CORPORATE CONTROL AND OWNERSHIP NETWORKS

Mirko Di Giacomo *, Marisa Cenci **

* *Corresponding author* Department of Business Studies, University of Rome III, Italy
Contact details: Via Silvio D'Amico, 77 - 00145, Rome, Italy
** Department of Business Studies, University of Rome III, Italy

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Abstract

In this paper, authors consider ownership networks to quantify the ease with which a company can be controlled due to the shareholding relationships in which it is involved. These networks have been usually considered in a descriptive perspective, either to quantify the control exerted by an ultimate shareholder, especially in presence of complex patterns of indirect control, or as a subject of topological analysis. Recently, a new stream of literature arose, solving optimization problems on ownership networks. Among these tools, authors explicitly refer to the Indirect Control Problem (IC) (Martins & Neves, 2017), which determines the minimum cost control strategy of a set of Target company, namely a strategy to build a robust investment fund which includes the corporate control on one or more companies. In this paper, we combine the descriptive and the optimization approach, introducing a linear programming model, namely Cheapest Control Problem (CCP), contributing on both the descriptive and the optimization approach. In particular, authors propose CCP overcome some of the IC main limitations, i.e. the overestimation of control in presence of mutual cross-shareholdings. Furthermore, CCP solutions allow computing three indexes that measure the ease with which a company can be controlled depending on its ownership relationships. Finally, a case study is incorporated to compare IC and CCP solutions, discussing the informative power of the indices introduced.

Keywords: Corporate Governance, Ownership Network, Network Optimization, Mixed-Integer Linear Programming.

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1. INTRODUCTION

Complex ownership patterns may make difficult the identification of the owner of a company. According to the literature, four reasons can motivate the existence of complex ownership patterns (Levy & Szafarz, 2016): i) to reinforce the collaboration between companies, that control each other; ii) As an element of implicit collusion and increase of the market power of all the companies that are part of it; iii) As an element to make the companies attractive to external shareholders, as there is no dominant subject that controls every single company; iv) As an element to extract private benefits of control to the detriment of minority shareholders (i.e. tunnelling). Among these reasons, the last turns out to be the most accredited one for the existence of the pyramids (Wolfenzon, 1999; La Porta et al., 2000; Dorofeenko et al., 2007; Levy, 2009; Paligorova & Xu, 2012; Romei et al., 2015; Levy & Szafarz, 2016).

As a consequence, the exact quantification of the control represents a fundamental aim from a policymaking or regulatory point of view. Once ownership and control relations are clear, for example, a minority shareholder can evaluate the possibility of the tunnelling to happen; similarly, a policymaker can carry out a better analysis and propose effective rules to defend minority shareholders.

Thus, the precise measurement of control is required, but the literature still suffers a lack of consensus on this topic, due to methodological discrepancies concerning the definition of control and the way in which cross-shareholdings are taken into account. As a consequence, the correct estimation of control is still an open issue.

This paper is based on a very intuitive idea: the shareholding relationships in which a company is involved have an impact on the ease with which it can be controlled. For example, gaining control is relatively more expensive and difficult in a company that is currently owned by a 51% shareholder than in a company with a widely dispersed shareholding structure. Furthermore, the higher the control exerted by a majority shareholder, the higher should be the cost in order to get control of a company.

The aims of this paper are twofold: the measurement of the ease with which changes in the current shareholding structure can occur and the identification of the more convenient strategy to control a Target company, when the focus is on saving as much as possible and the strategic options are either direct or indirect (e.g. through pyramids) control. Once such questions are answered, it is possible to identify the companies that can be easier get controlled, verifying whether a society is controlled and, in such a case, the ultimate owner. Then, as an innovative element, we propose to measure the control exerted by each shareholder on a company, starting from the company and its shareholding relationships, i.e. its position in the ownership network, rather than from the percentage of shares held by each shareholder.

Indeed, with the same aim, other measures have been used in literature, as, for example, Herfindahl index (Rotundo & D'Arcangelis, 2014); the innovative element of this paper consists in the introduction of measures that consider the location of a listed company in an ownership network. In other words, in the estimation the control exerted by any shareholder, for each company we consider its current shareholding structure and all ownership ties in which it is involved, both directly and indirectly.

Threats to the current shareholder structure can originate from who is already a shareholder of the company or from an external subject. For this reason, once one or more Target companies have been identified within the market, we consider two kinds of Actors, i.e. an Insider and an Outsider. An Actor is referred to as an Insider if a path exists between its node and the Target company in the ownership network. If this does not occur, it is referred to as an Outsider.

All the measures we introduce rely on the concept of minimum cost control strategy, namely the percentage of shares to buy on the market in order to get control on a set of target company at minimum cost.

The Outsider Fragility Index (OFI_i) and the Insider Fragility Index (IFI), represent the ease with which control over a Target company can be obtained by an Outsider or an Insider, respectively.

The third measure we introduce is the Outsider Strategy Adaptability Index (OSAI_i) to control company i, which represents a measure of the 'adaptability' of the cheapest strategy to control a Target company.

