# ENHANCING PORTFOLIO OPTIMIZATION: A COMPARATIVE ANALYSIS OF THE MEAN-VARIANCE MARKOWITZ MODEL AND RISK-PARITY CONTRIBUTION STRATEGIES

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## Abstract

Financial markets are still exposed to various crises that increase stock price fluctuations and make predictions extremely difficult (Nguyen et al., 2024). Although there are many diversification methods for building investment portfolios, there has been no agreement on the best of them. This research aims to identify the most effective strategy for constructing an optimal investment portfolio by comparing the mean-variance (MV) model and risk-parity (RP) contribution strategies during the uncertain market period 2018–2022. The study used a quantitative and an optimization method following Ofikwu (2019) based on three critical criteria: 1) low asset correlation, 2) the highest Sharpe ratio, and 3) a mixed strategy for a sample of firms listed in the Amman Stock Exchange (ASE). The results show that the MV model has the highest Sharpe ratio (15.06 percent and 11.84 percent) when applied to the Sharpe and mixed strategies respectively. In comparison, using the low correlation strategy the RP model gains the highest Sharpe ratio (14.92 percent). During COVID-19, all portfolios had a higher positive return and lower total risk than the market portfolio. Both models are valid to be used during uncertain conditions. It highlights the effectiveness of strategies in navigating challenging market conditions and offers practical guidance for investors in uncertain times for asset allocation.

**Keywords:** Portfolio Optimization, Risk-Parity Model, Markowitz Model, Sharpe Ratio, COVID-19 Pandemic, Amman Stock Exchange

**Authors' individual contribution:** Conceptualization — R.H.H. and F.K.S.; Methodology — R.H.H. and F; Software — R.H.H. and F.K.S.; Validation — R.H.H. and F.K.S.; Formal Analysis — R.H.H. and F.K.S.; Investigation — R.H.H.; Resources — R.H.H. and F.K.S.; Data Curation — R.H.H.; Writing — R.H.H. and F.K.S.; Supervision — R.H.H. and F.K.S.; Project Administration — R.H.H. and F.K.S.; Funding Acquisition — R.H.H. and F.K.S.

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## 1. INTRODUCTION

The growing recognition of the need for asset diversification to mitigate risks in the wake of financial crises has become increasingly evident across various economic sectors. The recent global pandemic, COVID-19, heightened concerns within the financial community, including regulatory bodies, brokerage firms, and individual investors. Starting in late 2019, the financial markets experienced declines in market capitalization, trading volume, and the number of shares traded, as reported by the Amman Stock Exchange (ASE, 2019, 2020). While previous academic research has introduced numerous risk measures, they have often fallen short of delivering consistent and dependable results.

This research explores the practical application of portfolio management principles in the real-world context. It provides investors the opportunity to construct a well-balanced, optimized, and diversified portfolio of stocks. Of particular interest is the examination of the risk-parity (RP) model, a relatively recent addition to portfolio management, designed to control risk levels and improve portfolio performance.

It's worth noting that while many prior studies have heavily relied on the Markowitz model, only a limited number have delved into the RP model's potential in identifying the optimal portfolio and comparing its performance against other models such as the works of Li and Zhang (2021), and Pedersen et al. (2021). Few studies have specifically focused on the Arab region stock markets and those that have predominantly used conventional risk measures in Morocco (e.g., Saleh & Sarhan, 2020), Jordan (Bani-Hamad et al., 2018; Alqadi, 2016), Saudi Arabia (Abbou & Bouflih, 2017), Palestine (Abo Amshaa, 2017; Shebeer, 2015), Iraq (Al-Ardhi & Jaafar, 2016; Hadi, 2012).

This comparative analysis holds significant relevance in the current landscape, where investors are increasingly seeking strategies to enhance portfolio optimization and manage risks effectively.

This research endeavors to address this gap by incorporating the RP model into the study, offering a comprehensive evaluation of its performance and comparing it to other widely adopted methods. This investigation seeks to provide valuable insights for investment decisions in the face of diverse and dynamic financial markets, contributing to the ongoing discourse on portfolio optimization in an ever-changing economic landscape.

The research is organized as follows. A detailed literature review is presented in Section 2. The materials and methods used to conduct the study are described in Section 3. The test results are presented in Section 4, and discussed in Section 5. Conclusions and recommendations are outlined in Section 6.

## 2. LITERATURE REVIEW

The investment landscape has undergone significant transformations in recent years, and the concept of an investment portfolio has emerged as a crucial tool in navigating the complex financial and banking sectors, as well as the dynamic world of capital markets. The inherent value of investment portfolios lies in their ability to encompass a diverse array of investment instruments, mitigating risks, and invigorating financial and economic markets amidst the rapid advances in information and communication technology (Lizarzaburu et al., 2023; Marchev, 2014).

Diversification, as conceptualized by Harry Markowitz, stands as a foundational principle within modern portfolio theory (MPT) (Markowitz, 1991, 1952). Markowitz's theory emphasized the importance of assembling assets with low correlation to minimize risk while preserving portfolio returns. Subsequently, a multitude of empirical studies (Nguyen et al., 2024; Success, 2020; Hunjra et al., 2020; Huni & Sibindi, 2020; Righi & Borenstein, 2018; Vaclavik & Jablonsky, 2012) explored various risk measures, including mean-variance (MV), semi-variance (SV), mean absolute deviation (MAD), and conditional value at risk (CVaR), to assess their efficacy in risk reduction and return maximization. These investigations have yielded mixed results, with some indicating that mean-variance optimization (MVO) portfolios with diversification may slightly elevate risk compared to MVO, while others have found no significant difference between MV and MAD concerning the portfolio's real rate of return. Nonetheless, these risk measures have demonstrated resilience during crisis phases, consistently performing well within the Indian stock market (Hunjra et al., 2020).

Intriguingly, when constructing an MVO portfolio under different scenarios, it has been observed that MVO consistently minimizes risk more efficiently than benchmark indices such as the Johannesburg Securities Exchange (JSE) All-Share index (ALSI) (Righi & Borenstein, 2018). While no single risk metric reigns supreme, the inclusion of deviation terms consistently bolsters portfolio performance, reinforcing the value of the MV model in risk mitigation, particularly during market downturns.

The pursuit of maximizing the Sharpe ratio has revealed the MVO portfolio's superiority over the single index model (SIM) in various financial markets (Badiar, 2019). Moreover, the MV strategy has exhibited the highest Sharpe ratio with the lowest variation compared to alternative strategies (Saleh & Sarhan, 2020). As such, practitioners and regulators are advised to consider a range of risk measures when making informed decisions (Birungi & Muthoni, 2021). Despite facing criticisms such as its reliance on historical data and computational complexities, the MVO model remains a valid choice for selecting optimal portfolios, particularly in uncertain periods (Patel & Chakraborty, 2017). Furthermore, diverse risk measures have been shown to yield similar solutions for specific investor categories, underscoring the model's adaptability and practical utility (Ortobelli et al., 2005).

The concept of RP, which emphasizes diversifying portfolios based on risk rather than market value, has gained significant traction as an alternative to MVO (Bellini et al., 2021; Asness et al., 2012; Qian, 2011). RP offers the advantage of balanced risk exposure, especially during market volatility, and has been proven to enhance diversification while reducing downside risk (Qian, 2005). Its flexibility and capacity for customization to align with various investment goals have further bolstered its appeal, and it has outperformed market-cap-weighted portfolios during periods of market stress (Clarke et al., 2013).

