

Quorum Rules and Shareholder Voting*

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Abstract

This paper completely characterizes the equilibria of a costly voting game where shareholders of a firm may vote for or against a proposed resolution, or not vote. Two types of shareholders are considered: unconditionals who always vote and partisans who vote strategically. It is shown that the presence of a minimum quorum and of unconditionals have similar effects as they create an incentive for voting coalitions. Equilibria in favor of the resolution and against may co-exist. A high minimum quorum favors the occurrence of (1) equilibria where coalitions of small partisans of the proposed resolution vote and the resolution is adopted (2) an equilibrium against where no partisan votes. For smaller values of the minimum quorum, other equilibria for and against exist where coalitions of partisans vote. Such coalitions gather only large shareholders. Finally, the equilibrium outcome may or may not correspond to the preference of the dominant shareholder.

Keywords: General assembly, Strategic voting, Unconditionals, Quorum rule.

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1 Introduction

The annual meeting is the only regular occasion where all shareholders have the opportunity to express themselves directly on important issues regarding their company. The only other case in which they have a direct say is a tender offer, which is a rare and disruptive event in a company's life. Yet, shareholder meetings received relatively less attention than other governance mechanisms such as takeovers, boards of directors or management's compensation from legislators and academics. Still the power of shareholders may be quite important in some countries where they can sponsor resolutions in favor of the replacement of management (Charl  ty, Chevillon and Messaoudi, 2009).

Voting models developed in public economics for large electorates usually assume that each elector has one vote, and that votes are simultaneous. One objective of these models is to explain the turnout in elections. In essence, with no voting cost, all electors should vote for their preferred alternative. On the other hand, even with a very small voting cost, most people should not vote since the probability to affect the outcome in large elections is infinitesimal. Feddersen and Pesendorfer (1996) and Feddersen (2004) explain why abstention may be strategic even when voting entails no cost: if some voters are better informed about the best alternative, it is rational to delegate the choice to the better informed and abstain. They also show that it may be rational for electors to vote against their private but imperfect information when they know that other voters know what is best.

Based on a version of Feddersen and Pesendorfer (1998), Maug and Rydqvist (2009) analyze the strategic voting of shareholders in annual meetings under the following assumptions: (1) each shareholder has one vote; (2) all shareholders have the same objective, i.e. select resolutions which increase profits; (3) information is asymmetric: although they share the same initial priors, they receive a private signal; (4) the only two possibilities are to vote for or against a resolution proposed by the management. In line with Feddersen and Pesendorfer, they show that shareholders may vote against their private information; they also derive an interesting result concerning the ef-

fects of majority voting rules. More stringent majority rules (for example a majority of $2/3$ of votes rather than $1/2$ to pass a resolution) induce more shareholders to vote in favour of the resolution. Indeed, understanding that a higher majority may prohibit the adoption of a good resolution, shareholders compensate this bias by voting more often for. As a result, their model predicts that the number of votes for increases with the required majority and that the adoption rate is independent of the rule in equilibrium.

The aforementioned model assumes all stockholders have one vote, share the same objective and that voting is costless. In practise, none of these hypotheses holds. In many countries the presence of large voting blocks besides smaller ones is the rule and shareholders differ in their voting power (Becht and Roell (1999) or Becht and Mayer (2002)). Also shareholders often have conflicting interests (e.g. the State versus hedge funds or employees).

Ritzberger (2005) analyzed voting in annual meetings when stockholders with different voting shares disagree on the resolutions, some being in favour of the proposal, others against (or equivalently for the status quo). Information is symmetric and voting entails a small cost. He concludes that there exists an equilibrium if and only if the largest (or dominant) shareholder is for the resolution (assuming that when nobody votes the status quo prevails). In equilibrium, only one shareholder votes, and the resolution is adopted. The equilibrium outcome therefore always corresponds to the dominant shareholder's preference. This result is easily explained. Since voting is costly, a shareholder votes only if: (1) his vote is necessary to obtain his preferred outcome; (2) no opponent shareholder may rock the result. This can only happen when the shareholder voting for the resolution (not necessarily the largest) commands more votes than any partisan of the status quo. It should be noted that, in equilibrium, the quorum is the share of the only voter, and the resolution passes with a majority of 100%. Thus the majority rule plays no role.

In line with Ritzberger, we analyze the case where shareholders who do not agree on the relevance of resolutions, differ in their voting power, incur a (small) voting cost and vote strategically. In addition to these partisans, we assume the presence of unconditional shareholders who always vote for

or against the resolutions; or equivalently that voting is mandatory for some shareholders. Finally, as specified by law in most countries, or defined by the corporate charter, we suppose that a minimum number of shares must be present or represented (quorum rule) in assembly, and that a resolution must obtain a minimum of favourable votes (majority rule) to be adopted.

We investigate the role of the quorum rule and the presence of unconditional shareholders under these hypotheses.

Our results differ substantively from Ritzberger's. In particular, we show the existence of equilibria where: (1) coalitions of shareholders for (and against) the resolution win even though the dominant shareholder is against (for); (2) no shareholder (except the unconditionals) votes and the resolution is not adopted even though the dominant shareholder is for the resolution. Moreover, for some shareholding structures and preferences, equilibria leading to a different outcome (adoption or rejection) may coexist.

Obviously, the existence and nature of the equilibria depend on the shareholding structure and the quorum rule. Let us take an example. Suppose a company has two large shareholders in favour of a resolution, holding respectively 20% and 15% of the voting shares, one large shareholder against the resolution with 30% of the voting shares. All the remaining shares are widely held and the unconditionals altogether represent a very small part of capital. Let the minimum quorum be 25%. The situation where only the two large shareholders partisan of the resolution vote in addition to the (negligible) unconditionals is an equilibrium: if either one of the two shareholders does not vote, the minimum quorum does not hold and the resolution is not adopted; no opponent to the resolution should vote since he cannot (alone) change the outcome; and no other shareholder for should vote since he is satisfied with the outcome. However, the case where no partisan (for or against) votes and the resolution does not pass is also an equilibrium: holding less than the minimum quorum each, no shareholder for can change the outcome by voting, and the dominant shareholder who is a partisan of against is satisfied with the result. Interestingly, a low minimum quorum does not always favour the dominant shareholder when he supports against. For any minimum quorum below 20%, no equilibrium remains in the previous example so

the outcome of the assembly becomes random.

Our results suggest that the choice of the minimum quorum is quite strategic. In some countries (the United States), the quorum is defined in the corporate charter and may be modified in assemblies. In other countries it is fixed by law.¹ Respondents to consultation of the European Commission in 2007 suggested that quorums should be reduced or abolished. According to our model, this potentially means reinforcing the power of the largest blockholder by precluding coalitions of smaller shareholders.

Also, the presence of unconditionals does not amount to lowering the minimum quorum. Actually with no minimum quorum, the largest partisan's preferred alternative is selected in some equilibria when the group of unconditional shareholders *opposed* to him is sufficiently larger than the group of unconditionals on his side; a relatively large unconditional electorate destroys the incentive to vote for partisans on the same side. The asymmetry between the two groups of unconditionals matters. Mandatory voting disclosure laws for institutionals which discourage them from staying away, as they must explain why to investors, may in this respect have unexpected effects on assemblies' results.

The paper is organized as follows. The model is presented in section 2. Preliminary results and equilibrium conditions are detailed in section 3. Equilibria are characterized and discussed in section 4. Section 5 concludes.

2 The model

The main role of annual meetings is to adopt or reject resolutions on the agenda which are usually sponsored by the board of directors and sometimes by one or several shareholders. We analyze the result of corporate assemblies

¹Actually in the United States, under NASDAQ rules, companies are required to have a minimum quorum of 33 1/3% of the holders of common stock for shareholder meetings. In Europe, the minimum quorum varies across countries (no minimum quorum in Germany, 20% of capital in France for AGM, 2 shareholders regardless of the level of their aggregate share ownership in the UK...) and may be different for AGM and EGM.

in a context where unanimity among shareholders does not hold and in the presence of large voting blocks besides smaller ones.