In order to determine the minimum cost control strategy, we introduce the Cheapest Control Problem, a linear programming problem, which is deeply inspired by Indirect Control problem (Martins & Neves, 2017). Even if CCP and IC share the same aim, they differ from a mathematical and formal perspective. As we will discuss, CCP is proposed to overcome the main drawbacks of IC, including the overestimation of control in presence of mutual cross-shareholdings. It is important here to underline that the presence of mutual cross-shareholdings have always represented in literature an obstacle in the correct estimation of control, either in terms of the correct value to assign to each shareholder or in terms of computational complexity. We contribute to literature proposing a method to correctly estimate the control exerted by each shareholder. The method we propose needs a negligible computational effort.

The paper is structured as follows. Next section introduces the literature review, then, in the third section, we discuss on corporate control, with a particular focus on IC problem and its drawbacks. In the fourth and fifth section, we mathematically define the CCP problem and its features and we define the OFI_i, IFI_i^j and OSAI_i, respectively. In the sixth section, a case study is considered and a discussion of the proposed features and any additional information they may provide is given.

2. LITERATURE REVIEW

This paper can be considered the intersection of three different streams of literature: ownership networks, quantification of corporate control, mathematical programming on graphs.

Qualitatively speaking, an ownership network is a graph where nodes represent shareholders and arcs represent shareholding relations. A weight is associated with each oriented arc, expressing the percentage of shares held by each shareholder. The use of ownership networks is very common in complex networks literature, where they have been studied in order to discover their topological properties. These studies share some features: they are mainly empirical papers, focused on particular geographical contexts and for these reasons we call this set of works 'descriptive approach'. For example, the first paper to introduce a topological analysis on ownership networks focuses on German firms (Kogut & Walker, 2001). Other Authors followed, focusing on Japan (Souma et al., 2005), Czech (Dietzenbacher & Temurshoev, 2008), European (Vitali et al., 2011; Pecora & Spelta, 2015), Italian (Bertoni & Randone, 2006; Corrado & Zollo, 2006; Rotundo & D'Arcangelis, 2014), Spanish (Sacristán-Navarro & Gómez-Ansón, 2007), and Chinese (Li et al., 2014; Chang & Wang, 2017) companies or banks(Flood et al., 2017).

It is interesting to note that some works refer either to control or ownership networks, meaning interlocking directorates network, i.e. graphs in which firms are connected by common directors, rather than shareholding relationships (Battiston et al., 2003; Robins & Alexander, 2004; Milaković et al., 2010; Takes & Heemskerk, 2016; Sapinski & Carroll, 2017; Esposito De Falco et al., 2018). Nevertheless, a common result of these studies is that such networks are featured in a so-called 'small world' property. In a small world network, a high number of companies are connected to others by means of a small number of ties. An immediate consequence of such a property is that the information spreads among its component quite fast (Christiano Silva & Zhao, 2016). Ownership networks have been used also for other purposes, as, for example, to study the consequences of the distress of one or more companies in the network, i.e. the so-called financial contagion effect (Elliott et al., 2014; Bardoscia et al., 2017; Dastkhan & Gharneh, 2017, 2018).

It is interesting to note that a relatively small body of literature has used ownership networks to study control-related topics. In particular, most of them were focused on the identification of the ultimate owner or on the analysis of control distribution within the network, i.e. a description of the flow of control (Battiston, 2004; Chapelle & Szafarz, 2005; Dorofeenko et al., 2007; Glattfelder & Battiston, 2009; Soltaninejad, 2016) or control concentration (Brancaccio et al., 2018).

Beside of famous theories concerning complex ownership patterns (Wolfenzon, 1999; Almeida & Wolfenzon, 2006), other streams of literature studied topics related to the separation between ownership and control and the quantification of control. Among them, we consider the econometric approach (Claessens et al., 2000; Laeven & Levine, 2008; Paligorova & Xu, 2012; Ben-Nasr et al., 2015; Lemma & Negash, 2016; Lingmin, 2016; Saghi-Zedek, 2016; Napoli, 2018) and the game theory approach. The main contribution of the econometric approach consists in the introduction of a rule aimed to separate ownership from control rights, the so-called 'weakest link principle' (Claessens et al., 2000). This rule can be summarised as follows: if company A owns 5% of shares of company B and company B holds 20% of company C, then company A controls 5% of company C. An extensive discussion of the drawbacks and limitations of the weakest link principle can be found in Dietzenbacher and Temurshoev (2008).

The game theory approach analyses the voting game in the race for control. To this aim, the so-called 'power indices' have been introduced, namely the Banzhaf and Shapley indices. An extensive literature review on applications of power indexes to corporate governance voting games can be found in Crama and Leruth (2013). The game theory approach to corporate control quantification represents a heterogeneous set of works, which differ for aims and scope of analysis. For example, in Levy (2009) the focus is in the comparison of existing methods and algorithms aimed to identify the owners and controllers of a firm in a pyramidal structure without cross-ownership. In Karos and Peters (2015) a general abstract model is introduced to study the voting game in the area of corporate governance and new power indices are developed, with the aim to represent the real power of the involved players. In Aminadav et al. (2011) game theory and networks are both used to achieve a binary matrix in which it is stated whether a subject controls a company or not. In Levy and Szafarz (2016) is proposed a game-theoretical method to measure the extent of shareholder expropriation through cross-ownership and in Rungi et al. (2017) game theoretical tools are used on ownership networks with the same aim. Power indices suffer from several drawbacks; as Levy and Szafarz (2016) noted, they didn't succeed in financial literature for two reasons: first, the determination of Banzhaf and Shapley values require a huge computational effort, especially in presence of a high number of shareholders; second, their interpretation is not easy, especially if compared to the intuitiveness of graph theoretical tools.