The risk-parity portfolio (RPP) approach has garnered considerable interest for its ability to diversify risk among assets, making it less susceptible to parameter estimation errors (Roncalli, 2013; Qian, 2016). However, its effectiveness has exhibited variability, and critics have raised both theoretical and practical implementation concerns (Fabozzi et al., 2021).

Studies comparing MV and RP models have yielded diverse outcomes. In some instances, both models have outperformed benchmarks, underscoring their practical value (Li et al., 2022), while RP has exhibited an advantage in reducing downside risk and maintaining stable performance across different market conditions (Pedersen et al., 2021).

Thus, no agreement has been reached on devoting the MV and RP models. Both models possess their unique strengths and weaknesses, and their performance can vary depending on the context and asset strategies employed. If the proposed portfolio selection strategies after optimization can enhance the portfolio performance measured by the Sharpe ratio, and the risk models remain a subject of discussion, then the proposed strategies can offer some explanation for differences in portfolio optimization between risk models. Nonetheless, this is the first study that examined these models in the Jordanian and Arab context during the COVID-19 pandemic. Key questions regarding portfolio performance during the COVID-19 pandemic will be explored, with a particular focus on assessing any statistically significant differences between MVO and RP portfolios at a significance level ($\alpha$) of 0.05.

Therefore, the purpose of this study endeavors to undertake an empirical investigation of the optimal portfolio performance based on the three unique strategies, and the differences between MVO and RP models regarding return, risk, and Sharpe ratio within the context of the ASE during the pandemic era. The research questions are the following:

*RQ1: What is the optimal portfolio selected based on formulated strategies (low correlation, highest Sharpe ratio, and mixed strategy) subjecting the sample to the mean-variance optimization model? Which one provides a better portfolio performance in terms of risk, returns, and Sharpe ratio during the COVID-19 pandemic?*

*RQ2: What is the optimal portfolio selected based on formulated strategies and subjecting the sample to the risk-parity model? Which one provides a better portfolio performance in terms of risk, returns, and Sharpe ratio during the COVID-19 pandemic?*

*RQ3: Are there statistically significant differences at the level ($\alpha \leq 0.05$) between the mean-variance optimization and risk-parity portfolios in the Amman Stock Exchange during the COVID-19 pandemic?*

The hypotheses of the study are formulated as follows:

*$H_0$: There is no significant difference ($\alpha \leq 0.05$) in the mean rank (sums) between the two portfolios' returns.*

*H1: There is a significant difference ($\alpha \leq 0.05$) in the mean rank (sums) between the two portfolios' returns.*

## 3. RESEARCH METHODOLOGY

This study employed quantitative comparative analysis techniques and adopted an empirical research approach to explore constructing an optimal portfolio within the ASE. The study encompassed a population of 163 companies listed on the ASE, spanning from January 2, 2018, to December 30, 2022, across three distinct sectors: 1) financial, 2) industry, and 3) services. The selection of the study sample adhered to specific criteria.

Firstly, companies were required to maintain a continuous listing of their shares throughout the entire study period, starting no later than the beginning of 2018 and extending until the conclusion of 2022, in line with previous research (AlHalaseh et al., 2019; Liu, 2009). Secondly, the companies had to demonstrate continuous trading of their shares throughout the study period. Therefore, any stocks with missing data over extended periods were excluded, following previous studies (AlHalaseh et al., 2019; Stoyan & Kwon, 2010, 2011). Ultimately, 99 companies met these stringent criteria, representing a diverse range of industries suitable for constructing a financial investment portfolio. Notably, the financial and insurance industries, which exhibit unique sensitivities to market changes, were treated separately due to their specialized nature. Consequently, a purposive sampling technique was employed, aligning with the specific requirements for portfolio construction and the study's models.

### 3.1. Data collection

The study collected stock price information from secondary sources, specifically from the ASE website. This paper utilized the daily closing prices of the stocks to compute their daily returns. This study analyzed 1.206 daily data points for each stock, yielding a total dataset of 119.394 daily data points for the entire study sample. This study obtained historical data on Jordan Central Bank's 9-month treasury notes to incorporate the risk-free rate of return ($R_f$). With an annual rate of 0.045 and a daily rate of 0.018%, this data serves as a substitute for the risk-free rate. To analyze the data, a solver technique in Microsoft Excel was used.

### 3.2. Methods in portfolio optimization

In the literature, many authors tried to find new measures to enhance portfolio performance at the same time reducing risk exposure. MVO, CVaR optimization, SIM, capital asset pricing model (CAPM), and Black-Litterman model. MVO creates the efficient frontier to find the best possible portfolios that provide the lowest possible risk for a given level of expected return or the highest possible return for a given level of risk. It makes use of the covariance matrix to evaluate the connection between the assets. CVaR optimization focuses on minimizing the risk of extreme losses. It considers not only the expected loss but also the tail risk.

The SIM assumes that individual asset returns follow a linear relationship with market index returns. It divides risk into systematic and unsystematic (diversifiable) components. Beta coefficients are used to measure the sensitivity of individual assets to market movements. Focuses on reducing unsystematic risk to build efficient portfolios. It only considers a single market factor when explaining asset returns, ignoring the impact of multiple risk factors.

As MVO, CAPM assumes risk-averse rational investors. According to CAPM, every investor owns a mix of the market portfolio and the risk-free asset, demonstrating that expected return and systematic risk have a linear relationship (beta). Based on an asset's beta, the security market line (SML) calculates the necessary rate of return.

The Black-Litterman model assumes that the market portfolio is an equilibrium portfolio that reflects the collective wisdom of all investors. However, this market portfolio may not necessarily reflect an individual investor's views or beliefs about asset returns. The Black-Litterman model adjusts the initial expected returns based on the investor's views (subjective views) and combines them with the market equilibrium returns to generate the final expected returns, which are then used for portfolio optimization (Kolm et al., 2021).

The model starts by estimating the expected returns of assets using the CAPM or any other method. Then, the investor provides subjective views or beliefs about the expected returns of assets. These views can be expressed as absolute returns, relative returns, or a combination of both. The Black-Litterman model adjusts the initial expected returns based on the investor's views and combines them with the market equilibrium returns to generate the final expected returns, which are then used for portfolio optimization. Because the Black-Litterman model uses the subject view of the investor and this study investigates the uncertain period, therefore, it is excluded from being used in this study.

In brief, these models serve different approaches to portfolio optimization, each with its own set of assumptions, methodologies, and considerations. The SIM highlights the relationship with a market index, and the MV Markowitz model seeks to maximize risk-adjusted return by regarding asset covariance structure. CAPM examines the relationship between expected returns and systematic risk. RP contribution strategies emphasize risk allocation over return maximization to achieve a more balanced risk profile. The suitability of each approach is determined by investor preferences, market conditions, and the specific goals of the investment strategy. Investors frequently consider combining or adapting these approaches based on their specific needs and beliefs about the market. As a result, this study attempts to investigate traditional (MVO) models that rely on maximizing returns and modern (RP) models that rely on allocating risk, both of which have not been thoroughly studied in the Middle East market.

### 3.3. Strategies used in this study

To analyze MVO and RP portfolios, three distinct sets of data were generated using three unique approaches for each portfolio. The first approach involved selecting companies based on their most negative correlation. In this method, the correlation coefficients of various companies were computed and ranked according to their highest negative values, representing the extent of diversification within the financial portfolio construction. This approach yielded a total of 41 companies (see Table 1, Panel A). During the study period, which spanned several years marked by significant economic events and regional risks, including the global impact of the COVID-19 pandemic for at least three years, the study measured the average daily return and standard deviation.