More precisely, we assume there exist two types of shareholders:

The first group, the *unconditionals*, always vote whatever their anticipations about the outcome. Some of them (representing altogether α_U^F of voting rights) systematically vote for, while others (representing altogether α_U^A of voting rights) systematically vote against the proposed resolution. Indeed, typically some shareholders are close to the management for different reasons and support their propositions. For example, mutual funds often support management of client firms in which they have a stake.² The contrary (systematic opposition) may also exist; obviously, when resolutions are sponsored by active shareholders and directed against the management,³ those close to management vote systematically against. Unconditionals may also account for mandatory voting⁴ and the presence of ethical voters who consider voting as a duty.

Other shareholders are either *partisans* of for (F) or against (A) and vote strategically. It is often assumed that the common objective of shareholders is to adopt resolutions increasing the value of the company.⁵ Actually, even in this case shareholders may disagree on the relevance of resolutions in several instances. When a stockholder holds simultaneously shares in two business related companies, he may be favorable to a value decreasing resolution in a company that has a positive impact on the value of his holdings in the other company, while other shareholders disagree with the resolution.⁶ Other

²For example Davis and Kim (2007) document "a positive relation between business ties and the propensity to vote with management".

³In France, for example, shareholders representing at least 5% of voting rights may propose an entire new board of directors.

⁴Introducing compulsory voting for some shareholders is sometimes mentioned (see the Wall Street Journal, 9 September 2009). It formally exists in politics but takes the form of having non voters pay a fine. One could argue that not voting for institutionals imposes a cost on them since they must explain why to their clients.

⁵See for example Maug and Rydqvist (2009).

⁶Matvos and Ostrovsky (2006) show for example that "in mergers with negative acquirer announcement returns, cross-owners are significantly more likely to vote for the merger". Charléty, Fagart and Souam (2009) endogenize such private benefits in the case

types of private benefits (ethical considerations,...) may explain opposite views. Partisan shareholders incur a small voting cost and vote strategically, contrary to the unconditionals. We represent a partisan supporting K ($K = F, A$) by his share of voting rights denoted α_i^K . There are N^F partisans of F belonging to $\mathcal{P}^F = \{\alpha_1^F, \alpha_2^F, \alpha_3^F, \dots, \alpha_{N^F}^F\}$ and N^A partisans of A belonging to $\mathcal{P}^A = \{\alpha_1^A, \alpha_2^A, \alpha_3^A, \dots, \alpha_{N^A}^A\}$; $\alpha_1^K \geq \alpha_2^K \geq \alpha_3^K \geq \dots \geq \alpha_{N^K}^K > 0$.

In practice, shareholders have four possibilities: if they vote, they may approve (vote for), disapprove (vote against) or abstain; they may also decide not to participate to the vote. We exclude the possibility of voting and abstaining for partisans since, with a small voting cost, abstaining may not change the outcome and is always strictly dominated by either voting according to one's preference or not voting.⁷ We therefore consider only three possible strategies for shareholders: vote for (F) the proposed resolution, or against (A) the resolution, or not vote (except for the unconditional supporters who always vote for or against).

Finally, two conditions must be verified for a resolution to be adopted. First, a minimum number of shareholders must be present or represented (quorum rule) in the assembly. We call Q the minimum proportion of shareholders that must vote for a resolution to be adopted. When the minimum quorum Q is not reached, the resolution cannot pass. Second, the resolution must obtain a minimum of favourable votes (majority rule). We assume that a resolution cannot pass unless F represents strictly more than 50% of the votes. If either the quorum rule or the majority rule do not hold, the resolution does not pass and A prevails. Thus the result of an insufficient quorum is considered to be equivalent to a valid vote against, and may be interpreted as the "Status Quo".

We model the assembly as a simultaneous game in which each partisan decides to vote or not based on his anticipations about the other partisans'

of horizontal partial acquisitions.

⁷The same conclusion would apply to indifferent shareholders whom, for this reason, we do not consider.

strategies. All shareholders are assumed to know all others' voting shares and preferences (information is perfect). We look for the pure strategy Nash equilibria of this game. Throughout the paper, equilibrium \mathcal{F} (resp. \mathcal{A}) refers to a pure strategy Nash equilibrium in which F (resp. A) passes. "Inexistence" refers to a situation where no pure strategy equilibrium exists.⁸

We investigate the role played by the presence of unconditionals and the quorum rule on the strategies adopted by partisans and the result of the assembly under these hypotheses.

3 Partisans' equilibrium strategies and equilibrium conditions

In this section we present in a first step some preliminary results regarding the partisans' equilibrium strategies. These considerations enable us to restrict in a second step the examination of the Nash equilibrium conditions to the only admissible strategies.

Partisans' equilibrium strategies

By assumption, the group of unconditionals α_U^A (resp. α_U^F) always votes against (resp. for).

Since voting is slightly costly, the best outcome for any partisan in favour of the resolution $\alpha_i^F \in \mathcal{P}^F$ (resp. any partisan against $\alpha_i^A \in \mathcal{P}^A$) is F (resp. A) without participation, the second best is F (resp. A) with participation, which is better than A (resp. F) without participation, and the worst is outcome A (resp. F) with participation.

Consequently, two properties hold in equilibrium:

⁸When no pure strategy Nash equilibrium exists, the outcome of the meeting cannot be predicted. Since the game is finite, we know (Nash) that a mixed strategy equilibrium always exists. The outcome of the assembly is random in that case.

(P1) No partisan of A (resp. F) votes in equilibrium \mathcal{F} (resp. \mathcal{A}).

Indeed suppose a non-voting partisan of against α_i^A (resp. α_i^F) expects F (resp. A) to result from the assembly. If, given the others' actions, he can rock the result, his best strategy is to vote against (resp. for), which means that the initial set of shareholders' actions was not an equilibrium. Therefore in equilibrium \mathcal{F} (resp. \mathcal{A}) no partisan of A (resp. F) is able to rock the result and since voting is costly does not vote.

(P2) In equilibrium \mathcal{F} (resp. \mathcal{A}), a partisan for $\alpha_i^F \in P^F$ (resp. a partisan against $\alpha_i^A \in P^A$) participates to the vote if and only if he is pivotal, i.e. his vote is necessary to obtain his preferred outcome.

Effectively suppose a voting partisan of for α_i^F (resp. α_i^A) expects the assembly to decide F (resp. A). If, given the others' actions, the result of the assembly remains F (resp. A) if he does not vote, his best strategy is to abstain from voting since voting is costly; this means that the initial set of shareholders' actions was not an equilibrium. Therefore in equilibrium \mathcal{F} (resp. \mathcal{A}) no partisan participates when his preferred outcome F (resp. A) emerges without his vote. In other words, with a small voting cost, a

partisan votes in equilibrium only if he anticipates that his vote is useful (if he is pivotal): he does not vote when his preferred outcome obtains without his vote, nor does he vote when his vote cannot change the outcome he dislikes.

Equilibrium conditions

We successively examine under which conditions the (Nash equilibrium) result of the assembly is to adopt (equilibrium for or \mathcal{F}) or reject (equilibrium against or \mathcal{A}) the resolution.

Equilibrium \mathcal{F}

Let $V^F \subset \mathcal{P}^F$ represent the set of partisans who vote for in equilibria \mathcal{F} .