In addition, as noted in Edwards and Weichenrieder (2009), in general, Banzhaf and Shapley indices can assign different power values to the same shareholder, in the same context. Then, the choice of the index introduces in itself a bias in the analysis. Finally, we note that the use of game theoretical tools doesn't allow to consider threats that originate from the external boundaries of the company. In other words, power indices can measure of the control exerted by each shareholder but cannot measure the ease with which corporate control changes may occur due to an Outsider threat, which is one of the results of this paper.

Some optimization models have been introduced very recently to address problems related to ownership networks and control issues (Romei et al., 2015; Martins & Neves, 2017). This paper is deeply related to such a very small stream of literature. In Romei et al. (2015) introduce three optimization problems: i) the integrated ownership problem, i.e. the quantification of shares owned by a shareholder either directly or indirectly through other companies; ii) the dividend problem, i.e. how much yearly dividends of a company a shareholder receives either directly or indirectly through other companies; iii) corporate group problem, i.e. what are the groups of companies controlled by a common parent shareholders. The Indirect Control problem (IC), introduced by Martins and Neves (2017), identifies a minimum cost strategy in order to control a set of Target companies. IC problem deeply inspired CCP, even if the former has a completely different aim from the latter, i.e. to suggest a sound investment strategy that includes the control of some Target companies. Next section provides a discussion of with the similarities and differences between these two linear programming models, with a specification of the IC drawbacks and the way in which CCP to overcome them.

3. CONTROL DEFINITIONS AND PREVIOUS APPROACHES

In this paper, we say that an Actor directly controls company i if he holds enough voting shares to do so, i.e. the percentage of i shares he owns is higher than a fixed threshold, which we shall refer to as θ_i. According to Eurostat (2010), a shareholder who owns between 10% and 50% of a company is someone with influence, rather than control. Nevertheless, we acknowledge the possibility of effective minority control as a de facto direct control, obtainable with a participation share of lower than 50% (Eurostat, 2010). For these reasons, and despite the fact we do not formally refer to explicit values of θ_i, we believe that, for practical purposes, only values of $\theta the_i = 0{,}51$ for each company i should be considered.

Company i is indirectly controlled by an Actor if the sum of participation in i of the Actor and other companies, as different from i and either directly or indirectly controlled by the Actor, is higher than θ_i. For example, consider the case in which the Actor owns 11% of company i and already has direct control over companies A and B. Company i is indirectly controlled by the Actor if the sum of i shares held by companies A and B is strictly higher than 39%.

The idea that gaining indirect control of a Target company can be cheaper than directly controlling it was first explored by Martins and Neves (2017), who introduced the Indirect Control

(IC) problem. Here, Authors explored the problem facing IC and were able to determine the cheapest strategy that could be used to control a set of Target companies. They then went on to discuss the validity of such an approach through case-oriented studies.

Martins and Neves (2017) realised that the main issue facing IC was that of control overestimation when presented with cross-ownership structures, i.e. cycles in the shareholding network. Based on this, they provided three versions of the problem, which are differentiated by the way in which they exclude mutual cross-shareholdings. In their study, prevention of circular ownership connections involving more than two companies was not included, meaning overestimation of control is still an open issue. In the next section, we introduce the Cheapest Control Problem (CCP) and explore whether such an approach is effective in finding the cheapest strategy to control a set of Target companies while correctly estimating control.

A minor issue of the model proposed by Martins and Neves (2017) is that it allows the purchasing of shares held by the relevant shareholders at the market price. From our point of view, it is difficult to assume that a shareholder with significant ownership of a company would be inclined to sell his shares at the market price asked for by a very small investor since such a price would be too low. Such an issue has been resolved in CCP by allowing the Actor to only buy shares owned by very small investors.

4. THE CHEAPEST CONTROL PROBLEM (CCP)

In this section, we provide a mathematical formulation of the Cheapest Control Problem (CCP) and explain how to identify the cheapest strategy starting with CCP's optimal solutions.

In CCP we consider an oriented graph $G = (V, A)$, where V is the set of selected listed companies, with $|V| = n$, and A is the set of arcs, i.e. the shareholding relationships among the companies. As is common throughout the literature, we only consider arcs that represent top-holder (i.e. qualified) positions; in other words, arc (i, j) exists only if i owns at least 1% of a company j. In this way, we omit the most volatile shares and we implicitly assume that the shareholding positions in the network are stable. The Actor is called company A, i.e. its node is labelled A.

We decompose the strategy used to control a set of Target companies via several logical steps, whereby $t = 1, 2, \dots, n-1$. It is easy to note that no more than $n-1$ logical steps are needed. Indeed, considering that $|V| = n$ and that one of its elements is the Actor, no more than $n-1$ companies can be controlled. Assuming that in each t exactly one and only one company falls under an Actor's control, no more than $n-1$ steps are required.