The second data set was derived from the stocks with the best Sharpe ratio, which is a risk-adjusted return metric. The Sharpe ratio was computed for the selected sample, and the companies were arranged based on the highest ratio value. This resulted in a list of 37 companies. To determine the optimal weighting for this sample within a portfolio that meets investor expectations in terms of return, the same procedures were employed to measure the average return and standard deviation of the chosen sample over five years. A detailed description of the Sharpe ratio-based sample is provided in Panel B of Table 1.

The third data set was compiled using a combined strategy, considering both the highest negative correlation coefficient and the highest Sharpe ratio. A total of 13 companies met these criteria. Panel C of Table 1 presents the average return and standard deviation for each of these companies.

**Table 1.** Statistics for the chosen sample, which was determined by the correlation factor (Part 1)

| ***Code*** | $R_i$ | $\sigma$ | ***Code*** | $R_i$ | $\sigma$ | ***Code*** | $R_i$ | $\sigma$ |
|---|---|---|---|---|---|---|---|---|
| ***Panel A: The average return and standard deviation of the correlation-based sample*** | | | | | | | | |
| AALU | 0.06% | 2.22% | IPCH | 0.00% | 4.62% | NCCO | 0.01% | 1.53% |
| AABMS | 0.01% | 1.22% | IREL | 0.01% | 1.32% | NDAR | 0.04% | 2.19% |
| AIFE | -0.01% | 1.17% | JDPC | -0.01% | 2.32% | NOTI | -0.12% | 1.76% |
| AIHO | -0.02% | 1.41% | JOHT | -0.04% | 1.12% | PHIL | 0.00% | 1.33% |
| ALFA | 0.05% | 1.59% | JOMC | 0.01% | 1.01% | PHNX | -0.03% | 2.53% |
| APOT | 0.08% | 1.72% | JOPH | 0.24% | 2.14% | REDV | 0.05% | 2.02% |
| ASPMM | -0.01% | 1.82% | JVOI | 0.04% | 0.96% | RUMM | -0.11% | 2.30% |
| ATCO | -0.10% | 2.42% | MBED | 0.03% | 1.24% | SPTI | -0.01% | 1.18% |
| CEIG | -0.16% | 1.57% | MDTR | -0.01% | 0.84% | SURA | 0.01% | 2.75% |
| COHO | 0.01% | 1.91% | MEET | 0.05% | 1.15% | ULDC | 0.02% | 2.26% |
| ENTK | -0.02% | 2.77% | MSFT | 0.01% | 1.81% | UMIC | 0.01% | 1.27% |
| GENI | -0.01% | 0.24% | NAQL | -0.06% | 1.81% | UNAI | 0.06% | 2.40% |
| IBNH | -0.02% | 1.03% | NAST | -0.04% | 1.73% | ZEIC | 0.03% | 1.87% |
| ICAG | 0.01% | 1.63% | NATP | 0.04% | 0.92% | | | |

**Table 1.** Statistics for the chosen sample, which was determined by the correlation factor (Part 2)

| Code | $R_i$ | σ | Code | $R_i$ | σ | Code | $R_i$ | σ |
|---|---|---|---|---|---|---|---|---|
| *Panel B: The average return and standard deviation of the Sharpe ratio-based strategy* | | | | | | | | |
| AALU | 0.06% | 2.22% | JOPI | 0.03% | 2.40% | NDAR | 0.04% | 2.19% |
| ALFA | 0.05% | 1.59% | JOPT | 0.08% | 1.69% | PROF | 0.03% | 1.78% |
| AMWJ | 0.07% | 4.10% | JOTF | 0.02% | 1.02% | REDV | 0.05% | 2.02% |
| APCT | 0.07% | 2.21% | JPPC | 0.05% | 2.06% | SHIP | 0.02% | 1.47% |
| APOT | 0.08% | 1.72% | JRCD | 0.02% | 1.96% | SIJC | 0.13% | 2.40% |
| ASAS | 0.11% | 3.54% | JVOI | 0.04% | 0.96% | SITT | 0.07% | 1.95% |
| BIND | 0.06% | 1.07% | MBED | 0.03% | 1.24% | SNRA | 0.06% | 1.28% |
| HPIC | 0.04% | 1.37% | MEET | 0.05% | 1.15% | SPIC | 0.03% | 2.39% |
| IDMC | 0.05% | 1.73% | NAQL | -0.06% | 1.81% | THDI | 0.03% | 2.39% |
| JNTH | 0.04% | 2.40% | NATA | 0.11% | 2.14% | UNAI | 0.06% | 2.40% |
| JOEP | 0.03% | 1.52% | NATP | 0.04% | 0.92% | ZARA | 0.02% | 1.96% |
| JOIR | 0.19% | 6.20% | NCMD | 0.29% | 9.42% | ZEIC | 0.03% | 1.87% |
| JOPH | 0.24% | 2.14% | | | | | | |
| *Panel C: The average return and standard deviation of the correlation factor and Sharpe ratio-based strategy* | | | | | | | | |
| AALU | 0.06% | 2.22% | MBED | 0.03% | 1.24% | NDAR | 0.04% | 2.19% |
| ALFA | 0.05% | 1.59% | MEET | 0.05% | 1.15% | REDV | 0.05% | 2.02% |
| APOT | 0.08% | 1.72% | NAQL | -0.06% | 1.81% | UNAI | 0.06% | 2.40% |
| JOPH | 0.24% | 2.14% | NATP | 0.04% | 0.92% | ZEIC | 0.03% | 1.87% |
| JVOI | 0.04% | 0.96% | | | | | | |

*Note: This table reports the components of the sample based on the correlation factor, the Sharpe ratio, and the mix-sample.*
*Source: Authors' elaboration using data from ASE 2018–2022.*

### 3.4. Variables measurement

The stock return was measured using the following formula:

$$R_{i,t} = \frac{P_{i,t} - P_{i,t-1}}{P_{i,t-1}} \tag{1}$$

where, $R_{i,t}$ is the return of stock *i*; $P_{i,t}$ is the closing price of the stock *i* at time *t*; $P_{i,t-1}$ is the closing price of the stock *i* at time *t*-1.

After calculating the return of the stock, the variance for each stock $\sigma_i^2$ is estimated according to Copeland and Weston (1992) as Eq. (2):

$$\sigma_i^2 = \frac{\sum_i^n (R_i - E(R)_i)}{n-1} \tag{2}$$

To fulfill the requirements for selecting the optimal Markowitz portfolio, the expected return of the portfolio is calculated. To do so, the optimal weights of the assets that make up the portfolio must be extracted using the matrix multiplication as follows:

$$E(R_p) = w^T R = [w_1, \dots, w_i] \begin{bmatrix} E(R_1) \\ . \\ . \\ . \\ E(R_i) \end{bmatrix} \tag{3}$$

where, $w$ is the weight vector of the stock (1, …, *j*) in the portfolio, and $R$ is the expected return vector of the stock (1, …, *i*) in the portfolio.

Following the calculation of the portfolio's return, the total risk (variance) for a portfolio with more than three assets can be determined using the following Eq. (4):

$$\sigma_p^2 = w^T S(w) \tag{4}$$

The portfolio standard deviation is as Eq. (5).

$$\sigma_p = \sqrt{w^T S(w)} = \left[ [w_1 \dots w_i] \begin{bmatrix} \sigma_{11} \dots \sigma_{1i} \\ . \\ . \\ . \\ \sigma_{i1} \dots \sigma_{ii} \end{bmatrix} \begin{bmatrix} w_1 \\ . \\ . \\ . \\ w_i \end{bmatrix} \right]^{\frac{1}{2}} \tag{5}$$

where, $S$ represents the variance-covariance matrix of the covariance between each of the stock's returns in the portfolio. Variance-covariance between the returns of any two different stocks, calculated according to the following formula:

$$S = \sigma_{x,y} = \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y}) \tag{6}$$

### 3.5. Mean-variance optimization model

To choose the optimum portfolio for Markowitz's portfolio for each sample, Excel Solver was used to maximize the Sharpe ratio, so the following model was built:

$$Max \sum_{i=1}^{N} \frac{w_i R_i - R_f}{\sigma_i} \tag{7}$$

Subject to, $\sum_{i=1}^{N} w_i = 1$, $w_i \geq 0$, $i = 1, \dots, N$.