For an equilibrium \mathcal{F} to exist, four conditions must hold (from (P1) no partisan of A ever votes in equilibrium \mathcal{F} therefore α_U^A represents all votes against):

(F1) Shareholders in favor of F must represent a (strict) majority (**simple majority rule**):

$$\sum_{V^F} \alpha_i^F + \alpha_U^F > \alpha_U^A$$

(F2) The minimum quorum must be reached (**quorum rule**):

$$\sum_{V^F} \alpha_i^F + \alpha_U^F + \alpha_U^A \geq Q$$

(F3) The winning coalition may not be overturned by any partisan of A , therefore by the largest α_1^A (**non contestability condition** for a Nash Equilibrium):

$$\sum_{V^F} \alpha_i^F + \alpha_U^F > \alpha_U^A + \alpha_1^A$$

(F4) All voting partisans α_j^F of the coalition must be pivotal (**pivotal voting partisan condition** for a Nash Equilibrium):

$$\text{for any } \alpha_j^F, \text{ either (a) } \sum_{V^F} \alpha_i^F + \alpha_U^F - \alpha_j^F \leq \alpha_U^A$$

$$\text{or (b) } \sum_{V^F} \alpha_i^F + \alpha_U^F + \alpha_U^A - \alpha_j^F < Q$$

Against obtains the majority (ties are in favor of the Status Quo which is equivalent to against) or the minimum quorum is not reached (in which case the Status Quo prevails) if any $\alpha_j^F \in V^F$ does not participate. Condition (F4) can be rewritten as:

$$\sum_{V^F} \alpha_i^F + \alpha_U^F - \text{Min}_{V^F} \alpha_j^F \leq \text{Max}(\alpha_U^A, Q^- - \alpha_U^A)$$

where Q^- represents the minimum quorum minus one vote.

Since (F3) implies (F1), only the three conditions (F2) to (F4) remain for the existence of an equilibrium \mathcal{F} . They are summarized in (F5):

$$\text{Max}(\alpha_U^A + \alpha_1^A, Q^- - \alpha_U^A) - \text{Min}_{V^F} \alpha_j^F < \sum_{V^F} \alpha_i^F + \alpha_U^F - \text{Min}_{V^F} \alpha_j^F \leq \text{Max}(\alpha_U^A, Q^- - \alpha_U^A)$$

Remarks and additional assumption:

Remark 1 *First note that (F2) can never hold if $\sum_{\mathcal{P}^F} \alpha_i^F + \alpha_U^F + \alpha_U^A < Q$, similarly (F3) can never hold if $\sum_{\mathcal{P}^F} \alpha_i^F + \alpha_U^F \leq \alpha_1^A + \alpha_U^A$.*

We therefore assume that $\sum_{\mathcal{P}^F} \alpha_i^F + \alpha_U^F > \text{Max}(\alpha_1^A + \alpha_U^A, Q^- - \alpha_U^A)$ for non triviality.

Remark 2 *If unconditional shareholders against the resolution represent more than 50% of the minimum quorum, any equilibrium voting coalition for must gather only partisans larger than the largest partisan against. This is not necessarily the case when unconditionals against represent strictly less than 50% of the minimum quorum.*

Indeed, when $2\alpha_U^A \geq Q$, the pivotal partisan condition amounts to (F4a) which together with (F3) imply:

$$\text{Min}_{V^F}(\alpha_i^F) > \alpha_1^A$$

Equilibrium \mathcal{A}

\mathcal{A} prevails whenever voters against obtain a majority (first type) or the minimum quorum is not reached (second type).

First type Let $V^A \subset \mathcal{P}^A$ represent the set of partisans who vote against in equilibrium \mathcal{A} in the first case.

For an equilibrium \mathcal{A} to exist, four conditions must hold (from (P1) no partisan of F ever votes in equilibrium \mathcal{A} therefore α_U^F represents all votes for):

(A1) Shareholders in favor of A must represent a majority (**simple majority rule**):

$$\sum_{V^A} \alpha_i^A + \alpha_U^A \geq \alpha_U^F$$

(A2) The winning coalition may not be overturned by any partisan of F , therefore by the largest α_1^F (**non contestability condition** for a Nash Equilibrium):

$$\sum_{V^A} \alpha_i^A + \alpha_U^A \geq \alpha_U^F + \alpha_1^F$$

(A3) All voting partisans α_j^A of the coalition must be pivotal (**pivotal voting partisan condition** for a Nash Equilibrium):

$$(a) \sum_{V^A} \alpha_i^A + \alpha_U^A - \underset{V^A}{Min}(\alpha_j^A) < \alpha_U^F$$

$$\text{and (b) } \sum_{V^A} \alpha_i^A + \alpha_U^F + \alpha_U^A - \underset{V^A}{Max}(\alpha_j^A) \geq Q$$

If any $\alpha_j^A \in V^A$ does not participate, for obtains a strict majority (a) (ties are in favor of the Status Quo which is equivalent to against) and the minimum quorum is still reached (b).⁹

Since (A2) implies (A1), only the three conditions (A2), (A3a) and (A3b) remain for the existence of an equilibrium \mathcal{A} in which some partisans of A vote. These conditions together are summarized in (A4):

$$Max \left\{ \alpha_U^F + \alpha_1^F, Q - \alpha_U^F + \underset{V^A}{Max}(\alpha_j^A) \right\} - \underset{V^A}{Min}(\alpha_j^A) \leq \sum_{V^A} \alpha_i^A + \alpha_U^A - \underset{V^A}{Min}(\alpha_j^A) < \alpha_U^F$$

⁹Condition (A3b) guarantees that the same result (against when the minimum quorum is not reached) cannot be obtained without the vote of any, therefore the largest shareholder in the voting coalition (P2).

Remarks and additional assumptions

Remark 3 (A2) can never hold when $\sum_{\mathcal{P}^A} \alpha_i^A + \alpha_U^A < \alpha_1^F + \alpha_U^F$ since any possible coalition of shareholders against could then be challenged by the largest partisan for.

We therefore assume that $\sum_{\mathcal{P}^A} \alpha_i^A + \alpha_U^A \geq \alpha_1^F + \alpha_U^F$ for non triviality.

Remark 4 If an equilibrium voting coalition against exists, it gathers only partisans larger than the largest partisan for.

This follows from (A2) and (A3a) which together imply:

$$\text{Min}_{V^A}(\alpha_i^A) > \alpha_1^F$$

Remark 5 A voting coalition of partisans against may exist only if the group of unconditionals for represents at least 50% of the minimum quorum and is larger than the group of unconditionals against.

As indeed (A3a) and (A3b) imply:

$$\begin{aligned} \alpha_U^F &> \alpha_U^A \\ 2\alpha_U^F &> Q \end{aligned}$$

Remark 6 The total of votes non marginally exceeds the minimum quorum in equilibrium, the difference represents at least the votes of the largest partisan for.

Since from (A3b) and remark 4:

$$\sum_{V^A} \alpha_i^A + \alpha_U^F + \alpha_U^A - Q \geq \text{Max}_{V^A}(\alpha_j^A) > \alpha_1^F$$

Second type In the second case where equilibrium \mathcal{A} prevails because the minimum quorum is not reached, from (P1) and (P2), no partisan votes. The necessary and sufficient conditions are therefore:

(A'1) The minimum quorum is not reached:

$$\alpha_U^F + \alpha_U^A < Q$$

(A'2) No partisan of F (therefore the largest α_1^F) may rock the result, (**non contestability condition** for a Nash Equilibrium):

$$\begin{aligned} \alpha_U^F + \alpha_U^A + \alpha_1^F &< Q \\ \text{or } \alpha_U^F + \alpha_1^F &\leq \alpha_U^A \end{aligned}$$

which are summarized as $\alpha_U^F + \alpha_U^A + \alpha_1^F \leq \text{Max}(Q^-, 2\alpha_U^A)$

4 Equilibrium analysis

The objective of this section is to characterize the equilibria of the voting game. In particular, we predict whether (at least) an equilibrium for, an equilibrium against, or both types co-exist for different shareholding structures. The presence of unconditionals and the minimum quorum turn out to play a crucial role and two cases emerge naturally from the analysis. In the first case, the unconditionals alone represent more than the minimum quorum. In the second case, the quorum rule cannot be met unless some partisans vote.

Large unconditionals ($\alpha_U^F + \alpha_U^A \geq Q$)

In this sub-section, we consider the case where the unconditionals have enough voting rights to reach the minimum quorum; either Q is small, or the unconditionals represent a large share of total voting rights.

Let Δ^A represent the set of voting coalitions of partisans against that cannot be challenged, i.e. $\Delta^A = \{V^A \subset \mathcal{P}^A \mid \sum_{V^A} \alpha_i^A + \alpha_U^A \geq \alpha_1^F + \alpha_U^F\}$ and define:

$$V_m^A = \text{Arg Min}_{V^A \in \Delta^A} \left\{ \sum_{V^A} \alpha_i^A - \text{Min}_{V^A}(\alpha_j^A) \right\}$$

$$\text{and } \alpha_m^A = \sum_{V_m^A} \alpha_i^A - \text{Min}_{V_m^A}(\alpha_j^A)$$

α_m^A represents the minimum, among all coalitions against that cannot be challenged, of total partisan votes against less the share of the smallest partisan of the coalition.