To maintain consistency, and to ensure a connection between the current study and the previous literature, we will express certain relations in relation to a set of Target companies, T.

We start describing CCP from sets, parameters and variables.

Sets

C_{Act}: set of companies currently controlled, directly or indirectly, by the Actor.

T: set of Target companies.

Parameters

α_i : market price of 1% of company i, for each $i \in V$ ($\alpha_i > 0$)

θ_i : percentage of shares needed to control company i, for each $i \in V$.

s_{ij} : percentage of j shares held by i, for each $(i, j) \in A$.

Variables

$x_i^{(t)} \equiv$ percent of company i bought on the market at step t by the Actor, for each $i \in V$

$$y_i^{(t)} = \begin{cases} 1 & \text{if company } i \text{ is under control at step } t \\ 0 & \text{otherwise} \end{cases}$$

Based on the aforementioned variables, CCP is defined as follows:

$$\min \sum_t \sum_i \alpha_i x_i^{(t)}$$

s.t.

$$y_i^{(0)} = 1 \qquad \forall i \in C_{Act} \tag{1}$$

$$y_i^{(0)} = 0 \qquad \forall i \notin C_{Act} \tag{2}$$

$$\sum_{t=1}^{n-1} y_i^{(t)} \geq 1 \qquad \forall i \in T \tag{3}$$

$$\sum_{j \neq i} y_j^{(t-1)} s_{ji} + s_{Ai} + \sum_{j=0}^{t-1} x_k^{(j)} \geq \theta_i y_i^{(t)} \qquad \forall i \neq A, \quad t = 1, \dots, n-1 \tag{4}$$

$$y_i^{(t)} \geq y_i^{(t-1)} \qquad \forall i \neq A, \quad t = 1, \dots, n-1 \tag{5}$$

$$\sum_{t=0}^{n-1} x_i^{(t)} \leq 100 - \sum_j s_{ji} \qquad i = 2, \dots, n \tag{6}$$

$$y_i^{(t)} \in \{0,1\} \qquad \forall i \neq A, \quad t = 0, \dots, n-1$$

$$x_i^{(t)} \geq 0 \qquad \forall i \neq A, \quad t = 0, \dots, n-1$$

Objective function represents the cost of strategy that has to be implemented by the Actor.

Constraints (1) and (2) are initial conditions relating to the companies that are under an Actor's control before the problem is solved. Constraints (3) state that each Target company has to be under control.

Finally, constraints (4) describe the mechanism that allows CCP to correctly estimate control.

In each step t, a company i is 'under the Actor's control' if one of the two following conditions holds:

– The sum of the percentage of i shares owned by companies under the Actor's control at step $t-1$ plus the percentage of i shares bought by the Actor in all previous steps, is at least θ_i;

– Company i was under control in previous steps.

More formally, we say that company i is under control at step t, i.e. $y_i^{(t)} = 1$, if:

$$\sum_{j \neq i} y_j^{(t-1)} s_{ji} + s_{Ai} + \sum_{j=0}^{t-1} x_k^{(j)} \geq \theta_i$$

Where s_{Ai} are the i shares held by the Actor.

In this way, cross-ownership relations do not affect the estimation of control, because the percentage of shares still needed to control company i, if any, depends exclusively on the companies that are already under control and on the shares formerly bought in previous steps.

Constraints (5) require that if i is 'under an Actor's control' at step t, then it must be 'under control' also in the subsequent steps.

Constraints (6) state that shares owned by a qualified participant (i.e. someone who owns at least 2% of the company) cannot be bought out. As a consequence, if $\exists\, i | s_{ij} \geq \theta_j$ then company j cannot be controlled by the Actor directly, and there is the potential that the optimal strategy cost ends up higher than the cost of direct control. The presence of constraints (4) and (6) is a distinctive feature of our model when compared to previous literature. In order to analyse whether and how optimal strategy changes, in Section 4 we will solve CCP by exploring one of the case studies previously studied in the literature.

Let $x_i^{*(t)}$, $y_i^{*(t)}$ denote, respectively, the optimal values of $x_i^{(t)}$, $y_i^{(t)}$.

Let us call x_i^* the optimal percentage of company i shares bought by the Actor in the cheapest strategy, and y_i^* a binary variable equal to 1 if, in the cheapest strategy, company i is under the Actor's control (and 0 otherwise). Then, for each $i \in V$, the cheapest strategy to control a set of Target companies is described by the following variables:

$$x_i^* = \sum_t x_i^{*(t)}$$

and:

$$y_k^* = \begin{cases} 1 & if \sum_t y_k^{*(t)} \geq 1 \\ 0 & otherwise \end{cases}$$

The next section introduces measures of network effects on corporate control. We will call Z_j^i the objective function optimal value when the Actor is the company i (i.e. either one of the Insiders or an Outsider) and the Target is the company j.