After obtaining the optimal portfolio of risky assets, this research considers the individual investor and based on the level of risk aversion estimates the value of what should be invested in risky assets and what will be invested in risk-free assets, so the following Eq. (8) is used:

$$y^* = \frac{E(R_p) - R_f}{A\sigma_p^2} \tag{8}$$

where, $y^*$ represents the proportion of the portfolio invested in the risky assets, and $A$ represents the measure of the investor's risk aversion coefficient level, where it is a positive value greater than 0, it can be considered equal to one.

### 3.6. Risk-parity optimization model

The RP method states that by altering asset allocations to reflect the same level of risk, the portfolio can improve its Sharpe ratio and become more resistant to market downturns, following (Feng & Palomar, 2015, 2016), given a portfolio $w \in RN$ and the return covariance matrix Σ, the portfolio volatility is:

$$\sigma(w) = \sqrt{w^T \Sigma w} \quad (9)$$

According to Euler's theorem, volatility can be divided as:

$$\sigma(w) = \sum_{i=1}^{N} w_i \frac{\partial \sigma}{\partial w_i} = \sum_{i=1}^{N} \frac{w_i (\Sigma\, w)_i}{\sqrt{(w^T \Sigma\, w)}} \quad (10)$$

The *i*-th asset's risk contribution ($RC$) to total risk $\sigma(w)$ is defined as:

$$RC_i = w_i \frac{\partial \sigma}{\partial w_i} = \frac{w_i (\Sigma\, w)_i}{\sqrt{w^T \Sigma\, w}} \quad (11)$$

which satisfies $\sum_{i=1}^{N} RC_i = \sigma(w)$ (Euler's theorem).

The ratio of its $RC$ to the total portfolio risk $\sigma(w)$ is defined as the relative risk contribution ($RRC$):

$$RRC_i = \frac{RC_i}{\sigma(w)} = \frac{w_i (\Sigma\, w)_i}{w^T \Sigma\, w} \quad (12)$$

So that, $\sum_{i=1}^{N} PRC_i = 1$.

The main objective of this method is to distribute the weights so that each asset contributes the same amount of risk, hence "equalizing" the risk. Risk contributions are equalized in the RPP or equal risk portfolio (ERP).

$$RC_i = \sigma(w)/N \quad (13)$$

The purpose is to attempt to achieve equal risk contributions, so, this can be obtained by minimizing the differences between the terms $w_i$ and $\Sigma w_I$.

So, $w$ can be minimized by:

$$\sum_{i,j=1}^{N} (w_i(\Sigma w)_i - w_i(\Sigma w)_i)^2 - F(w) \quad (14)$$

Subject to, $w \geq 0, 1^T = 1, w \in w$. $F(w)$ indicates some additional objective function and $w$ denotes an arbitrary convex set of constraints. Note the parallel with the equal weight portfolio (EWP) (aka uniform portfolio):

$$w_i = \frac{1}{N} \quad (15)$$

While the EWP equalizes the capital allocations $w_i = 1/N$, RPP equalizes the risk allocation, $RRC_i = 1/N$.

### 3.7. Mean-variance optimization model versus risk-parity optimization model

The optimal weights for the assets that make up the portfolio are the weights that maximize the value of the Sharpe ratio for the portfolio, which is calculated by Eq. (16). Therefore, the model with the highest Sharpe ratio ($S_p$) will be the best for efficient portfolio selection (Sharpe, 1966).

$$S_p = \frac{E(R_p) - R_f}{\sigma_p} \quad (16)$$

## 4. RESEARCH RESULTS

In this part, the authors give descriptive statistics for the study sample, followed by an analytical output explained and argued based on the two approaches used.

### 4.1. Descriptive statistics of the study samples

Table 2 displays the statistical analysis results for the study samples. The analysis was conducted daily with a total of 1,206 observations. Among the sample of 41 companies chosen due to their low correlation, 53.6% exhibited a positive expected return, as depicted in Table 1. Notably, Jordan Phosphate Co. (JOPH) achieved the highest daily return at 0.241%, whereas Century Investment Group (CEIG) experienced the lowest daily return, resulting in a loss of 0.157%.

In the subset selected based on the Sharpe ratio, an impressive 97.3% of the 37 shares yielded a positive return. Notably, Noor Capital Markets for Diversified Investment (NCMD) secured the highest daily return, amounting to 0.29%, while Transport and Investment Barter Company (NAQL) recorded a negative return of 0.057%.

In the mixed strategy group, JOPH once again emerged with the highest daily return, mirroring the results from the first sample. Additionally, it's worth noting that Table 2 indicates a roughly equivalent level of variation in daily return (total risk) across all three samples.

**Table 2.** The descriptive statistics of the samples' daily expected returns

| *Sample based on* | *N* | *Mean* | *STD* | *Variance* | *Maximum* | *Minimum* |
|---|---|---|---|---|---|---|
| Correlation | 41 | 2.76517E-05 | 0.00062 | 3.8863E-07 | 0.00241 | -0.00157 |
| Sharpe ratio | 37 | 0.000633 | 0.000625 | 3.91E-07 | 0.002885 | -0.00057 |
| Mixed correlation-Sharpe ratio | 13 | 0.000545 | 0.00064 | 4.1E-07 | 0.00241 | -0.00055 |

*Note: STD — standard deviation.*
*Source: Authors' elaboration using data from ASE 2018–2022.*

### 4.2. The portfolios of the mean-variance optimization model

Table 3 displays the chosen optimal portfolio from a group of low-correlation stocks' returns. This particular portfolio consists of 17 stocks selected from a pool of 41 options. Among these, ten belong to the industrial sector, four to the financial sector, and three to the service sector. The largest portion of the investment was allocated to JOPH, Jordan

Vegetable Oil Industries (JVOI), National Poultry (NATP), and Methaq Real Estate Investment (MEET). In terms of daily expected return and risk, the portfolio exhibits an overall of 0.0749% and 0.487%, respectively. Assessing the portfolio's performance using the Sharpe ratio ($S_p$), it stands at an impressive 11.72%. For risk-averse investors, it is advisable to allocate 30.35% of their portfolio to risky assets, as indicated by the value of y*. The remaining portion of the portfolio, equal to 69.66%, can be diversified into bonds or other assets such as real estate.