α_m^F is defined symmetrically: Δ^F represents the set of coalitions of partisans for that cannot be challenged, i.e. $\Delta^F = \{V^F \subset \mathcal{P}^F \mid \sum_{V^F} \alpha_i^F + \alpha_U^F > \alpha_1^A + \alpha_U^A\}$ and

$$V_m^F = \text{Arg Min}_{V^F \in \Delta^F} \left\{ \sum_{V^F} \alpha_i^F - \text{Min}_{V^F}(\alpha_j^F) \right\}$$

$$\text{and } \alpha_m^F = \sum_{V_m^F} \alpha_i^F - \text{Min}_{V_m^F}(\alpha_j^F)$$

Proposition 1 fully characterizes the conditions of existence and the nature (for, against; involving the vote of some partisans or not).

Proposition 1 *The equilibria of the game are given by :*

(1) When $\alpha_U^A < \alpha_U^F$, a unique equilibrium \mathcal{F} exists where no partisan votes if and only if $\alpha_1^A < \alpha_U^F - \alpha_U^A$.

There exists at least one equilibrium \mathcal{A} where some partisan(s) against vote(s) if and only if $\alpha_m^A < \alpha_U^F - \alpha_U^A$.

(2) Symmetrically when $\alpha_U^A \geq \alpha_U^F$, a unique equilibrium \mathcal{A} exists where no partisan votes if and only if $\alpha_1^F \leq \alpha_U^A - \alpha_U^F$.

There exists at least one equilibrium \mathcal{F} if and only if $\alpha_m^F \leq \alpha_U^A - \alpha_U^F$.

Proof. See appendix. ■

In what follows we examine the case where the largest group of unconditional is for the resolution (i.e. $\alpha_U^F > \alpha_U^A$). For large groups of unconditionals, the case where $\alpha_U^A \geq \alpha_U^F$ is symmetric. This will not be true when the unconditionals represent less than the minimum quorum.

When the largest unconditional group holds a stake larger than the sum of the shares of the other unconditional and of the largest opponent ($\alpha_U^F > \alpha_1^A + \alpha_U^A$), there always exists a unique equilibrium (\mathcal{F}) where this largest unconditional "controls" the assembly (under the simple majority rule) since no opposed partisan (supporting A) may change the result and partisans of the same side (supporting F) obtain their preferred outcome without voting.

However, even in these situations where the dominant unconditional group cannot be challenged, there may also exist equilibria where coalitions of the opposed outcome (A) win. These coalitions must represent enough votes to prevent opposition from the largest partisan of the opposite side (they belong to Δ^A); and they must be composed of large enough partisans who otherwise might be tempted not to vote: the smallest partisan of the voting coalition (against) must be larger than the largest opposed partisan (α_1^F) as stated in the previous section (remark 4).¹⁰ The following example illustrates this case.

Example 1 *Co-existence of a unique equilibrium \mathcal{F} and multiple coalitions supporting equilibrium \mathcal{A} : $Q = 25\%$, $\alpha_U^F = 15\%$, $\alpha_U^A = 10\%$, $\alpha_1^F = 1\%$, $\mathcal{P}^A = \{4\%, 3\%, 2\%, 2\%, 1\%, \dots\}$.*

A unique equilibrium \mathcal{F} where only the unconditionals vote exists, since $\alpha_U^F > \alpha_1^A + \alpha_U^A$. There also exist equilibria \mathcal{A} . Δ^A is the set of coalitions against representing at least 6% of voting rights, $V_m^A \in \{\{4\%, 3\%\}, \{4\%, 2\%\}\}$, $\alpha_m^A = 4\% < 15\% - 10\%$, $V^A = \{3\%, 2\%, 2\%\}$ is another equilibrium voting coalition.

¹⁰Symmetrically, based on remark 2, winning coalitions for necessarily gather partisans larger than α_1^A when $\alpha_U^A \geq \alpha_U^F$ since condition $2\alpha_U^A > Q$ holds with large unconditionals ($\alpha_U^A + \alpha_U^F \geq Q$).

Interestingly, when the largest group of unconditionals represents less votes than the coalition of the opposed largest partisan and unconditionals ($\alpha_U^F \leq \alpha_1^A + \alpha_U^A$), the result of the assembly is always the preferred outcome of the *smallest* unconditional (α_U^A) when an equilibrium exists. Indeed, start from a hypothetical situation where only the unconditionals vote. This is clearly not an equilibrium since α_1^A has an incentive to vote and rock the result as $\alpha_U^F \leq \alpha_1^A + \alpha_U^A$. If $\alpha_1^A + \alpha_U^A \geq \alpha_1^F + \alpha_U^F$, this new situation where only the largest partisan of A and the unconditionals vote against is an equilibrium. No partisan for considers voting since he alone cannot change the outcome of the assembly. Nor does any other partisan against since his favourite result already obtains. This could be interpreted as a *generalized dominant shareholder rule* where the "dominant shareholder" is the largest coalition of the group of unconditionals and largest partisan on the same side. This equilibrium may not be unique. For instance, if $\alpha_2^A + \alpha_U^A \geq \alpha_1^F + \alpha_U^F$, the game has a second equilibrium in which only α_2^A and the unconditionals vote. A coalition of partisans of A , say $V^A \subset \mathcal{P}^A$, might also vote in equilibrium if it gathers enough voting rights to overcome the dominant shareholder for (i.e. $\sum_{V^A} \alpha_i^A + \alpha_U^A \geq \alpha_1^F + \alpha_U^F$) and any member of the coalition is pivotal (i.e. $\sum_{V^A} \alpha_i^A - \alpha_j^A + \alpha_U^A < \alpha_U^F$ for all $\alpha_j^A \in V^A$). Multiple equilibria are therefore feasible, however the result of the assembly is identical (the generalized dominant shareholder rule holds). Again an equilibrium voting coalition necessarily gathers voters whose voting rights are high enough (that is $\alpha_j^A > \alpha_1^F$ for all $\alpha_j^A \in V^A$ as shown in the previous section).

There exists no equilibrium if the largest group of unconditionals is "too small" ($\alpha_U^F > \alpha_U^A$ but $\alpha_U^F \leq \alpha_U^A + \alpha_1^A$) as well as all partisans opposed to this group ($\alpha_i^A \leq \alpha_1^F$ for all i). In the case where the game has no pure strategy equilibrium the outcome of the assembly is random from an ex ante point of view.

The following example illustrates these points.

Example 2 Suppose $Q = 25\%$ and:

$$\begin{aligned}
\alpha_U^F &= 15\% \text{ and } \alpha_U^A = 11\% \\
\text{Case 1} &: \alpha_1^F = 5\% \text{ and } \alpha_1^A = 10\% \\
\alpha_i^A &< 5\% \text{ for all } i > 1 \\
\text{Case 2} &: \alpha_1^F = 5\% \text{ and } \alpha_1^A = 4.5\%
\end{aligned}$$

There exists a unique equilibrium \mathcal{A} where α_1^A is the only partisan who votes in *Case 1* despite the fact that the largest shareholder α_U^F is for the resolution. The outcome of the assembly is random (no pure strategy equilibrium) in *Case 2*.

Coalitions, the role of unconditional shareholders and shareholding structure

The role of unconditional shareholders

With large unconditionals, the minimum quorum constraint is not binding and the situation is equivalent to the case where there is no minimum quorum ($Q = 0$). The presence of unconditionals and specifically the asymmetry between the two groups explain the emergence of equilibrium coalitions in the assembly. Indeed, when $\alpha_U^F = \alpha_U^A$, there exists an equilibrium for with one partisan voting if and only if $\alpha_1^F > \alpha_1^A$ and no equilibrium exists when $\alpha_1^F \leq \alpha_1^A$. This is equivalent to the result of Ritzberger (2005). Only when the shares of the unconditionals for and against are sufficiently different do other types of equilibria exist.

