AIC criteria
X1, X14	Clayton	1.71	0.46	-19.752
X14, X4	Joe	108.8	0.98	-215.349
X13, X5	Gaussian	-0.46	-0.31	-4.470
X5, X4	Joe-Clayton	2.22, 2.12	0.61	-35.435
X3, X4	Clayton	1.23	0.38	-13.158
X2, X4	Clayton	2.74	0.58	-37.321
X4, X8	Frank	10.03	0.67	-43.278
X16, X12	Clayton	1	0.33	-10.927
X9, X12	Frank	3.67	0.36	-8.698
X12, X10	Frank	-4.6	-0.43	-15.267
X6, X10	Rotated Clayton 90 degrees	-0.91	-0.31	-8.738
X10, X8	Frank	-3.38	-0.34	-6.285
X8, X11	Clayton	0.9	0.31	-4.719
X7, X15	Independence	---	---	---
X15, X11	Gaussian	0.42	0.28	-3.863

Table 4. Estimation results of dependency structure of risks with C-Vine copulas

Risks names	Bivariate copulas	Parameter(s)	Kendall's τ	AIC criteria
X6, X8	Frank	2.49	0.26	-2.470
X15, X8	Clayton	0.87	0.3	-4.334
X3, X8	Frank	3.65	0.36	-7.635
X13, X8	Independence	---	---	0.000
X4, X8	Frank	10.03	0.67	-43.278
X12, X8	Independence	---	---	0.000
X1, X8	Clayton	0.84	0.3	-3.998
X7, X8	Independence	---	---	0.000
X10, X8	Frank	-3.38	-0.34	-6.285
X11, X8	Clayton	0.9	0.31	-4.719
X9, X8	Frank	2.25	0.24	-1.934
X2, X8	Clayton	2.22	0.53	-23.379
X5, X8	Clayton	1.67	0.46	-14.717
X14, X8	Survival Gumbel	2.69	0.63	-33.119
X16, X8	Independence	---	---	0.000

Table 5. Estimation results of dependency structure of risks with D-Vine copulas

Risks names	Bivariate copulas	Parameter(s)	Kendall's τ	AIC criteria
X16, X15	Survival Joe	1.23	0.12	0.672
X15, X14	Clayton	0.67	0.25	-2.952
X14, X13	Rotated Gumbel 90 degrees	-1.23	-0.19	-1.694
X13, X12	Gaussian	-0.17	-0.11	1.158
X12, X11	Survival Clayton	0.23	0.11	1.162
X11, X10	Rotated Clayton 270 degrees	-1	-0.33	-9.978
X10, X9	Frank	-2.88	-0.3	-5.803
X9, X8	Frank	2.25	0.24	-1.934
X8, X7	Clayton	0.52	0.21	-0.485
X7, X6	t-student	0.29, 2.03	0.19	-2.379
X6, X5	Joe	1.39	0.18	-1.984
X5, X4	Joe-Clayton	2.22, 2.12	0.61	-35.435
X4, X3	Clayton	1.23	0.38	-13.158
X3, X2	Clayton	0.64	0.24	-2.587
X2, X1	Survival Joe	1.65	0.27	-3.470

Figures 1, 2 and 3 illustrate the dependency structure of business lines (risks) graphically. These figures represent the firs tree of the specified R-Vine, C-Vine and D-Vine for available data sample with pair-copula families and empirical Kendall's τ values corresponding to pair-copula parameters as edge labels.

Figure 1. Dependency structure of risks modelled with R-Vine

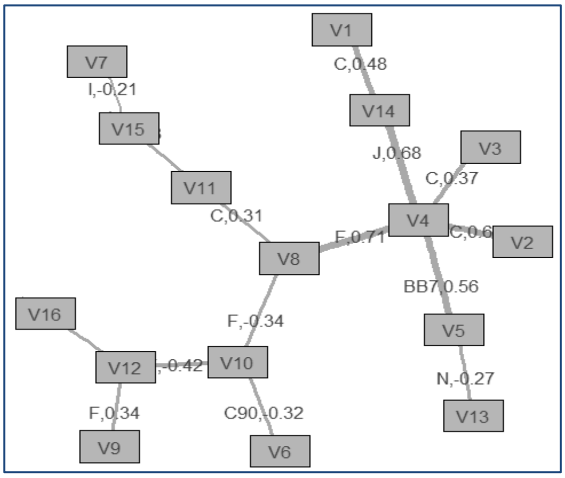


Figure 2. Dependency structure of risks modelled with C-Vine.