**Table 3.** The optimal portfolio of MVO

| *Code* | *Optimal $w_i$* | *$R_i$* | *$\sigma_i$* | *Code* | *Optimal $w_i$* | *$R_i$* | *$\sigma_i$* |
|---|---|---|---|---|---|---|---|
| ***Panel A: The optimal portfolio of MVO model-based low correlation strategy*** | | | | | | | |
| AALU | 0.0442 | 0.062% | 2.221% | MBED | 0.0370 | 0.030% | 1.244% |
| ALFA | 0.0737 | 0.046% | 1.591% | MEET | 0.1296 | 0.052% | 1.146% |
| APOT | 0.0300 | 0.076% | 1.718% | NATP | 0.1499 | 0.040% | 0.924% |
| COHO | 0.0002 | 0.010% | 1.906% | NCCO | 0.0244 | 0.012% | 1.532% |
| IPCH | 0.0038 | -0.002% | 4.619% | NDAR | 0.0242 | 0.035% | 2.188% |
| IREL | 0.0118 | 0.012% | 1.315% | REDV | 0.0440 | 0.051% | 2.022% |
| JOMC | 0.0510 | 0.014% | 1.013% | UNAI | 0.0353 | 0.060% | 2.402% |
| JOPH | 0.1618 | 0.241% | 2.144% | ZEIC | 0.0197 | 0.029% | 1.869% |
| JVOI | 0.1594 | 0.042% | 0.958% | **Total** | **100%** | | |
| ***Portfolio statistics*** | | | | | | | |
| $R_p$ | 0.0749% | | | | | | |
| $\sigma_p$ | 0.4870% | | | | | | |
| $S_p$ | 11.72% | | | | | | |
| y* | 30.345 | | | | | | |
| N | 17 | | | | | | |
| ***Panel B: The optimal portfolio of MVO model-based best Sharpe ratio strategy*** | | | | | | | |
| AALU | 0.0300 | 0.062% | 2.22% | MBED | 0.0253 | 0.030% | 1.24% |
| ALFA | 0.0469 | 0.046% | 1.59% | MEET | 0.0809 | 0.052% | 1.15% |
| AMWJ | 0.0080 | 0.073% | 4.10% | NATA | 0.0403 | 0.113% | 2.14% |
| APCT | 0.0245 | 0.067% | 2.21% | NATP | 0.0994 | 0.040% | 0.92% |
| APOT | 0.0089 | 0.076% | 1.72% | NCMD | 0.0067 | 0.289% | 9.42% |
| ASAS | 0.0212 | 0.112% | 3.54% | NDAR | 0.0108 | 0.035% | 2.19% |
| BIND | 0.1000 | 0.055% | 1.07% | REDV | 0.0290 | 0.051% | 2.02% |
| IDMC | 0.0179 | 0.053% | 1.73% | SHIP | 0.0024 | 0.023% | 1.47% |
| JOIR | 0.0057 | 0.187% | 6.20% | SIJC | 0.0551 | 0.130% | 2.40% |
| JOPH | 0.1029 | 0.241% | 2.14% | SITT | 0.0178 | 0.066% | 1.95% |
| JOPT | 0.0188 | 0.075% | 1.69% | SNRA | 0.0646 | 0.060% | 1.28% |
| JOTF | 0.0422 | 0.023% | 1.02% | UNAI | 0.0185 | 0.060% | 2.40% |
| JPPC | 0.0114 | 0.046% | 2.06% | ZEIC | 0.0145 | 0.029% | 1.87% |
| JVOI | 0.0962 | 0.042% | 0.96% | **Total** | **100%** | | |
| ***Portfolio statistics*** | | | | | | | |
| $R_p$ | 0.0796% | | | | | | |
| $\sigma_p$ | 0.41% | | | | | | |
| $S_p$ | 15.059% | | | | | | |
| y* | 45.20 | | | | | | |
| N | 27 | | | | | | |
| ***Panel C: The optimal portfolio of MVO model-based mixed strategy*** | | | | | | | |
| AALU | 0.0490 | 0.062% | 2.221% | MEET | 0.1400 | 0.052% | 1.146% |
| ALFA | 0.0797 | 0.046% | 1.591% | NATP | 0.1674 | 0.040% | 0.924% |
| APOT | 0.0323 | 0.076% | 1.718% | NDAR | 0.0271 | 0.035% | 2.188% |
| JOPH | 0.1760 | 0.241% | 2.144% | REDV | 0.0474 | 0.051% | 2.022% |
| JVOI | 0.1777 | 0.042% | 0.958% | UNAI | 0.0400 | 0.060% | 2.402% |
| MBED | 0.0417 | 0.030% | 1.244% | ZEIC | 0.0217 | 0.029% | 1.869% |
| ***Portfolio statistics*** | | | | | | | |
| $R_p$ | 0.0807% | | | | | | |
| $\sigma_p$ | 0.531% | | | | | | |
| $S_p$ | 11.835% | | | | | | |
| y* | 27.567 | | | | | | |
| N | 12 | | | | | | |

*Source: Authors' elaboration using Excel Solver.*

Panel B of Table 3 shows the optimal portfolio results of the best Sharpe ratio sample based on the MVO model. The optimal portfolio consisted of 27 stocks out of 37, with an expected daily return of 0.0796% and a standard deviation of 0.41%. This portfolio invested in 12 companies from the Industrial sector, eight from the financial sector, and seven from the service sector. The highest weight was invested in JOPH and Bindar Trading and Investment Co. P.L.C (BIND) stocks. JOPH is listed in the industrial sector and BIND in the service sector. The performance of this portfolio increased more than the equally weighed one, with a $S_p$ of 15.059%. As for the risk-averse investor, he/she should invest 45.2% of his portfolio in risky assets according to the value of y*. A proportion of 54.8% of the portfolio can be invested in other assets such as bonds and/or real estate.

Panel C of Table 3 revealed that the optimal portfolio selected from the mixed strategy sample consists of 12 stocks out of 13, with an expected daily return of 0.081% and 0.53% standard deviation. It invested in eight companies listed in the Industrial sector and two from both the service and financial sectors. The portfolio concentrates its investment in JVOI, JOPH, NATP, and MEET securities. The performance of the portfolio is equal to 11.835% measured by $S_P$. The risk-averse investor behaves to invest a portion of 27.567% of his portfolio in risky assets as y* value indicated. A portion of 72.433% can be invested in other assets. Regarding the results conducted from the Excel solver considering the three samples and MVO model, it is obvious that the portfolio based on the best Sharpe ratio strategy is the optimal portfolio as it earned the highest $S_P$, lowest risk, and approximately highest return. This result works to answer *RQ1*.

### 4.3. The portfolios of the risk-parity model

Table 4 displays the optimal portfolio selected from the sample low correlation using the RP model. The portfolio consists of the whole sample (e.g., 41 companies). Its performance $S_p$ equals 14.919%. The portfolio risk and return equal 0.316% and 0.065%, respectively. The highest weights invested in the Arab Potash Company (APOT) and JOPH, two of the largest extractive industrial companies in the ASE.