In order to illustrate partisans' behavior in equilibrium, let us consider the case where the group of unconditionals for dominates, $\alpha_U^F > \alpha_U^A$ (the other case is symmetric).

Let us first remark that partisans vote only when they are opposed to the largest group of unconditionals. Indeed larger unconditionals unambiguously destroy the incentive for shareholders on the same side to vote. Suppose all shareholders anticipate that only the unconditionals vote. Partisans of F then find it useless to vote. Only partisans of A have an incentive to vote

if they can rock the result. Thus equilibrium coalitions are always formed against the largest group of unconditionals. Note also that the difference between the two groups of unconditionals ($\alpha_U^F - \alpha_U^A$) matters: the highest it is, the more likely the existence of an equilibrium coalition against.

Who belongs to the voting coalition?

Recall that in this case where the minimum quorum is not binding, partisans belonging to any voting coalition must be larger than the opposed largest partisan. Moreover, if there exists a voting coalition the size of a voting shareholder is necessarily strictly smaller than $\alpha_U^F - \alpha_U^A$. Indeed, we have $\sum_{V^A} \alpha_i^A - \text{Min}_{V^A}(\alpha_j^A) < \alpha_U^F - \alpha_U^A$ (pivotal condition) and $\sum_{V^A} \alpha_i^A - \text{Min}_{V^A}(\alpha_j^A) \geq \alpha_j^A$ for each $j \in V^A$. So $\alpha_j^A < \alpha_U^F - \alpha_U^A$.

In a voting coalition, the shares of the voters are bounded: $\alpha_1^F < \alpha_j^A < \alpha_U^F - \alpha_U^A$. So a necessary condition for the existence of an equilibrium where a coalition votes against is $\alpha_1^F < \alpha_U^F - \alpha_U^A$.

Does the largest partisan vote?

From above, the largest partisan against (α_1^A) can be a member of a voting coalition (at least two partisan voters) if he is small enough ($\alpha_1^A < \alpha_U^F - \alpha_U^A$). A unique equilibrium \mathcal{F} where only the unconditionals vote simultaneously exists in this case. To the contrary, he never belongs to a voting coalition when $\alpha_1^A \geq \alpha_U^F - \alpha_U^A$ since no other member of the potential coalition is pivotal. However, he can vote alone if he is really large (i.e. $\alpha_1^A + \alpha_U^A \geq \alpha_1^F + \alpha_U^F$), in which case there is no \mathcal{F} equilibrium. For an intermediate share ($\alpha_1^F + \alpha_U^F - \alpha_U^A > \alpha_1^A \geq \alpha_U^F - \alpha_U^A$), it is interesting to note that he never votes. To sum up, the largest partisan against votes alone when his share is very large, does not vote for intermediate values, and votes in a coalition when his share, although the largest, is small.

Structure of a winning coalition

Let n^A be the size of an \mathcal{A} equilibrium coalition. The following double inequality is verified: $(n^A - 1)\alpha_1^F < \sum_{V^A} \alpha_i^A - \text{Min}_{V^A}(\alpha_j^A) < \alpha_U^F - \alpha_U^A$. The first

inequality comes from the fact that all members should be larger than the largest partisan for. The second means that all the members of the coalition are pivotal. We thus obtain an upper limit for the size of the coalition : $n^A < \frac{\alpha_U^F - \alpha_U^A}{\alpha_1^F} + 1$. Moreover, $\alpha_U^F \leq \sum_{V^A} \alpha_i^A + \alpha_U^A \leq n^A \alpha_1^A + \alpha_U^A$ since the coalition represents a simple majority (see condition A1) and α_1^A is the largest partisan. Therefore we have:

$$\frac{\alpha_U^F - \alpha_U^A}{\alpha_1^A} \leq n^A < \frac{\alpha_U^F - \alpha_U^A}{\alpha_1^F} + 1.$$

This double inequality gives a range for the size of the equilibrium coalitions which is not necessarily unique; in example 1 we obtain $n^A \in \{2, 3\}$. In the special case where all partisans against are identical ($\alpha_i^A = \alpha^A$), there is a unique equilibrium size characterized by: $(n^A - 1)\alpha^A < \alpha_U^F - \alpha_U^A$ (each voting partisan is pivotal) and $n^A \alpha^A + \alpha_U^A \geq \alpha_U^F$ (simple majority rule). Thus¹¹

$$\frac{\alpha_U^F - \alpha_U^A}{\alpha^A} \leq n^A < \frac{\alpha_U^F - \alpha_U^A}{\alpha^A} + 1.$$

To summarize,

- The presence of unconditional shareholders creates an incentive for coalition formation from partisans opposed to the largest group of unconditionals, provided the difference between the two groups is high enough.
- Equilibrium voting coalitions (including more than one partisan) involve few, sufficiently but not too large members. No equilibrium coalition ever gathers atomistic partisans opposed to the largest group of unconditionals.
- The largest partisan opposed to the largest group of unconditionals votes alone if he holds a really large share; in this case there exists no equilibrium where the side of the largest group of unconditionals

¹¹When $\alpha_A^U \geq \alpha_F^U$, the symmetric condition is $\frac{\alpha_A^U - \alpha_F^U}{\alpha_F^U} < n^F \leq \frac{\alpha_A^U - \alpha_F^U}{\alpha_F^U} + 1$. One can notice that $n^F = 1$ for $\alpha_A^U = \alpha_F^U$.

wins. He may belong to a voting coalition if he is sufficiently small, in which case the unique (non voting) equilibrium of the opposite side always exists. He never votes for intermediate sizes, which however does not preclude the existence of equilibrium voting coalitions of smaller shareholders.

Small Unconditionals ($Q > \alpha_U^A + \alpha_U^F$)

We now consider the case where the unconditionals alone do not have enough voting rights to reach the minimum quorum. Recall A is selected when the minimum quorum is not reached (A is the status quo). Therefore if an equilibrium \mathcal{F} exists it necessarily involves partisans for voting. It is not the case of equilibria \mathcal{A} in which partisans may or may not vote.

Let us consider Ω^A the set of coalitions of partisans of A that cannot be challenged and for which the minimum quorum is still reached whenever a member leaves:

$$\Omega^A = \left\{ V^A \subset \mathcal{P}^A \mid \sum_{V^A} \alpha_i^A + \alpha_U^A \geq \alpha_U^F + \alpha_1^F \text{ and } \sum_{V^A} \alpha_i^A + \alpha_U^F + \alpha_U^A - \text{Max}(\alpha_j^A) \geq Q \right\}.$$

$$\text{Let } V_M^A = \text{Arg Min}_{V^A \in \Omega^A} \left\{ \sum_{V^A} \alpha_i^A - \text{Min}_{V^A}(\alpha_j^A) \right\} \text{ and } \alpha_M^A = \sum_{V_M^A} \alpha_i^A - \text{Min}_{V_M^A}(\alpha_j^A).$$

The following proposition gives the conditions under which there exist equilibria \mathcal{A} and \mathcal{F} .

Proposition 2 *The equilibria of the game are given by:*

(1) When $\alpha_U^A < \alpha_U^F$, a voting equilibrium \mathcal{F} exists if and only if $Q > Q^F = \alpha_m^F + \alpha_U^F + \alpha_U^A$.

A non voting equilibrium \mathcal{A} exists if and only if $Q > Q^A = \alpha_1^F + \alpha_U^F + \alpha_U^A$.

If $Q < 2\alpha_U^F$, a voting equilibrium \mathcal{A} exists if and only if $\alpha_M^A < \alpha_U^F - \alpha_U^A$.

Conversely if $Q \geq 2\alpha_U^F$, there exists no voting equilibrium \mathcal{A} .

(2) When $\alpha_U^A \geq \alpha_U^F$, there exists no equilibrium \mathcal{A} where partisans vote.

A non voting equilibrium \mathcal{A} exists if and only if $Q > Q^A = \alpha_1^F + \alpha_U^F + \alpha_U^A$ or $\alpha_1^F \leq \alpha_U^A - \alpha_U^F$.