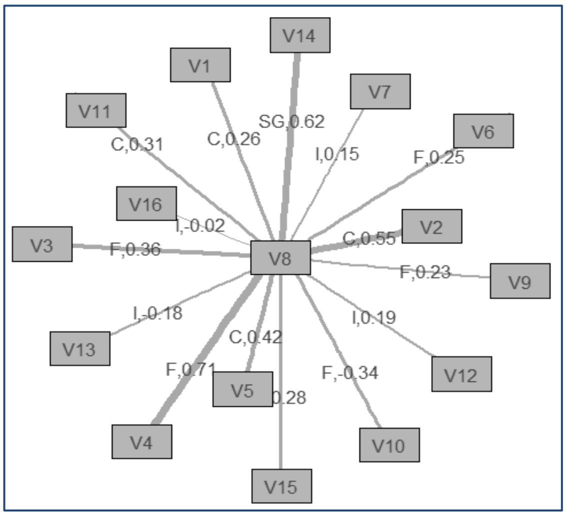
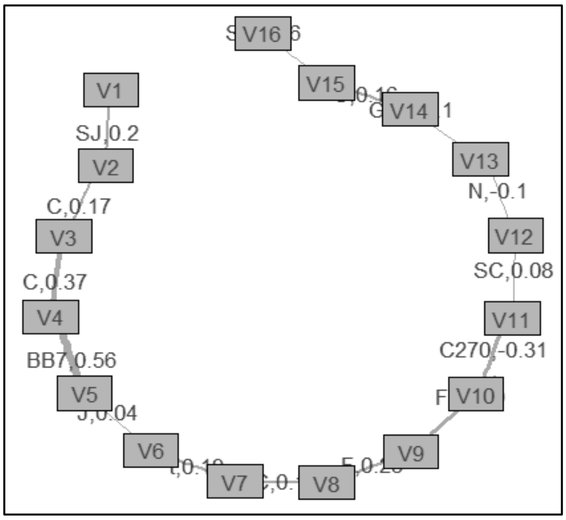


Figure 3. Dependency structure of risks modelled with D-Vine



It is apparent from Figure 2 and Table 4 that business line "Oil and energy insurance" is the root node of C-Vine copula dependency structure; however, its share in the total premium incomes is about 1.5%.

3.3 Vine Copula selection

The log likelihoods obtained by maximum likelihood estimation, Schwarz's Bayesian information criterion (BIC) and Akaike information criterion (AIC) with respect to R-Vine, C-Vine and the D-vine copula models are shown in Table 6. Based on the results of this table, it appears that the R-Vine copula model best matches the loss ratio data set.

Table 6. Compare the log- likelihood, AIC and BIC criteria to select the best model

Copula model	AIC	BIC	Log likelihood
R-Vine	-488.5774	-441.9977	272.2887
C-Vine	-288.7615	-235.5275	176.3808
D-Vine	-268.5802	-62.29856	258.2901

Besides the classical AIC and BIC criteria, Vuong (2007) and Clarke (1989) tests allowed pairwise comparison of three competing models; R-Vine, C-Vine and D-vine. Below, Table 7 summarizes the Vuong and Clarke tests to determine the best model. The large p values corresponding to the Vuong test indicate that pair models cannot be distinguished statistically. According to the Clarke Test's results, model R-Vine is preferred to both C-Vine and D-Vine models, whereas, C-Vine and D-Vine models are statistically equivalent.

Table 7. Vuong and the Clarke tests to select the models

Pairwise model	Vuong Test statistics	Clarke Test statistics
R-Vine, C-Vine	0.0909 (0.927)	5 (0.000)
R-Vine, D-Vine	0.0132 (0.989)	5 (0.000)
C-Vine, D-Vine	1.4435 (0.148)	23 (0.336)

Note: numbers in parentheses refer to p-value of test statistics. The null hypothesis shows that both models are statistically indistinguishable.

3.4 Aggregated loss ratio

To obtain the aggregated loss ratio, sample size of 1000 data of loss ratio was simulated for each business line from the specified R-Vine, C-Vine and D-vine copula model using the R programming language and software (see algorithms of simulating in Aas et al. [2009]). Based on Equation 1, the simulated loss ratios were aggregated. That is, data of simulated loss ratio for each business line multiplied by corresponding weight of earned premium. Figure 4 shows the overlap of the density functions of a sample of 1000 data of aggregated loss ratio obtained by R-Vine, C-Vine and D-vine copula model.