5. MEASURING NETWORK EFFECTS ON CORPORATE CONTROL

Henceforth, we consider optimal control strategies aimed at controlling only one Target company, i.e. $|T| = 1$. When measuring network effects on the ease with which Target corporate control changes can occur, it is important to verify the relation between the cost of the Outsider optimal strategy controlling the Target and the cost of direct control. In such a case, the cheaper the indirect control, the higher the network effect becomes; a phenomenon we shall call 'Outsider Fragility'.

As Martins and Neves (2017) noted, an indirect control strategy can be partially hidden due to an indirect and diluted investment, for such a process avoids the expected reaction of the stock's market when feeling the Outsider's willingness to control the Target company. For this reason, such a kind of analysis can be useful from several perspectives. Indeed, in addition to a real Outsider, who is more than likely to be interested in knowing the cost needed to control the Target company, the actual controlling shareholders may be interested in understanding whether threats to their own control exists and if so, the extent to which they can exert their own strength. Furthermore, from a policy-making point of view, these analyses could allow a better supervision of the market for corporate control, since it allows one to understand, for any given network, which companies are fragile and which companies are strong. Let Z_i^{Out} be the CCP objective function optimal value when company i is the Target company and the Outsider is the Actor. Such a value represents the cost of the Outsider's cheapest strategy when attempting to control the Target company by considering the network to which it belongs.

Let $\alpha_i\theta_i$ be the market value of the percentage of shares that ensures an Outsider's control over the Target company. Here, three cases should be considered:

- $Z_i^{Out} = \alpha_i\theta_i$;
- $Z_i^{Out} < \alpha_i\theta_i$;
- $Z_i^{Out} > \alpha_i\theta_i$.

In the first case, the cost of the cheapest strategy needed to control the Target company is equal to the cost of direct control. As a consequence, there is no network effect on the ease with which corporate control changes can occur, i.e. the relations in which the Target company is involved do not have any influence on the current shareholding structure. In this case, we define the network effect as 'neutral' with respect to network effects.

In the second case, it could be possible for an Outsider to gain indirect control over the Target company at a cost lower than that of direct control. In particular, the cheaper such a strategy is, the higher the intensity of the network effect becomes in relation to potential corporate control changes. In other words, the more convenient the indirect control strategy is, the more influential the relations of the Target company become regarding the current shareholding structure. In this case, we define the Target as 'fragile' with respect to network effects.

In the last case, the cost an Outsider incurs is higher than the cost of direct control. This is due to a highly concentrated ownership structure, which makes it difficult to realise any change in the company control. In this case, we define the Target company as 'sturdy' with respect to network effects.

For these reasons, we consider the following OFI_i value as the most appropriate measure of the ease with which company i control changes can occur due to Outsider threats:

$$\mathrm{OFI}_i = \frac{\alpha_i\theta_i - Z_i^{Out}}{\alpha_i\theta_i}$$

Note that Z_i^{Out} is always strictly positive and, for this reason, $\mathrm{OFI}_i \in (-\infty, 1)$.

In particular, value $\mathrm{OFI}_i = 0$ describes a neutral Target company. Positive OFI_i values describe a fragile Target, i.e. the higher the OFI_i value, the higher the fragility of the current shareholding structure.

Finally, negative OFI_i values denote a sturdy Target company, meaning that it is very difficult for control changes to occur. Each possible Target company in a network can have one associated OFI_i value. In order to ascertain the most fragile company in the network, i.e. the company in which control changes are easier, the OFI_i value of each company has to be determined. The most fragile company in the network is the one that has the highest OFI_i value.

In a real environment, the realisation of the strategy suggested by CCP could be subjected to unforeseen difficulties, i.e. a company that was set to be controlled in the Outsider optimal strategy can no longer be controlled. For example, the optimal strategy could require that the Outsider buys enough shares of company k to control it directly. If during the execution of the cheapest strategy a shareholder v of company k increases its participation to 50%, then the previous CCP solution is no more feasible, since the Outsider cannot buy enough shares to directly control k due to constraints (6). Broadly speaking, an Outsider might be interested in evaluating the additional cost incurred when the situation reveals that it is not possible to control, either directly or indirectly, one of the companies belonging to the optimal solution of CCP. We propose the Outsider Strategy Adaptability Index (OSAI_i) to control company i as a measure of the 'adaptability' of a strategy, i.e. the ease with which the CCP optimal strategy can be adapted to a new context.

OSAI_i provides relevant information to both the Outsider and the current shareholding structure in that it outlines the expected cost increase given that the cheapest strategy cannot be realised due to an uncontrollable company. While such information is useful to an Outsider wanting to evaluate a contingency plan, it is also beneficial for the current shareholding structure, since it allows one to consider, as a defensive strategy, the possibility of buying shares of one (or more) company in order to increase the OSAI_ivalue as much as possible.

In order to define OSAI_i we recall the value Z_i^{Out} as the CCP objective function optimal value when company i is the Target company and the Outsider is the Actor. In particular, we define S as the set of companies, different from the Target, that have to be controlled in the optimal solution of CCP in order to control the Target, i.e. $S = \{i \neq A, T | y_i^* = 1\}$.