If $Q \leq 2\alpha_U^A$, a voting equilibrium \mathcal{F} exists if and only if $\alpha_m^F \leq \alpha_U^A - \alpha_U^F$

Conversely, if $Q > 2\alpha_U^A$, a voting equilibrium \mathcal{F} exists if and only if $Q > Q^F = \alpha_m^F + \alpha_U^F + \alpha_U^A$.

- Equilibria \mathcal{F}

In the first case ($\alpha_U^A < \alpha_U^F$), Q is always higher than $2\alpha_U^A$ and an equilibrium \mathcal{F} exists if the minimum quorum is sufficiently high. Actually if $Q \leq Q^F$, the smallest shareholder of any non contestable coalition has an incentive to leave since the minimum quorum is still reached without him. Such coalitions are therefore unstable. A high minimum quorum actually provides an incentive to vote in favor of the resolution. In this equilibrium, voting shareholders need not be large. Atomistic shareholders may vote in which case the turnout equals the minimum quorum.

In the second case ($\alpha_U^A \geq \alpha_U^F$), the same reasoning and results hold for $Q > 2\alpha_U^A$. These are actually the only cases where the voting coalition may include such small shareholders. Conversely when $Q \leq 2\alpha_U^A$ the minimum quorum plays no role since any non contestable coalition reaches the minimum quorum. Therefore the condition for existence of an equilibrium \mathcal{F} is exactly the same as in Proposition 1 in this last case.

- Equilibrium \mathcal{A}

In the first case ($\alpha_U^A < \alpha_U^F$), there may be voting and non voting equilibria \mathcal{A} .

Let us first consider the equilibrium \mathcal{A} where no partisan votes. A passes since the status quo prevails when the minimum quorum is not reached. This is the case here since unconditionals are small. Suppose all partisans anticipate that no partisan votes. A partisan of A clearly should not vote since his preferred outcome prevails. And if $Q > Q^A = \alpha_1^F + \alpha_U^F + \alpha_U^A$, no partisan in \mathcal{P}^F has alone enough voting rights to satisfy the quorum rule, so his vote can never change the decision. Thus the situation where no partisan votes is stable. As expected, a high minimum quorum relative to the total voting rights of unconditionals discourages voting and favours the rejection of resolutions.

The second type of equilibrium \mathcal{A} involving coalitions is similar to the voting equilibrium \mathcal{A} in the previous section: as the minimum quorum is automatically reached for coalitions that cannot be challenged (since $Q < 2\alpha_U^F$), it really plays no role in that case. When $Q \geq 2\alpha_U^F$, the largest voting shareholder, who must be small enough to ensure that the minimum quorum is still reached if he leaves the coalition, actually has an incentive to leave the coalition since A would still obtain the majority.¹²

In the second case ($\alpha_U^A \geq \alpha_U^F$), there exists no voting equilibrium \mathcal{A} (cf. Remark 5 in section 3). A non voting equilibrium \mathcal{A} always exists when $Q > Q^A$ as in the first case. Moreover a non voting equilibrium also exists when $\alpha_1^F \leq \alpha_U^A - \alpha_U^F$ since, as in Proposition 1, no partisan for is large enough to rock the result, whatever Q .

The following example illustrates the second type of voting equilibrium \mathcal{A} for intermediary minimum quorum.

Example 3 $Q = 25\%$, $\alpha_U^F = 15\%$, $\alpha_U^A = 7\%$, $\alpha_1^F = 1\%$, $\mathcal{P}^A = \{4\%, 3\%, 2\%, 1\%, \dots\}$.

Note that every coalition that cannot be contested should involve at least $\alpha_U^F + \alpha_1^F - \alpha_U^A = 9\%$ which implies that the minimum quorum is reached ($\alpha_U^F + \alpha_U^A + 9\% > 25\%$). The coalition $\{4\%, 3\%, 2\%\}$ does support a voting equilibrium \mathcal{A} .

The example below illustrates the multiple equilibria case where the minimum quorum may either deter voting (when partisans anticipate a too small turnout) resulting in a non voting equilibrium \mathcal{A} , or on the contrary gives an incentive to vote to make sure that the minimum quorum is reached and the resolution is adopted (voting equilibrium for).

Example 4 *Suppose the minimum quorum $Q = 25\%$ and consider the following shareholding structure:*

$$\begin{aligned}\alpha_U^A &= \alpha_U^F = 6\% \\ \mathcal{P}^A &= \{15\%, \dots\} \\ \mathcal{P}^F &= \{10\%, 8\%, \dots\}\end{aligned}$$

¹²Conditions (A3a) and (A3b) are contradictory.

From Proposition 2, a non voting equilibrium \mathcal{A} exists since $Q^A = 22\% < Q$. There also exists a voting equilibrium \mathcal{F} since $Q^F \leq Q^A < Q$. Indeed the coalition $\{\alpha_1^F, \alpha_2^F\}$ cannot be contested, which implies $\alpha_m^F < \alpha_1^F$. This is actually an equilibrium coalition. The resolution may be rejected (equilibrium \mathcal{A}) or adopted (equilibrium \mathcal{F}). The largest shareholder α_1^A may thus be countered.

Coalitions and the role of the quorum rule

With a high minimum quorum, the presence of unconditionals affects the results only marginally. Actually, when there are no unconditionals, the results are similar to those stated in Proposition 2; the only difference is that no equilibrium coalition of voters against may exist. Obviously, with no unconditional for, there is no more incentive to vote against a resolution since the same result (against) obtains at no cost without voting.

Corollary 1 *Suppose there are no unconditional shareholders but the minimum quorum rule applies. In that case,*

- (1) There exists a non voting equilibrium \mathcal{A} if and only if $Q > \alpha_1^F$,
- (2) Equilibria \mathcal{F} exist if and only if $Q > \alpha_m^F$.

Therefore the minimum quorum requirement by itself increases the incentive to form voting coalitions in favor of the resolution with possibly very small shareholders. This never exists in equilibrium without a minimum quorum where at most one shareholder votes. Moreover, with a high enough minimum quorum the situation where no partisan votes becomes an equilibrium.

The following last example shows the importance of having a minimum quorum on the existence and nature of equilibria (coalitions of atomistic partisans for).

Example 5 Consider the following shareholding structure:

$$\begin{aligned}\alpha_U^F &= \alpha_A^F = 0 \text{ (no unconditionals)} \\ \mathcal{P}^A &= \{14\%, \dots\} \\ \mathcal{P}^F &= \{9\%, 8\%, 7\%, 4\%, 3\%, 2\%, 1\%, 1\%, 1\%, \dots\}\end{aligned}$$

With no quorum requirement, no equilibrium exists. With a minimum quorum $Q = 8.5\%$, the coalition $\{8\%, 7\%\}$ supports an equilibrium for. With a minimum quorum $Q = 25\%$, both equilibria exist: the coalitions $\{9\%, 8\%, 7\%, 4\%\}$, $\{9\%, 8\%, 7\%, 1\%\}$ and $\{9\%, 8\%, 3\%, 2\%, 1\%, 1\%, 1\%\}$ support a voting equilibrium \mathcal{F} , the two last ones just reach the minimum quorum; there also exists an equilibrium \mathcal{A} where no shareholder votes.

To summarize,

- A sufficiently high minimum quorum creates an incentive for coalition formation from partisans in favor of the resolution. The equilibrium coalitions for may gather atomistic shareholders.
- Simultaneously, a high enough minimum quorum insures the existence of an equilibrium against where no partisan votes.
- For smaller values of the minimum quorum, there exist voting equilibria (for or against) where few sufficiently but not too large partisans opposed to the largest group of unconditionals vote provided the difference between the two groups of unconditionals is high enough.

Let us finally remark that in the case where the minimum quorum is "really high", the results are asymmetric (equilibria for are voting and may involve atomistic shareholders while the equilibrium against is non voting). It is also quite different from the case of intermediate values which is comparable to the low minimum quorum case, previously examined.