Figure 4. Aggregated Loss Ratio Distribution with R-Vine, C-Vine and D-Vine Copulas

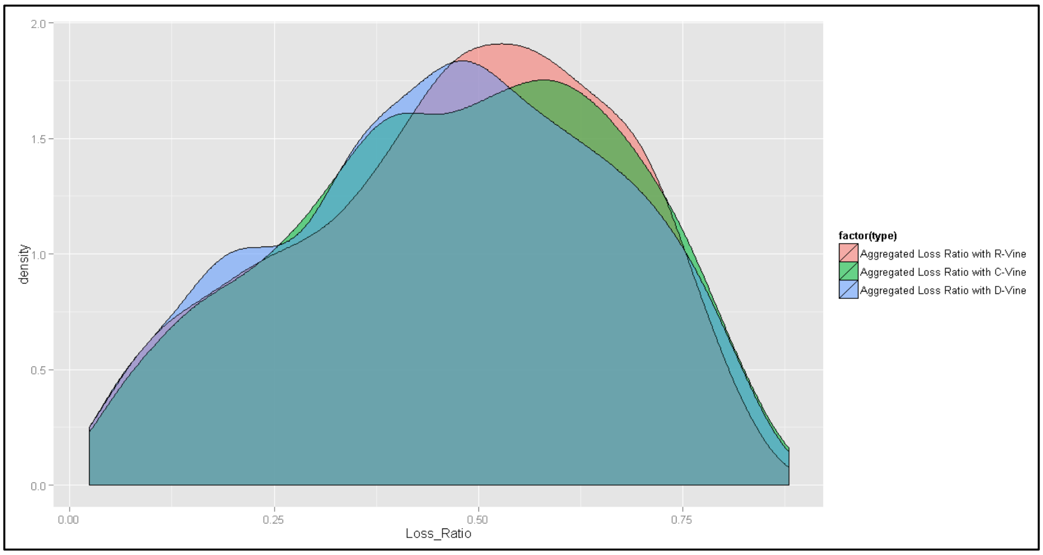


Table 8 provides a comparison of distribution for the aggregated loss ratio simulated with each copula model. As perceived from this table and observed from Figure 4, there are differences in the resulting distributions for the different vine copula class.

Table 8. Summary Statistics of Aggregate Loss Distributions

Statistic	R-Vine	C-Vine	D-Vine
Mean	0.47	0.47	0.46
SD	0.19	0.2	0.2
Mode	0.21	0.22	0.22
Skewness	-0.33	-0.23	-0.16
Kurtosis	-0.75	-0.81	-0.81
Minimum	0.02	0.04	0.03
1st Quantile	0.29	0.29	0.31
Median	0.49	0.49	0.47
3rd Quantile	0.62	0.60	0.58
Maximum	0.85	0.86	0.88

3.5 Aggregated capital requirements

In order to quantify the aggregate capital requirements, we calculated the VaR[25] and ES[26] of the aggregated loss ratio at a confidence level of 95% in the first instance[27]. The resulting VaR 95% and ES 95% was multiplied by the net premium income of the insurance industry so as to obtain the capital charge in the next step. The confidence level (95%)was chosen arbitrarily; however, computing the economic capital requirement through VaR 99.5% and ES 99% gives the required capital by Solvency II and by the Swiss Solvency Test (SST).

Tables 9 and 10 show the aggregated capital requirement for each vine copula model and their corresponding diversification benefits quantified by the risk measures VaR and ES in the year 2013. For comparison of the result of risk aggregation with Vine copula models and risk aggregation based on the internal model, the total risk-based capital (RBC) of the insurance industry was assessed and the assessment results of the total RBC are shown in Table 7. In addition, the Diversification effect is defined as the reduction rate of the aggregated VaR or ES from the total risk-based capital (RBC) of the insurance industry based on the internal model.

As seen from Table 9, the R-Vine copula model gave the lower level of aggregated capital requirement compared to the C-Vine and D-Vine copula models. This can be explained by the fact that using R-Vine copula leads to a model dependency structure of business lines (risks) appropriately. Furthermore, the R-Vine copula model gave higher level of calculated aggregate capital requirement that corresponds to the severity of the losses using the coherent risk measure

[25]The value at risk (VaR) of the portfolio at the given confidence level $\alpha \in (0,1)$ is given by the smallest number l such that the probability that the loss L exceeds l is no larger than (1-α). Formally,
$VaR_\alpha = \inf(l \in R: P(L > l) \le 1 - \alpha) = \inf(l \in R: F_L(l)$
[26]The expected shortfall (ES) at confidence level α is the expected loss conditional on losses being greater than VaR_α. In other words, ES is the expected loss in the upper tail of the loss distribution. $ES_\alpha = \mathrm{E}(L|L > VaR_\alpha)$.
[27]Note that VaR is not a coherent risk measure in general because of dissatisfaction of the subadditivity condition of coherency, but the ES is a coherentent risk measure and reflects severity of the losses(Artzner et al, 1997).