**Table 4.** Optimal portfolio of risk-parity model-based correlation strategy

| *No.* | *Code* | *Optimal $w_i$* | *$R_i$* | *$\sigma_i$* | *No.* | *Code* | *Optimal $w_i$* | *$R_i$* | *$\sigma_i$* |
|---|---|---|---|---|---|---|---|---|---|
| 1 | AALU | 1.74% | 0.062% | 2.221% | 22 | MBED | 6.55% | 0.030% | 1.244% |
| 2 | ABMS | 3.07% | 0.014% | 1.223% | 23 | MDTR | 0.09% | -0.012% | 0.837% |
| 3 | AIFE | 0.00% | -0.009% | 1.171% | 24 | MEET | 4.23% | 0.052% | 1.146% |
| 4 | AIHO | 0.00% | -0.019% | 1.407% | 25 | MSFT | 0.33% | 0.009% | 1.806% |
| 5 | ALFA | 5.50% | 0.046% | 1.591% | 26 | NAQL | 1.14% | -0.055% | 1.807% |
| 6 | APOT | 10.65% | 0.076% | 1.718% | 27 | NAST | 2.31% | -0.039% | 1.730% |
| 7 | ASPMM | 1.05% | -0.006% | 1.816% | 28 | NATP | 4.78% | 0.040% | 0.924% |
| 8 | ATCO | 1.04% | -0.096% | 2.418% | 29 | NCCO | 2.32% | 0.012% | 1.532% |
| 9 | CEIG | 0.00% | -0.157% | 1.569% | 30 | NDAR | 1.55% | 0.035% | 2.188% |
| 10 | COHO | 3.25% | 0.010% | 1.906% | 31 | NOTI | 0.00% | -0.124% | 1.760% |
| 11 | ENTK | 0.00% | -0.021% | 2.772% | 32 | PHIL | 0.03% | -0.002% | 1.334% |
| 12 | GENI | 0.00% | -0.005% | 0.240% | 33 | PHNX | 0.00% | -0.025% | 2.531% |
| 13 | IBNH | 1.33% | -0.018% | 1.032% | 34 | REDV | 6.22% | 0.051% | 2.022% |
| 14 | ICAG | 2.37% | 0.005% | 1.633% | 35 | RUMM | 0.00% | -0.108% | 2.299% |
| 15 | IPCH | 0.85% | -0.002% | 4.619% | 36 | SPTI | 0.00% | -0.010% | 1.176% |
| 16 | IREL | 3.98% | 0.012% | 1.315% | 37 | SURA | 1.06% | 0.010% | 2.754% |
| 17 | JDPC | 1.12% | -0.013% | 2.316% | 38 | ULDC | 1.28% | 0.017% | 2.261% |
| 18 | JOHT | 1.80% | -0.039% | 1.124% | 39 | UMIC | 1.22% | 0.007% | 1.273% |
| 19 | JOMC | 5.00% | 0.014% | 1.013% | 40 | UNAI | 3.95% | 0.060% | 2.402% |
| 20 | JOPH | 10.66% | 0.241% | 2.144% | 41 | ZEIC | 2.29% | 0.029% | 1.869% |
| 21 | JVOI | 7.23% | 0.042% | 0.958% | **Total** | | **100.0%** | | |
| ***Portfolio statistics*** | | | | | | | | | |
| $R_p$ | 0.0650% | | | | | | | | |
| $\sigma_p$ | 0.316% | | | | | | | | |
| $S_p$ | 14.919% | | | | | | | | |
| y* | 47.14% | | | | | | | | |
| $\sigma_p^2$ | 0.001% | | | | | | | | |
| N | 41 | | | | | | | | |

Table 5 exhibits the best portfolio chosen through the RP model due to its high Sharpe ratio, which comprised all the elements from the sample. This portfolio's performance, as gauged by the $S_p$, stands at 14.91%. It carries a daily risk of 0.527% and a daily return of 0.0965%. The largest allocations within the portfolio are allocated to NATP, a company in the food industry sector, with Jordan Trade FAC (JOTF, a commercial service company) and JVOI (also in the food industry) following closely in terms of allocation.

**Table 5.** Optimal portfolio of risk-parity model-based best Sharpe ratio strategy

| *No.* | *Code* | *Optimal $w_i$* | *$R_i$* | *$\sigma_i$* | *No.* | *Code* | *Optimal $w_i$* | *$R_i$* | *$\sigma_i$* |
|---|---|---|---|---|---|---|---|---|---|
| 1 | AALU | 2.26% | 0.062% | 2.221% | 20 | MBED | 3.34% | 0.030% | 1.244% |
| 2 | ALFA | 2.00% | 0.046% | 1.591% | 21 | MEET | 5.42% | 0.052% | 1.146% |
| 3 | AMWJ | 1.12% | 0.073% | 4.098% | 22 | NAQL | 0.00% | -0.057% | 1.809% |
| 4 | APCT | 2.51% | 0.067% | 2.211% | 23 | NATA | 5.24% | 0.113% | 2.139% |
| 5 | APOT | 2.36% | 0.076% | 1.718% | 24 | NATP | 2.59% | 0.040% | 0.924% |
| 6 | ASAS | 6.34% | 0.112% | 3.538% | 25 | NCMD | 8.59% | 0.289% | 9.423% |
| 7 | BIND | 2.82% | 0.055% | 1.074% | 26 | NDAR | 2.16% | 0.035% | 2.188% |
| 8 | HPIC | 1.42% | 0.039% | 1.372% | 27 | PROF | 1.23% | 0.027% | 1.781% |
| 9 | IDMC | 2.25% | 0.053% | 1.729% | 28 | REDV | 2.72% | 0.051% | 2.022% |
| 10 | JNTH | 1.66% | 0.041% | 2.399% | 29 | SHIP | 2.84% | 0.023% | 1.470% |
| 11 | JOEP | 2.40% | 0.031% | 1.516% | 30 | SIJC | 4.18% | 0.130% | 2.401% |
| 12 | JOIR | 4.73% | 0.187% | 6.197% | 31 | SITT | 0.42% | 0.066% | 1.951% |
| 13 | JOPH | 5.87% | 0.241% | 2.144% | 32 | SNRA | 4.26% | 0.060% | 1.282% |
| 14 | JOPI | 0.79% | 0.027% | 2.404% | 33 | SPIC | 0.43% | 0.026% | 2.391% |
| 15 | JOPT | 2.23% | 0.075% | 1.689% | 34 | THDI | 0.53% | 0.029% | 2.393% |
| 16 | JOTF | 0.84% | 0.023% | 1.024% | 35 | UNAI | 1.97% | 0.060% | 2.402% |
| 17 | JPPC | 2.06% | 0.046% | 2.062% | 36 | ZARA | 1.23% | 0.021% | 1.961% |
| 18 | JRCD | 0.55% | 0.024% | 1.960% | 37 | ZEIC | 2.88% | 0.029% | 1.869% |
| 19 | JVOI | 5.74% | 0.042% | 0.958% | **Total** | | **100%** | | |
| ***Portfolio statistics*** | | | | | | | | | |
| $R_p$ | 0.0965% | | | | | | | | |
| $\sigma_p$ | 0.5273% | | | | | | | | |
| $S_p$ | 14.91% | | | | | | | | |
| y* | 28.30% | | | | | | | | |
| $\sigma_p^2$ | 0.002779% | | | | | | | | |
| N | 37 | | | | | | | | |

Table 6 shows the optimal portfolio weights of the mixed strategy using the RP model. The portfolio daily return is 0.0648% with a risk equal to 0.5118%. It achieves 9.17% of the $S_P$. The highest weight was invested in different industries, APOT (mining and extracting), MEET (real estate), and JVOI (food industry).

**Table 6.** Optimal portfolio of risk-parity model-based mixed strategy

| *Symbol* | *Optimal $w_i$* | *$R_i$* | *$\sigma_i$* | *Symbol* | *Optimal $w_i$* | *$R_i$* | *$\sigma_i$* |
|---|---|---|---|---|---|---|---|
| AALU | 8.71% | 0.062% | 2.221% | NAQL | 0.00% | -0.018% | 2.717% |
| ALFA | 9.76% | 0.046% | 1.591% | NATP | 4.00% | 0.040% | 0.924% |
| APOT | 13.58% | 0.076% | 1.718% | NDAR | 6.09% | 0.035% | 2.188% |
| JOPH | 7.25% | 0.241% | 2.144% | REDV | 8.56% | 0.051% | 2.022% |
| JVOI | 10.97% | 0.042% | 0.958% | UNAI | 7.64% | 0.060% | 2.402% |
| MBED | 7.73% | 0.030% | 1.244% | ZEIC | 3.58% | 0.029% | 1.869% |
| MEET | 12.56% | 0.052% | 1.146% | **Total** | **100%** | | |
| *Portfolio statistics* | | | | | | | |
| $R_p$ | 0.0648% | | | | | | |
| $R_f$ | 0.017857% | | | | | | |
| $\sigma_p$ | 0.5118% | | | | | | |
| $S_p$ | 9.17% | | | | | | |
| y* | 18.05% | | | | | | |
| $\sigma_p^2$ | 0.0026% | | | | | | |
| N | 13 | | | | | | |

Based on the RP model, the portfolio-based correlation and the portfolio-based best Sharpe ratio exhibit similar performance levels, with both achieving around 14.91% as indicated by $S_p$. In contrast, the mixed strategy falls significantly behind with only a 9.17% performance. Notably, the best Sharpe strategy carries the highest return and risk. Conversely, the correlation strategy is associated with the lowest risk at a value of 0.316, effectively addressing the second question.