5 Conclusion

In this paper, we investigate the respective roles of the quorum rule and the presence of unconditional shareholders (or equivalently mandatory voting for some shareholders) in annual meetings. In a framework where information is perfect, shareholders differ in size and opinion, and voting is costly, we find that both the quorum rule and the presence of unconditional shareholders increase the incentive to vote for other shareholders who vote strategically. We show the existence of equilibria where coalitions of shareholders for (resp. against) the resolution win even though the largest shareholder is against (resp. for). For some shareholding structures, equilibria leading to a different outcome (adoption or rejection of a resolution) may coexist. Our results therefore differ substantively from Ritzberger's (2005) according to which a unique equilibrium exists if and only if largest (or dominant) shareholder is for a resolution in which case only one shareholder votes.

The concentration of the shareholding structure plays a key role in our analysis. In particular, equilibrium coalitions against a resolution necessarily involve sufficiently large shareholders, namely at least larger than the largest strategic shareholder in favor of the resolution. We therefore predict that a resolution may only be rejected when opponents hold large stakes. This is not always the case for equilibria in favor of a resolution. Indeed, when the minimum quorum is sufficiently high, a coalition gathering only small shareholders in favor of the resolution may vote in equilibrium. Widespread ownership is consistent with the adoption of resolutions but not with rejection. Voting equilibria in favor or against a resolution are thus fundamentally different. From our model, it should be easier to oppose the management by sponsoring and vote a resolution directed against him rather than vote against a management sponsored resolution. This somewhat highlights the importance of federating small shareholders in associations, through proxy voting, as an effective corporate control mechanism.

Even though the presence of unconditional shareholders and a minimum quorum rule both facilitate the existence of voting equilibria, these two parameters do not play the same role. The existence of unconditional shareholders

is not equivalent to a decrease of the minimum quorum rule. Rather than the total -for and against together- share held by unconditional shareholders, it is the difference between the respective shares the unconditional against and for the resolution that plays a key role for the existence and the type of the equilibria. Finally, it is worth noting that the introduction of voting constraints such as a quorum rule actually enriches the results in terms of number and nature of equilibria. It would be interesting in a future research to test empirically our theoretical findings, in particular whether voting equilibria against a resolution are more likely in the presence of large shareholders against a resolution.

6 Appendix

Proof of proposition 1 (large unconditionals)

Suppose $\alpha_U^F + \alpha_U^A \geq Q$, in other words unconditionals are large enough to reach the minimum quorum.

Case 1 - $\alpha_U^F > \alpha_U^A$

Existence of equilibrium \mathcal{F} . (F1) and (F2) always hold in case 1. Since the majority in favor of F is obtained without any partisan's participation, no partisan of F votes and (F4a) is never verified (condition (F4b) is irrelevant here). Thus the existence of an equilibrium \mathcal{F} is guaranteed if and only if $\alpha_U^F > \alpha_U^A + \alpha_1^A$ (F3). Note that only the unconditionals vote in equilibrium \mathcal{F} when it exists.

Existence of equilibrium \mathcal{A} . If it exists, it can only be an equilibrium where a subset of partisans of A vote since the minimum quorum is reached. Also (A3b) always holds in that case. The necessary and sufficient conditions for an equilibrium simplify to:

$$\alpha_U^F + \alpha_1^F - \underset{V^A}{Min}(\alpha_j^A) \leq \sum_{V^A} \alpha_i^A + \alpha_U^A - \underset{V^A}{Min}(\alpha_j^A) < \alpha_U^F$$

Let Δ^A represent the set of coalitions of partisans against that cannot be challenged, i.e. $\Delta^A = \{V^A \subset \mathcal{P}^A \mid \sum_{V^A} \alpha_i^A + \alpha_U^A \geq \alpha_1^F + \alpha_U^F\}$ (\iff (A2)) and define:

$$V_m^A = \underset{V^A \in \Delta^A}{\text{Arg Min}} \left\{ \sum_{V^A} \alpha_i^A - \underset{V^A}{\text{Min}}(\alpha_j^A) \right\}$$

$$\text{and } \alpha_m^A = \sum_{V_m^A} \alpha_i^A - \underset{V_m^A}{\text{Min}}(\alpha_j^A)$$

An equilibrium voting coalition against exists when $\alpha_U^F > \alpha_U^A$ if and only if $\alpha_m^A < \alpha_U^F - \alpha_U^A$. Indeed, if $\alpha_m^A < \alpha_U^F - \alpha_U^A$, α_m^A verifies the necessary and sufficient conditions and the corresponding coalition V_m^A is therefore an equilibrium voting coalition (it may not be unique). If conversely $\alpha_m^A \geq \alpha_U^F - \alpha_U^A$, the second inequality can never be verified and no voting equilibrium exists.

Case 2 - $\alpha_U^A \geq \alpha_U^F$

Case 2, is symmetric to case 1.

Existence of equilibrium \mathcal{A} . (A1) always holds in case 2. Since the majority in favor of A is obtained without any partisan's participation, no partisan of A votes (conditions (A3a) and (A3b) are irrelevant here). Thus the existence of an equilibrium \mathcal{A} is guaranteed if and only if $\alpha_U^A \geq \alpha_U^F + \alpha_1^F$ (A2). Note that only the unconditionals vote in equilibrium \mathcal{A} when it exists.

Existence of equilibrium \mathcal{F} . Some partisans of F must vote for F to pass in equilibrium. With large unconditionals, (F2) is always verified and (F4b) can never hold and is therefore irrelevant. The necessary and sufficient conditions simplify to:

$$\alpha_1^A + \alpha_U^A - \underset{V^F}{\text{Min}}(\alpha_i^F) < \sum_{V^F} \alpha_i^F + \alpha_U^F - \underset{V^F}{\text{Min}}(\alpha_i^F) \leq \alpha_U^A$$

By the same reasoning as before, an equilibrium with a voting coalition for exists when $\alpha_U^A \geq \alpha_U^F$ if and only if $\alpha_m^F \leq \alpha_U^A - \alpha_U^F$ where α_m^F represents

the minimum, among all coalitions for that cannot be challenged, of total partisan votes for less the share of the smallest partisan of the coalition.

Remarks

- The necessary and sufficient conditions for the existence of a voting equilibrium \mathcal{A} (resp. \mathcal{F}) in case 1 where $\alpha_U^F > \alpha_U^A$ (resp. in case 2 where $\alpha_U^A \geq \alpha_U^F$) imply $\text{Min}_{V^A}(\alpha_i^A) > \alpha_1^F$ (resp. $\text{Min}_{V^F}(\alpha_i^F) > \alpha_1^A$). Thus the smallest partisan of the voting coalition must be larger than the largest opposed partisan in equilibrium.

- If $\alpha_U^F > \alpha_U^A$ (resp. $\alpha_U^A \geq \alpha_U^F$) and $\alpha_1^A + \alpha_U^A \geq \alpha_1^F + \alpha_U^F$ (resp. $\alpha_1^F + \alpha_U^F > \alpha_1^A + \alpha_U^A$), then the situation where α_1^A (resp. α_1^F) votes for and all other partisans do not vote is an equilibrium \mathcal{A} (resp. \mathcal{F}). There may exist other voting equilibria \mathcal{A} (resp. \mathcal{F}). No equilibrium \mathcal{F} (resp. \mathcal{A}) exists in that case.

Proof of proposition 2 (small unconditionals)

Suppose $\alpha_U^F + \alpha_U^A < Q$, or unconditionals alone are not large enough to reach the minimum quorum.

Existence of equilibrium \mathcal{F} .

If it exists, it can only be an equilibrium where a subset of partisans of F vote. As no partisan of A ever votes in equilibrium \mathcal{F} (P1), if no partisan of F votes either, the quorum cannot be reached, so for never passes.