Let $Z_{i|j}^{Out}$ be the CCP objective function optimal value when company i is the Target company, the Outsider is the Actor and company j is no longer controllable. From a practical point of view, the $Z_{i|j}^{Out}$ value can be obtained as a solution of CCP by adding the constraint $\sum_t y_j^{(t)} = 0$. Then, assuming that $|S| = m$ for illustrative purposes, OSAI_i is defined as follows:

$$\mathrm{OSAI}_i = \frac{\sum_{j \in S} Z_{i|j}^{Out} - Z_i^{Out}}{m Z_i^{Out}}$$

OSAI_i represents the expected cost increase in relation to the optimal solution of CCP, where i is the Target company. This is based on the fact that a company that was formerly part of the cheapest control strategy is no longer controllable.

Note that $Z_{i|j}^{Out} \geq Z_i^{Out}$ and, for this reason, OSAI_i is always non-negative. In particular, a strategy where $\mathrm{OSAI}_i = 0$ is perfectly adaptable, since there is no cost variation due to a company becoming uncontrollable. Note that a strategy such that $\mathrm{OSAI}_i \geq 1$ means that if a company belonging to the optimal strategy becomes unavailable, then the expected cost increase of the strategy is at least equal to the cost of the previous optimal strategy, i.e. the higher the OSAI_i value is, the less adaptable the strategy becomes.

Clearly, if $Z_i^{Out}(1 + \mathrm{OSAI}_i) < \theta_i \alpha_i$ then the CCP strategy is still more convenient, on average, than the achievement of direct control.

The third analysis is interesting for both the market supervisor and the shareholder of the company i in that it enables one to understand the ease with which one of the Insiders can gain control over the Target company. In other words, we propose an index that measures the strengths of threats to the current shareholding structure due to Insiders' actions. I_i is the set of Insiders of company i, with $|I_i| = v,$ and we define Insider Fragility Index (IFI_i^j) of a company i due to an Insider j as:

$$IFI_i^j = \frac{(v-1) Z_i^j}{\sum_{k \in I_i / \{j\}} Z_i^k}$$

We will have one IFI_i^j value for each Insider of the Target company.

IFI_i^j compares the optimal control strategy cost that Insider j is expected to incur against the mean cost of the remaining Insiders. The lower such a ratio is, the stronger the threat posed by Insider j becomes, meaning that Insider j has to incur a cost lower than the average cost of the other Insiders to gain control over the Target company.

In more detail, note that $IFI_i^j \in (0, \infty)$ since $Z_i^j > 0$, and that value $IFI_i^j = 1$ implies that Insider j has no particular incentive to threaten the actual corporate governance structure, since the cost he would incur is exactly the same as the mean cost that the remaining shareholders would incur.

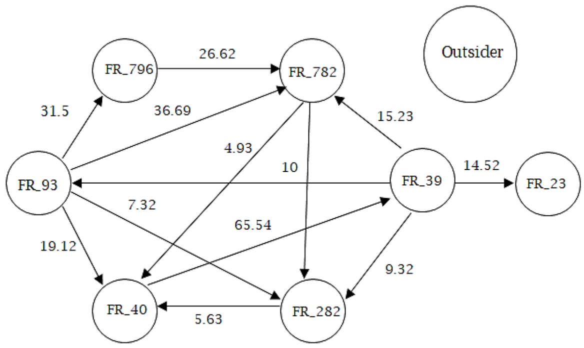
6. NETWORKS EFFECTS ON CORPORATE CONTROL: A CASE STUDY

In this section, we measure network effects on the controlling structure of a Target company. With this aim in mind, and in order to keep links with previous literature, we refer to the same dataset and stock prices considered in (Martins and Neves, 2017), where further details on the dataset and methodology used to build the graph can be found. Throughout their research, the Authors considered several case studies; we, however, have chosen to focus on one of them, namely the case study in which there is only one Target company, called FR_23.

This case is particularly interesting due to the complex ownership and cross-holding structure that can be found in the graph to which FR_23 belongs; such complexity enables a deep comparison between IC and CCP optimal strategies.

The dataset used by Martins and Neves (2017) consists of a shareholding graph $G = (N, A)$, with $|N| = 748$ and $|A| = 578$, where all the arc weights are greater or equal to 1%, that is $s_{ij} \in [1, \dots, 100]$ for all $(i, j) \in A$, due to the selection of qualified participation only.

Figure 1. Ownership network considered in the case study



We begin by considering the Insiders of company FR_23, i.e. all the nodes that contribute to the path leading to FR_23. In other words, we deal with the graph represented in Fig. 1, which allows us to empirically verify whether a shareholding portion of 10-50% is considered influential rather than controlling and, broadly speaking, whether ambiguity can be avoided when the necessary threshold value is higher than 50%. Indeed, two shareholders of company FR_782 hold more than 25% and it is not clear whether and who effectively controls FR_782, when $\theta_i = 25$, as previously assumed in literature.

This section is structured as follows. First, we solve CCP with FR_23 as the Target company by assuming, in the same way as (Martins and Neves, 2017), that $\theta_i = 25$ for each company in the network. We do this in order to compare IC and CCP solutions, and to prove that CCP correctly estimates control. Second, in order to avoid any kind of ambiguity, we solve CCP with $\theta_i = 51$ for each company in the network, finding the cheapest strategy that Outsiders can use to control FR_23. Finally, we compute OFI_i, OSAI_i and IFI_i^j, before discussing the results.