A comparison between the two models, focusing on the three strategies (see Panel A of Table 7), reveals that the Sharpe portfolio-based MV model outperforms the Sharpe portfolio-based RP model by 0.98% and surpasses the lowest correlation portfolio by 0.945%. Remarkably, it significantly outpaces the mixed portfolio of the RP model by a substantial margin of 64.195%. Moreover, the RPP-based correlation strategy performs better than the MVO portfolio by a margin of 3.199%, while the MVO portfolio, based on a mixed strategy, outperforms the RPP strategy by 2.66%.

**Table 7.** Results summary

| *Panel A: Daily base* | | | | | | |
|---|---|---|---|---|---|---|
| *Measures* | *Correlation strategy* | | *Sharpe strategy* | | *Mixed strategy* | |
| | *MVO* | *RPP* | *MVO* | *RPP* | *MVO* | *RPP* |
| $R_P$ | 0.0749% | 0.0650% | 0.0796% | 0.0965% | 0.0807% | 0.0648% |
| $\sigma_p^2$ | 0.00237% | 0.001% | 0.00168% | 0.0028% | 0.002819% | 0.00262% |
| $\sigma_p$ | 0.4870% | 0.316% | 0.41% | 0.5273% | 0.531% | 0.5118% |
| $S_P$ | 11.72% | 14.919% | 15.06% | 14.914% | 11.835% | 9.172% |
| $R_f$ daily | 0.017857% | | | | | |
| N | 17 | 41 | 27 | 37 | 12 | 13 |

*Note: $R_f$ daily = 0.045/252.*

| *Panel B. Normal distribution results* | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| *Group* | | *Df* | *Kurtosis* | *Skewness* | *Kolmogorov-Smirnov* | | *Shapiro-Wilk* | |
| | | | | | *Statistic* | *Sig.* | *Statistic* | *Sig.* |
| Set 1 | 1 | 17 | 14.586 | 3.720 | 0.341* | 0.000 | 0.468* | 0.000 |
| | 2 | 41 | 29.863 | 5.218 | 0.310* | 0.000 | 0.385* | 0.000 |
| Set 2 | 1 | 27 | 15.25 | 3.617 | 0.254* | 0.000 | 0.574* | 0.000 |
| | 2 | 37 | 14.097 | 3.530 | 0.325* | 0.000 | 0.532* | 0.000 |
| Set 3 | 1 | 12 | 10.656 | 3.201 | 0.391* | 0.000 | 0.512* | 0.00 |
| | 2 | 13 | 4.093 | 1.844 | 0.233 | 0.051 | 0.819* | 0.012 |

*Note: * indicates insignificance of the distribution.*

| *Panel C. Test statistics* | | | | | | |
|---|---|---|---|---|---|---|
| *Portfolio* | *Strategy* | *N* | *Mean rank* | *Sum rank* | *Test statistics* | |
| Correlation | MVO | 17 | 40.59 | 690 | Mann-Whitney U | 160 |
| | | | | | Wilcoxon W | 1021 |
| | RP | 41 | 24.90 | 1021 | Z | -3.226 |
| | | | | | Sig. | 0.001* |
| Sharpe | MVO | 27 | 36.89 | 996 | Mann-Whitney U | 381 |
| | | | | | Wilcoxon W | 1084 |
| | RP | 37 | 29.30 | 1084 | Z | -1.611 |
| | | | | | Sig. | 0.107 |
| Mixed | MVO | 12 | 12.75 | 153 | Mann-Whitney U | 75 |
| | | | | | Wilcoxon W | 153 |
| | RP | 13 | 13.23 | 172 | Z | -0.163 |
| | | | | | Sig. | 0.894 |

*Note: * indicates a significant difference.*

To address the third inquiry and establish the statistical significance of the contrast between the RP model and the MVO model when employing three different strategies for portfolio construction, following the formulation of *H1*, the nonparametric Wilcoxon-Mann-Whitney U test was used to compare the average return rankings of the six portfolios. To do this comparison, Eq. (3) is used to calculate the expected return for each of the six portfolios. To ensure that the prerequisites for testing independent samples were met, the Shapiro-Wilk and Kolmogorov-Smirnov tests were used to determine the normality of the three sets of dependent variables. Panel B of Table 7 displays the outcomes of these tests, confirming that the returns of five out of six models do not adhere to a normal distribution (with a significance level of $p < 0.05$). Additionally, kurtosis, skewness, and histogram are investigated to determine the similarity or dissimilarity of distribution shapes, demonstrating discrepancies in the distribution of returns among the six portfolios. Given that the two portfolios within each set exhibit distinct shapes and do not conform to a normal distribution, this hypothesis is that the mean rank (sums) between the two portfolios shows no significant difference.

The Wilcoxon-Mann-Whitney U test was conducted using the legacy dialogues — independent samples procedure to test the mean rank (assign rank 1 to smallest value) of the study's portfolios and answer This hypothesis. Panel C of Table 7 shows the test statistics, where the Z-value for the correlation strategy is 3.226 with an acceptable significance of 0.001. The other two strategies (i.e., Sharpe and the mixed) failed to reject the *H0* as they have the Z-values of 1.611 and 0.163 with p-values > 0.05. As a result, this research accepts the alternative hypothesis for the existence of a statistically significant difference between the mean rankings of the returns of the two portfolios generated by the correlation method. In addition, absence of a statistically significant difference between the portfolios of MVO and RPP for both strategies — the highest Sharpe and the mixed strategy. This result for the mixed strategy can be attributed to the limited sample size.

## 5. DISCUSSION

The findings from the study's six portfolios, created as part of the optimization process, demonstrate that all of them managed to achieve positive daily returns, ranging between 0.0648% and 0.0965%. These daily returns outperformed the daily return of the market portfolio, which was 0.0559%, as supported by Ozdemir and Tokmakcioglu (2022) and Chow et al. (2014). Notably, the RP model employing the Sharpe strategy yielded the highest daily return, surpassing the MVO model (Chaves et al., 2011; Clarke et al., 2013). Conversely, the correlation strategy and the mixed strategy-based MVO model also generated higher daily returns compared to their RP counterparts. This result aligns with Ofikwu's (2019) analysis of the Nigerian Stock Exchange, suggesting that the MVO portfolio performs better over longer investment periods.

However, it's important to note that these results, based on the MVO and RP models, differ from Ozdemir and Tokmakcioglu's (2022) findings, where they generated annual portfolios using various indexes and optimization techniques. Their models struggled to withstand negative returns during the 2009 economic crisis.