ES; that is about 53,153,152 million Rials of Iran (Table 10). .

The amount of calculated Aggregated Capital Requirements based on Vine copula models under both risk measure is less than the calculated RBC for the entire insurance industry based on the internal model. Furthermore, the diversification effect which indicates the crude measure of the magnitude of the diversification benefits, under different risk measures and vine copula models varies considerably between the range of 45.2 to 91.7%, respectively. Thus, variation in the amount of calculated Aggregated Capital Requirements and corresponding Diversification effect with each Class of Vine Copula and with each risk measure led to the following conclusion. The choice of dependency structure and risk measures have a paramount effect on the calculated Aggregated Capital Requirements as well as on the diversification benefits for an insurer.

Table 9. Aggregated Capital Requirements based on Vine Copula models using risk measure VaR

Class of Vine Copula	VaR 95%	Aggregated Capital Requirements	Diversification Benefits	Diversification effect %
R-Vine	0.06282	8,070,594.8	88,872,796.0	91.7
C-Vine	0.20230	25,987,595.1	70,955,795.6	73.2
D-Vine	0.07498	9,632,114.0	87,311,276.8	90.1

Note: VaR 95% corresponds to the 950^{th} value of the sample of 1000 simulated aggregated loss ratio data which distribution of this sample ranked in increasing size.

Table 10. Aggregated Capital Requirements based on Vine Copula models using risk measure ES

Class of Vine Copula	ES 95%	Aggregated Capital Requirements	Diversification Benefits	Diversification effect %
R-Vine	0.41376	53,153,151.7	43,790,239.1	45.2
C-Vine	0.19784	25,414,772.6	71,528,618.2	73.8
D-Vine	0.31088	39,936,480.8	57,006,910.0	58.8

Note: ES 95% is the average sample of data in the 5% tail of the aggregated loss ratio distribution.

3.6. Risk based capital in the internal model

On the basis of Regulation No. 69, Central Insurance of Iran (RN69), assessment of total risk-based capital (RBC) of industry is 96,943,391 million of Iran Rials for year 2013 (Table 11). In the RN69, the risks of business lines such as “Liability”, “Oil and energy” and “Credit” were not included in the risk aggregation. But the risks in the business lines of “Fire”, “Engineering”, “Third-party” and “Life insurance” were included twice with different coefficients to cover the catastrophic risks such as earthquake. These are some limits of using the RN69 instructions besides linear risks aggregation problems to assess the RBC of insurance companies in Iran.

Table 11. Aggregation of underwriting risks based on internal model of RN69

	Line of Business	Premium Income	Coefficient of Premium Risk	Claims Payment	Coefficient of Claims risk	$RBC(X_i)$
	Fire insurance	5,880,292	0.172	2,457,451	0.245	1,011,410.2
	Cargo insurance	1,598,649	0.123	369,908	0.175	196,633.8
	Accident insurance	1,898,604	0.678	656,572	0.969	1,287,253.4
Car insurance	Driver accident insurance	6,085,592	0.25	2,781,557	0.358	1,521,398.1
	The body of the car insurance	10,287,407	0.309	6,612,351	0.442	3,178,808.8
	Third-party insurance and excess	51,745,766	1.127	53,523,386	1.61	86,172,651.6
	Life insurance	4,639,439	1.164	3,322,801	1.663	5,525,818.6
	Premium treatment	30,500,851	0.815	35,801,123	1.165	41,708,308.3
	Ship insurance	1,726,179	2.181	683,091	3.116	3,764,797.2
	Aircraft insurance	1,349,454	1.017	69,415	1.453	1,372,394.8
	Engineering insurance	1,984,672	1.088	980,215	1.554	2,159,323.5
	Money insurance	103,187	2.923	20,015	4.176	301,615.2
	Other types of insurance	88,067	0.684	25,426	0.977	60,237.7
Catastrophic risks	Fire insurance	5,880,292	0.58	2,457,451	0.841	3,410,569.3
	Engineering insurance	1,984,672	0.051	980,215	0.074	101,218.3
	Third-party insurance and excess	51,745,766	0.16	53,523,386	0.232	12,417,425.6
	Life insurance	4,639,439	0.13	3,322,801	0.188	624,686.6
Total risk-based capital (RBC) for year 2013	$\sqrt{\sum_{i=1}[\boldsymbol{RBC}(\boldsymbol{X_i})]^2}$					96,943,391

Note: numbers in second, third and sixth column of the table, from left to right, represents the unit of million in terms of Iran Rials (IRR). The premium income and claims payment multiplied by the corresponding coefficient risk in each line gives a great measure, that is, the RBC's of line, while the square root of total squared risks gives the total RBC of industry.