6.1. CCP and IC problems: A comparison

The Indirect Control (IC) optimal strategy when $\theta_i = 25$ for each company in the network and $T = \{FR_{23}\}$ suggests that 10.48% of FR_23, and 15% of FR_93, should be bought. The cost of this strategy is €4,100,741,640, which is 49.06% less than the 25% purchase rate of FR_23 when obtaining direct control. However, the solution is ambiguous since it implies that the Outsider can control FR_93 with a shareholding portion of 15%, due to the presence of the cross-ownership structure. Indeed, if the Outsider buys 15% of FR_93, then it has no control over it and, consequently, cannot control the remaining 10% of FR_93 held by the company it controls indirectly (FR_39).

On the other hand, CCP correctly estimates control, as its following optimal solution reveals that the Outsider should:

– buy 0.95% of FR_40;
– buy 25% of FR_93;
– buy 10.48% of FR_23.

In this way, the Outsider gets direct control over FR_93 and consequently holds:

– Indirect control over FR_782 and FR_796, which are directly controlled by FR_93;
– Indirect control over FR_40, due to the shares owned by FR_93 (19.12%) and FR_782 (4.93%), as well as the shares bought on the market. Consequently, it indirectly controls FR_39, which is directly controlled by FR_40;
– Indirect control over FR_23, due to the shares owned by FR_39 (14.52%), plus the shares bought on the market (10.48);
– As a collateral effect, indirect control over company FR_282, which is directly controlled by FR_782.

Although the CCP optimal strategy costs more than the IC optimal solution (€4,653,384,330.76), the Outsider ultimately saves 42.19% when compared to buying 25% of FR_23.

6.2. Cheapest strategy to control FR_23

In this section, we solve CCP based on the when $\theta_i = 51$ for each company in the network. Based on the higher threshold value needed to directly control a company, it costs €16,421,646,162.00 to directly control FR_23. The CCP optimal strategy recommends:

– Buying 51% of FR_40, thus gaining direct control over it. Consequently, the Outsider has indirect control over FR_39 and indirectly holds 14.53% of FR_23;
– Buying 36.48% of FR_23 on the market; when taken with the 14.53% held by FR_39, this ensures indirect control of the Target company.

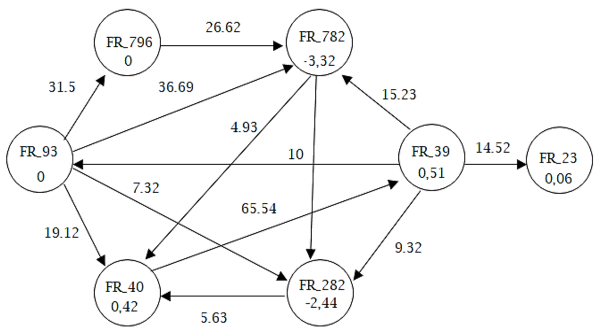
CCP optimal strategy has a cost of €15,422,069,681.76, i.e. it costs 6.09% less than direct control.

6.3. Measuring network effects: OFI value

In this section, we discuss the information provided by the OFI_i value.

First, we note that $\mathrm{OFI}_{FR\_23} \cong 0.06$, meaning that its shareholding structure is quasi-neutral with respect to Outsider threats. Then, in order to compare companies, we compute OFI_i values for all the companies in the network, i.e. we solve several instances of CCP, assuming that each company in the graph is the Target company. Results are given in Fig. 2, where OFI_i values are represented as node labels.

Figure 2. OFI_i values of each company in the network



A comparison between OFI_i values of each company in the network shows that companies FR_93 and FR_796 are neutral, while FR_782 and FR_282 are sturdy companies. The most fragile companies in the network are FR_39 and FR_40. However, it should be stressed that FR_40 is more relevant than FR_39, since the former directly controls the latter. In other words, Fig. 2 shows that, if FR_40 is fragile and directly controls FR_39, then the latter will be a fortiori fragile.

6.4. Measuring network effects: $OSAI_i$ value.

In this section, we analyse the adaptability of the optimal CCP strategy to control a Target company.

As mentioned previously, the minimum cost strategy for the Outsider to control FR_23 consists of buying 51% of FR_40 and 36.48% of FR_23. Then, control over the Target company (FR_23) is obtained by adding shares held by FR_39 (which is directly controlled by FR_40) to shares bought by the Outsider on the market.

As is easy to observe, two companies can create difficulties when attempting to realise the CCP optimal strategy, i.e. $S = \{\mathrm{FR\_40}, \mathrm{FR\_39}\}$. The OSAI_i value (in this case, where FR_23 is the Target company) is equal to 6%, meaning that, on average, if a company belonging to S becomes uncontrollable, then the new optimal strategy has a 6% higher cost than the previous CCP optimal strategy. This kind of analysis adds an extra layer of information to the simple solution of CCP and its cost evaluation. We must remember that, in cases whereby a company becomes uncontrollable, the CCP strategy is more convenient, on average, than direct control if $Z_i^{Out}(1 + \mathrm{OSAI}_i) < \theta_i \alpha_i$. By including the expected cost increase due to a company becoming uncontrollable in the determination of the cheapest strategy to control FR_23, the results reveal that the CCP optimal strategy is not more convenient with respect to gaining direct control, since $Z_i^{Out}(1 + \mathrm{OSAI}_i) = 0.51\alpha_i$.