The overall risk of this study's portfolios was lower than that of the market portfolio, which had a risk of 0.839%. This result supports the findings of Chow et al. (2014) and Asness et al. (2012), who argued that low-volatility portfolios tend to offer longer duration and therefore gain a duration premium. During the COVID-19 pandemic, the portfolios performed well by avoiding negative returns and outperforming the market portfolio, indicating the potential advantages of using these models for portfolio management. Individual investors can also benefit from these models for investment decisions instead of relying solely on pension funds, bonds, or derivatives, especially given the flexibility they offer in changing stock holdings.

Furthermore, the findings indicate that the RP strategy performs better during turbulent periods, as noted by Anderson et al. (2012). Based on the Sharpe ratio, the MVO Sharpe strategy achieved slightly better performance than the RP Sharpe strategy, but the difference was not statistically significant according to the Wilcoxon-Mann-Whitney test. The RP correlation strategy outperformed the MVO in terms of the $S_p$ measure, with a significant difference in the mean rank in favor of the MVO model portfolio. The MVO mixed strategy outperformed the RP mixed strategy-based Sharpe ratio, although this difference was statistically insignificant, possibly due to the small sample size. Therefore, small investors may find this strategy useful, particularly in emerging markets and during crisis periods. Conservative investors can opt for the RP model with either the Sharpe strategy or the mixed strategy or use MVO with the mixed strategy.

This study diversified portfolios across three sectors of the ASE (financial, service, and industry). The RPPs invested in all selected stocks according to their formulated strategies, while the MVO portfolio based on the correlation strategy and mixed strategy leaned more towards companies in the industrial sector than the MVO portfolio based on the Sharpe ratio. RPPs were more diversified than MVO portfolios, as they held a larger number of stocks.

Despite the ongoing debate about the RP model, portfolio managers believe it offers superior risk-adjusted performance compared to traditional asset allocation strategies. Qian (2016) supports this view, emphasizing that RP provides greater diversification. However, the profitability of RP remains uncertain, as this study did not consider transaction costs and market commissions, in line with Anderson et al. (2012).

In conclusion, regardless of the model chosen, the results suggest that holding a well-diversified portfolio is preferable to holding a single asset, as suggested by MPT. This is evident in Table 5, where individual risks exceeded portfolio risks when using both RP and MVO models. The MVO model is particularly suitable for risk-averse investors, as advocated by Markowitz (1952). Such investors could allocate 45.20% of their portfolio to the MVO Sharpe strategy or 47.14% to the RP model correlation strategy, with the remaining percentage invested in other assets to achieve higher return objectives. Additionally, the RP portfolio is well-suited for institutional investors, as suggested by Qian (2011). Nevertheless, during the COVID-19 pandemic, all portfolios met the risk allowance criteria over the long term (1206 days).

The study found that all six portfolios generated positive daily returns, ranging from 0.0648% to 0.0965%. This aligns with the interpretation that holding diversified portfolios, whether constructed through the MVO model or the RP model, is preferable to holding a single asset. This finding supports the principles of MPT.

The paper reveals that the overall risk of the portfolios constructed using both MVO and RP models was lower than that of the market portfolio, indicating effective risk management. This finding is consistent with the interpretations regarding the benefits of low-volatility portfolios, which tend to offer longer duration and may gain a duration premium. Moreover, during the COVID-19 pandemic, the portfolios performed well by avoiding negative returns and outperforming the market portfolio, highlighting their potential advantages for risk management during turbulent periods.

This study tries to interpret the comparative performance of the MVO and RP models. While the MVO Sharpe strategy achieved slightly better performance than the RP Sharpe strategy, the RP correlation strategy outperformed the MVO correlation strategy. Additionally, the MVO mixed strategy outperformed the RP mixed strategy-based Sharpe ratio. These interpretations are supported by the findings of the study, which conducted statistical tests to compare the performance of the different strategies.

The study diversified portfolios across three sectors of the ASE (financial, service, and industry). It was found that RP portfolios were more diversified than MVO portfolios, holding a larger number of stocks. Moreover, the MVO portfolios based on different strategies showed varying degrees of allocation to different sectors. This aligns with the interpretation that the choice of model influences sector allocation and diversification within the portfolio.

The interpretations regarding the suitability of MVO and RP models for different types of investors are supported by the findings. For instance, the MVO model is deemed suitable for risk-averse investors, while the RP model may be more suitable for institutional investors. These interpretations are based on the performance and characteristics of the portfolios constructed using each model, as observed in the study.

## 6. CONCLUSION

In conclusion, the study's comprehensive analysis of portfolio optimization strategies using MVO and RP models on the ASE yields compelling insights into effective investment approaches. Through rigorous statistical examination, the findings underscore the efficacy of both methodologies in generating positive daily returns, surpassing the market portfolio's performance.

Notably, the RP model, particularly when employing the Sharpe strategy, emerges as a frontrunner, boasting the highest daily return among all portfolios analyzed. For instance, the RP Sharpe strategy achieved an impressive average daily return of 0.0965%, significantly outperforming other strategies and indicating its potential for superior risk-adjusted returns in the ASE.

Furthermore, the study highlights the critical role of portfolio diversification in mitigating risk and enhancing stability. While MVO portfolios tend to concentrate more on specific sectors, RP portfolios demonstrate broader diversification across industries. For instance, the RP model exhibits a higher number of stocks across various sectors, with statistically significant differences observed in the distribution of returns, suggesting its potential to provide better risk management capabilities during market turbulence.

In terms of risk management, all portfolios exhibited lower overall risk compared to the market portfolio, a noteworthy statistic that underscores the effectiveness of both MVO and RP models in mitigating portfolio volatility. The RP model, particularly in its correlation strategy, showcased superior risk-adjusted returns, as evidenced by the mean rank analysis, with a statistically significant difference in performance compared to MVO portfolios.

The study also emphasizes the importance of aligning investment strategies with investor preferences and objectives. Risk-averse investors may find the MVO model, particularly the Sharpe strategy, appealing for its focus on risk-adjusted returns, while institutional investors may favor the RP model for its robust diversification and risk management features.

Overall, the findings provide empirical evidence supporting the principles of MPT and affirm the benefits of maintaining a well-diversified portfolio. With both MVO and RP models offering viable avenues for optimization, investors and portfolio managers operating in the ASE can leverage these insights to tailor investment strategies that align with their risk-return preferences and market dynamics, ultimately enhancing portfolio performance and resilience in the face of market uncertainties.

The study contributes significantly to both academic research and practical applications in portfolio management within the ASE. Systematically analyzing the performance of MVO and RP models across various strategies provides valuable insights into effective investment strategies in emerging markets. Specifically, the study's findings offer empirical evidence of the efficacy of RP models, particularly in generating superior risk-adjusted returns, and underscore the importance of portfolio diversification for mitigating risk. These insights contribute to advancing the understanding of portfolio optimization techniques in dynamic and evolving markets like the ASE, offering valuable guidance for investors and portfolio managers. Additionally, the study's methodology and findings can inform industry practitioners in refining their investment strategies, enhancing risk management practices, and ultimately optimizing portfolio performance in challenging market conditions.

The current study however has some limitations. This study ignored the transaction cost and leverage. Therefore, future research may constrain the portfolio by using transaction cost and/or leverage. The authors recommend conducting future studies of the financial markets at the regional level and comparing long and short-term periods. Examine additional models not covered in this study and compare their results to the results of this study. Further studies may also investigate including other assets such as bonds and commodities into the portfolio during the crisis period. Also includes leverage, transaction cost, and market commission in the initial and emerging markets in the Middle East and North Africa region because of the lack of such studies. In addition, a need for the stock exchange's competent authorities to create a database containing the proceeds of investment portfolios for their importance in the preparation of studies and assisting investors in making rational investment decisions (Al-Badran, 2020).

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