Recall that condition (F5) from section 2 summarizes the necessary and sufficient conditions for the existence of such an equilibrium:

$$\text{Max}(\alpha_U^A + \alpha_1^A, Q^- - \alpha_U^A) - \text{Min}_{V^F} \alpha_j^F < \sum_{V^F} \alpha_i^F + \alpha_U^F - \text{Min}_{V^F} \alpha_j^F \leq \text{Max}(\alpha_U^A, Q^- - \alpha_U^A)$$

Let Δ^F represents the set of coalitions of partisans for that cannot be

challenged, i.e. $\Delta^F = \{V^F \subset \mathcal{P}^F \mid \sum_{V^F} \alpha_i^F + \alpha_U^F > \alpha_1^A + \alpha_U^A\}$ and:

$$V_m^F = \text{Arg Min}_{V^F \in \Delta^F} \left\{ \sum_{V^F} \alpha_i^F - \text{Min}_{V^F}(\alpha_j^F) \right\}$$

Depending on the value of Q , three cases emerge.

Case 1 - $Q \leq 2\alpha_U^A$ The necessary and sufficient conditions (F5) simplify to:

$$\alpha_U^A + \alpha_1^A - \text{Min}_{V^F}(\alpha_j^F) < \sum_{V^F} \alpha_i^F + \alpha_U^F - \text{Min}_{V^F}(\alpha_j^F) \leq \alpha_U^A$$

V_m^F verifies the first inequality since by definition it belongs to Δ^F . It also verifies the second inequality if $\alpha_m^F = \sum_{V_m^F} \alpha_i^F - \text{Min}_{V_m^F}(\alpha_j^F) \leq \alpha_U^A - \alpha_U^F$. Moreover the second inequality can never hold if $\alpha_m^F > \alpha_U^A - \alpha_U^F$ since α_m^F minimizes the value of $\sum_{V^F} \alpha_i^F - \text{Min}_{V^F}(\alpha_j^F)$ in Δ^F . Note that the above conditions imply $\text{Min}_{V^F}(\alpha_j^F) > \alpha_1^A$. This case with relatively high α_U^A is actually similar to the large unconditionals case with $\alpha_U^A \geq \alpha_U^F$ since Q plays no role in case 1. V_m^F is an equilibrium coalition, but others may exist.

Case 2 - $2\alpha_U^A < Q \leq 2\alpha_U^A + \alpha_1^A$ The necessary and sufficient conditions simplify to:

$$\alpha_U^A + \alpha_1^A - \text{Min}_{V^F}(\alpha_j^F) < \sum_{V^F} \alpha_i^F + \alpha_U^F - \text{Min}_{V^F}(\alpha_j^F) < Q - \alpha_U^A$$

As in case 1, the first inequality is verified for V_m^F since it belongs to Δ^F . It also verifies the second inequality if

$$Q^F \equiv \sum_{V_m^F} \alpha_i^F + \alpha_U^F + \alpha_U^A - \text{Min}_{V_m^F}(\alpha_j^F) < Q$$

By definition of V_m^F , the coalition $V_m^F \setminus \{\text{Min}_{V_m^F}(\alpha_j^F)\}$ is contestable ($\sum_{V_m^F} \alpha_i^F + \alpha_U^F - \text{Min}_{V_m^F}(\alpha_j^F) \leq \alpha_U^A + \alpha_1^A$), therefore $Q \leq 2\alpha_U^A + \alpha_1^A$. Moreover, if $Q^F \geq Q$, the second inequality can never be verified for any non contestable coalition.

Thus there exists an equilibrium voting coalition for if and only if $Q^F < Q$. In other words, the effective quorum less the smallest share among voting partisans is below the minimum quorum which ensures that all shareholders are pivotal. V_m^F is an equilibrium coalition, but others may exist.

Case 3 - $2\alpha_U^A + \alpha_1^A < Q$ The necessary and sufficient conditions in that case are:

$$Q - \alpha_U^A - \underset{V^F}{\text{Min}}(\alpha_j^F) \leq \sum_{V^F} \alpha_i^F + \alpha_U^F - \underset{V^F}{\text{Min}}(\alpha_j^F) < Q - \alpha_U^A$$

$$\text{or } 0 \leq \left\{ \sum_{V^F} \alpha_i^F + \alpha_U^F + \alpha_U^A \right\} - Q < \underset{V^F}{\text{Min}}(\alpha_j^F)$$

Let Ω^F represent the set of coalitions of partisans for that reach the minimum quorum, i.e. $\Omega^F = \{V^F \subset \mathcal{P}^F \mid \sum_{V^F} \alpha_i^F + \alpha_U^F + \alpha_U^A \geq Q\}$. As $2\alpha_U^A + \alpha_1^A < Q$, no $V^F \in \Omega^F$ can be challenged ($\Omega^F \subset \Delta^F$). Define:

$$V_Q^F = \underset{V^F \in \Omega^F}{\text{Arg Min}} \left\{ \sum_{V^F} \alpha_i^F \right\}$$

It is then easy to see that V_Q^F is an equilibrium voting coalition for. Indeed, by definition V_Q^F reaches the minimum quorum since it belongs to Ω^F . It also verifies the second inequality which ensures that any shareholder belonging to V_Q^F is pivotal. Indeed, suppose this is not the case (i.e. $\left\{ \sum_{V_Q^F} \alpha_i^F + \alpha_U^F + \alpha_U^A \right\} - Q \geq \underset{V_Q^F}{\text{Min}}(\alpha_j^F)$). This means that the coalition $V_Q^F \setminus \underset{V_Q^F}{\text{Min}}(\alpha_j^F)$ belongs to Ω^F . This contradicts the fact that V_Q^F has the smallest size in this set.

Remarks: - When there exists a coalition V^F s.t. $\sum_{V^F} \alpha_i^F + \alpha_U^F + \alpha_U^A = Q$ (this is the case with small shareholders, each one having one vote), the voting coalition may match exactly the minimum quorum Q in equilibrium.

- If $\alpha_1^F + \alpha_U^F > \alpha_1^A + \alpha_U^A$ and $\alpha_U^F + \alpha_U^A + \alpha_1^F \geq Q$, there exists a voting equilibrium where only the largest shareholder for votes.

Existence of equilibrium \mathcal{A} .

The necessary and sufficient conditions (A2), (A3a) and (A3b) for a voting equilibrium against to exist are:

$$\begin{aligned} \sum_{V^A} \alpha_i^A + \alpha_U^A &\geq \alpha_U^F + \alpha_1^F \\ \sum_{V^A} \alpha_i^A + \alpha_U^A - \text{Min}_{V^A}(\alpha_j^A) &< \alpha_U^F \\ \sum_{V^A} \alpha_i^A + \alpha_U^F + \alpha_U^A - \text{Max}_{V^A}(\alpha_j^A) &\geq Q \end{aligned}$$

As mentioned in the text, these conditions imply together that

$$\text{Min}_{V^A}(\alpha_j^A) > \alpha_1^F$$

$$\alpha_U^F > \alpha_U^A$$

$$2\alpha_U^F > Q$$

So in the case where $Q \geq 2\alpha_U^F$, no voting equilibrium exists. However voting equilibria against may exist if $\alpha_U^F + \alpha_U^A < Q < 2\alpha_U^F$.

They are characterized as follows. Let us consider $\Omega^A = \{V^A \subset \mathcal{P}^A \mid \sum_{V^A} \alpha_i^A + \alpha_U^F + \alpha_U^A - \text{Max}_{V^A}(\alpha_j^A) \geq Q \text{ and } \sum_{V^A} \alpha_i^A + \alpha_U^A \geq \alpha_U^F + \alpha_1^F\}$.

Let $V_M^A = \text{Arg Min}_{V^A \in \Omega^A} \left\{ \sum_{V^A} \alpha_i^A - \text{Min}_{V^A}(\alpha_j^A) \right\}$. Thus if $\alpha_M^A = \sum_{V_M^A} \alpha_i^A - \text{Min}_{V_M^A}(\alpha_j^A) \geq \alpha_U^F - \alpha_U^A$, no voting equilibria exists. When $\alpha_M^A < \alpha_U^F - \alpha_U^A$, V_M^A is a voting coalition equilibrium against.

When $\alpha_U^F + \alpha_U^A < Q$, a non voting equilibrium against exists if and only if (A'2) holds i.e.

$$\begin{aligned} \alpha_U^F + \alpha_U^A + \alpha_1^F &< Q \\ \text{or } \alpha_U^F + \alpha_1^F &\leq \alpha_U^A \end{aligned}$$

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