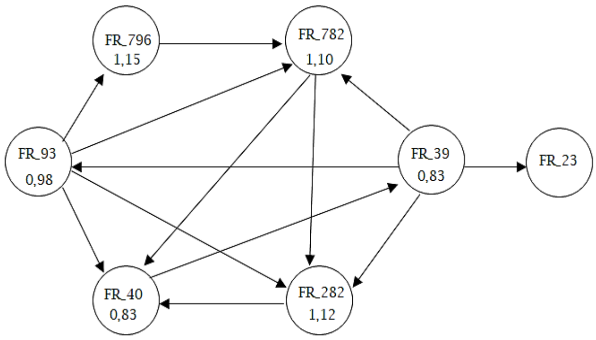
6.5. Measuring network effects: IFI_i^j value.

In this section, we discuss the information available through the IFI_i^j value.

We begin by computing IFI_i^jvalues for each Insider of FR_23. The results, represented in Fig. 3 as node labels, clearly show that, among the Insiders, the main threat to the current shareholding structure can be found.

Indeed, remember that the lower the IFI_i^jvalue associated to Insider j is, the stronger the threat it poses, and that $IFI_i^j \in (0, \infty)$, it is evident that FR_39 and FR_40 have the biggest incentive to gain control over FR_23 when compared to the other companies. In particular, companies FR_282, FR_796 and FR_782 would incur a cost higher than that of the average Insider if they attempted to gain control over FR_23. Thus, we can conclude that there is no particular incentive to gain control over the Target company. Furthermore, we can observe that FR_93 has very little incentive to gain control over FR_23, since the IFI_i^j value it has associated is roughly equal to 1, meaning that it would incur a cost equal to the average Insider cost to gain control over the Target company.

Figure 3. IFI_i^j values of each company in the network when T=FR_23



7. CONCLUSION

In this paper, we discussed and explored the idea that the shareholding relationships in which a company is involved have an impact on the ease with which it can be controlled. With the aim of measuring the influence of these relationships on the degree to which corporate control changes can occur, we adopted CCP, a linear programming model inspired by the Indirect Control (IC) problem and introduced three indices based on its optimal solution. In this paper, we outlined ways to overcome the IC issues relevant to our framework, e.g. the overestimation of control in the presence of mutual cross-shareholding, namely cycles in the ownership network. Once CCP was solved, we were able to obtain three measures pertaining to the impact of shareholding relationships on potential corporate control changes. The first was that of the OFI_i value, which measures the strength of threat from an Outsider, i.e. an Actor who has neither direct nor indirect shares in the Target company. The second was the OSAI_i value, which measures the adaptability of the optimal strategy for an Outsider wanting to gain control over the Target company. Such an index outlines the expected cost increase given that the cheapest strategy cannot be realised due to an uncontrollable company. Such information is useful both for the Outsider and for the current shareholding structure, enabling the former to evaluate a contingency plan, and allowing the latter to use the OSAI_i value to develop a defensive strategy.

The third index we introduced, i.e. the IFI, measures the strength of the threat from an Insider, i.e. an Actor who has either direct or indirect shares in the Target company when considering the current shareholding structure.

Furthermore, we incorporated a case study to compare the CCP and IC results and discussed the features of CCP in relation to the correct control estimation; furthermore, a discussion related to the informative power of the indices provided. It is important to underline here the limitations of this work, that are the mainly the limitations of CCP. Indeed, indices provided in this work can be computed on the basis of any model aimed to find a minimum cost strategy to control a Target company. As a consequence, the model used to obtain this result is extremely relevant. As for the CCP, three are the relevant: the time, the regulatory framework and other actors' reactions. Indeed, CCP implicitly assumes that all financial operations on the market are performed instantaneously. This is clearly possible, but the quoted price of the bought shares should react, depending on several factors as, for example, market liquidity and shareholder base wideness. This is not modelled in CCP since the price is considered as a constant parameter. On the contrary, if the buy operations suggested by the strategy are executed in a time window, for example, a year, then such a strategy should consider price movements, that should be modelled as a consequence. As a further element of improvement, other Actors' reaction should be taken into account as, for example, the current controlling shareholder who wants to obstacle the realization of the strategy.

Furthermore, in many countries, the regulatory framework requires an Actor to promote a takeover bid when he comes to hold, directly or indirectly, a percentage of shares, or control rights, above a fixed threshold. These and other regulatory aspects, which may vary across Countries, have a strong impact on the cost of any strategy to control a Target company. Such aspects have not been considered because the focus of the CCP formulation is the correct estimation of control in presence of cross-ownership relationships. Clearly, further developments of CCP, which overcome the above-mentioned drawbacks and the introduction of additional indices could be promising. Additionally, future research should investigate ways to deal with an unstable shareholding network, i.e. explore the possibility of the shareholding structures of the companies in the network changing during the execution of the optimal CCP strategy.

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