4. Conclusion

Risk is the nature of insurance business activities. To ensure solvency, insurers are required both for regulatory purposes and as a going business concern to hold capital to back their insurance liabilities. In aggregating losses from different business lines for the purpose of capital determination, insurers have traditionally either ignored the dependence structure between business lines or used simple linear correlations to model such dependence.

In this study, the underwriting risks of insurance industry of Iran was aggregated with taking account of likely dependecy among different insurance business lines. For this purpose, vine copula approach and the historical loss ratio data to model the dependency structure of underwriting risks was used. Subsequently, the capital requirement at 95% confidence level for the entire insurance industry based on the specified R-vine, C-vine and D-vine class of vine copula models using the value-at-risk (VaR) and expected shortfall (ES) risk measures was calculated. In addition, the total risk-base capital (RBC) of insurance industry assessed in accordance with the internal RN69 model for year 2013 to explore the advantage of using vine copula model instead of the internal linear risk aggregation method was represented as diversification effects.

The results in the risks aggregation process showed that estimated aggregated economic capital under different risk measures and different vine copula class vaies considerably. Futhermore, less than the amount of RBC was assessed under the internal RN69 model. Thus, based on the results of this research, it was concluded that taking account of the varity of dependency for pairs of risks in the process of risks aggregation by utilizing the R-vine copulas models led to overestimation of the economical capital of insurance firms in comparision with the estimated results of simple and linear aggregation method, such as the standard model RN69. Further, the choice of dependency structure and risk measures have a paramount effect on the aggregate economic capital and diversification benefits.

This study provides some empirical evidence supporting the importance of modeling the dependency structures in risk management, as it remains crucial to firms faced with multitude of risks in their business activities.

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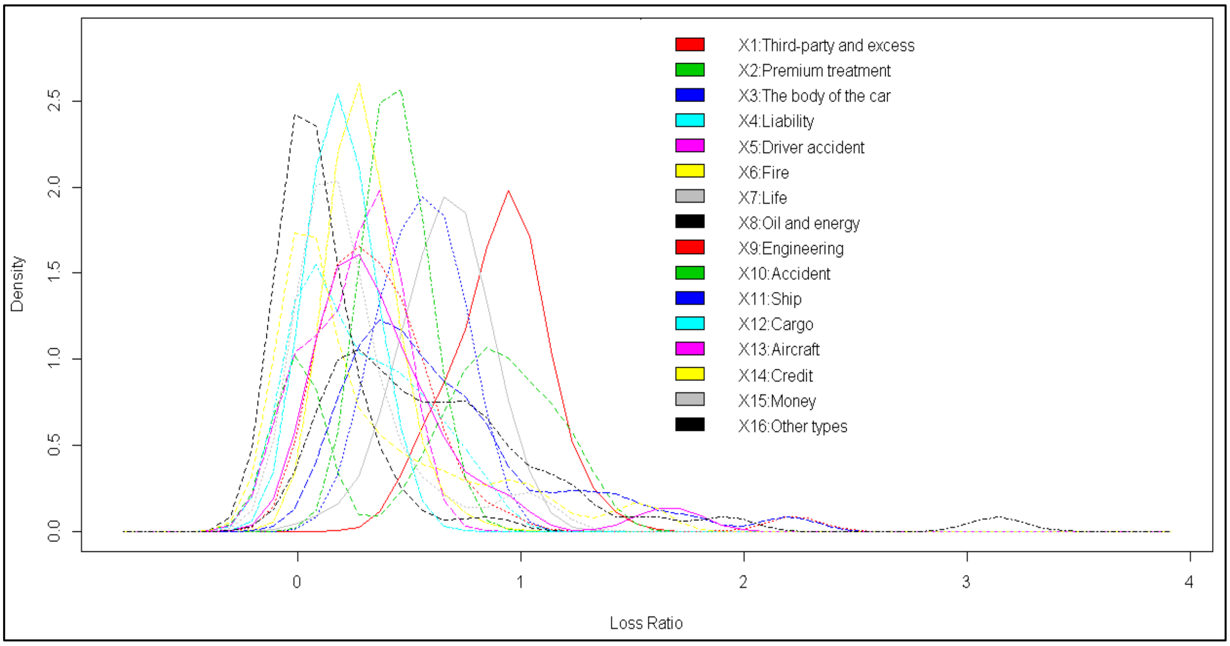
Appendix

A1. Compare the kernel densities of Loss Ratio in each line



A2. Scatter Plot of Loss Ratios