IS AN EQUALLY WEIGHTED GLOBAL INVESTMENT PORTFOLIO THE OUTPERFORMER?

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Abstract

The paper builds, in the first part, a benchmark index based on the optimal mix of indices for the global asset classes of equity, fixed-income securities, real estate, commodities, and currencies including cryptocurrencies so as to maximize the ex-post Sharpe ratio. The objective of the first part is to help investors across the globe compare portfolio performance with a uniform benchmark. In the second part, a comparison of portfolio performances is based on five methods of portfolio construction viz; 1) historical returns and variance matrix used along with Markowitz model to discover optimal weights for portfolio components, 2) modification to this approach by using autoregressive integrated moving average (ARIMA) based predicted returns in place of historical returns, 3) global minimum volatility (GMV) portfolio, 4) global market weight portfolio and 5) equal weight portfolio. The objective in the second part is to explore an easy-to-use and at the same time conceptually sound method to build portfolios for any investor worldwide even if such an investor does not have access to or does not wish to rely upon the views and opinions of investment experts. The ex-post performance of portfolios based on these five methods is compared with the ex-post performance of 207 global active and passive funds. This comparison suggests that an equal-weighted portfolio with periodical rebalancing gives the best Sharpe ratio for a global investor.

Keywords: Optimum Global Portfolio, Global Portfolio Benchmark, Equal Weight Portfolio, Periodic Rebalancing, Sharpe Ratio, ARIMA Prediction, Markowitz Optimisation, Exchange Traded Funds

Authors' individual contribution: Conceptualization — A.D. and N.V.; Methodology — A.D. and N.V.; Software — A.D. and N.V.; Validation — A.D. and N.V.; Formal Analysis — A.D. and N.V.; Investigation — A.D. and N.V.; Resources — A.D. and N.V.; Data Curation — A.D. and N.V.; Writing — Original Draft — A.D. and N.V.; Writing — Review & Editing — A.D. and N.V.; Visualization — A.D. and N.V.; Supervision — A.D. and N.V.; Project Administration — A.D. and N.V.

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1. INTRODUCTION

Constructing an optimum investment portfolio is relentlessly pursued by the investment community, including researchers and the industry. With the advent of millennial investors who are comfortable accessing and applying digital offerings, average investors across the globe are actively working on improving their individual portfolio performance. On the supply side, a plethora of investment options is available to choose from. Funds that are actively managed use various techniques and tools to generate alpha, that is, excess returns over the relevant benchmark (Dhanorkar, 2018).

Looking at the stock markets, investors are looking for more heuristic solutions that are computationally simple to implement and seem to be robust. What is lacking in this context is an absolute universal benchmark that every investor across the spectrum can use to compare his or her individual portfolio performance. As regards the "true" global benchmark, the authors debate that the current academic and practitioner know-how does not provide such a market portfolio, which can be readily used universally to compare with any actively managed investment portfolio. The current paper aims to fill this gap.

In addition, what is lacking is an easy-to-use and, at the same time, conceptually sound technique for any investor anywhere in the world to build an optimum portfolio within the available choices. There is no conclusive research on this aspect available in recent times, even though this area has been actively researched for a very long time, starting from the modern portfolio theory (Markowitz, 1952), followed by the value-weighted that is "market" portfolio as a central theme in asset pricing, i.e., the capital asset pricing model (CAPM) (Sharpe et al., 1964); and subsequently the arbitrage pricing theory (Ross, 1976) and the factor models. There are methods such as those developed by Black and Litterman (1991), which require building views on specific securities independently and may involve expert opinion/subjectivity which is not always readily available to an average investor. Such investors would resort to investing in actively managed funds. There are several actively managed mutual funds with varying styles and techniques. Their techniques, methods, and algorithms are not always made public. However, their performance is known in the public domain and hence can be compared with the performance of portfolios constructed using the methods recommended in the academic literature. Moreover, the omnipresent possibility of a market weight or equal-weighted portfolio outperforming all other portfolios does exist in the real world. Therefore this can also not be ignored.

The research objectives are:

- to evaluate and select a method to build an investment portfolio that is conceptually sound as well as easy to use for any global financial investor;
- to set a benchmark performance measure for any global financial investor.

This current study focuses on all investment asset classes across the world to maximise experienced return adjusted for the risk, i.e., Sharpe ratio. The asset classes covered include global equity, global fixed income, short-term, long-term, and sovereign, corporate bonds, global real estate, commodities, and currencies, including cryptocurrencies. The specific securities within the asset classes are selected such that they are representative of all the investible asset classes and at the same time readily available to any global investor. The combination of these securities, which gives the ex-post maximum Sharpe ratio, sets the benchmark for any global investor's portfolio performance. Further, the paper explores five methods to construct a portfolio namely, 1) historical returns and variance matrix used along with Markowitz model to discover optimal weights for the portfolio components; 2) modification to this approach by using autoregressive integrated moving average (ARIMA) (Box & Jenkins, 1970) based predicted returns in place of historical returns; 3) global minimum volatility (GMV) portfolio; 4) global market weight portfolio; 5) equal weight portfolio.

The ex-post performance of portfolios constructed using the techniques above is compared with the ex-post experiences of 207 large active and passive funds out of a total of 7481 funds based on the 2022 Investment Company Factbook (Investment Company Institute, 2022), which represent the bulk of global investment activity across the asset classes.

Finally, the paper establishes that the simple method of constructing an equally weighted portfolio using index-based global exchange traded funds (ETFs) covering all investible asset classes and rebalancing such a portfolio periodically is the most appropriate method for an average investor globally to optimise his reward-to-risk ratio.

The rest of this paper is organised as follows. In Section 2, the literature review is discussed. Section 3 covers the research methodology including the objectives of the study. It also covers the sample dataset details and sources of the same. Sections 4 and 5 give the results and deliberate the data findings based on the comparison with the experienced performance of 207 sampled global active funds. Section 6 concludes the paper.

2. LITERATURE REVIEW

Over the last few decades, multiple theories and models have been developed relating to investment and fund management. Starting from the modern portfolio theory (Markowitz model) to factor models, investors and fund managers have a fundamental goal of optimising the reward-risk ratio (Sharpe ratio). The underlying objective is to identify the "ideal" weights in the asset allocation to attain the optimum Sharpe ratio.

The path-breaking modern portfolio theory (MPT) by Markowitz (1952) laid the foundation for investors and fund managers to construct portfolios using the concept of diversification and the efficient frontier. The theory also discusses the global minimum variance portfolio, and thus, provides a range of efficient portfolios from a minimum risk and above for various investors. The theory has been cogitated since the birth of modern financial economics and has led to the development of many models with variations over the last seventy years. Markowitz's approach and refined versions are now commonly used by Institutional portfolio managers to build their portfolios and undertake performance evaluations (Rubinstein, 2002). Numerous studies over the last seven decades have seen the use of the MPT. Popescu (2022) applied the efficient frontier concept to a combination of digital assets and stocks from the New York Stock Exchange (NYSE) to study the impact on risk returns of highly volatile cryptos and traditional stocks. The findings based on the twenty years of historical returns (2000 to 2021) indicate that for the decade 2011–2021, the efficient portfolios gave better Sharpe ratios as compared to the previous decade in terms of the buy-and-hold strategy. Calvo et al. (2012) analysed the shape of the efficient frontier and found that the shape of the efficient frontier indicated multiple irregularities, which may affect the investor's investments due to risk variations; however, they suggested that if more assets are included in the portfolio, the frontier shape stabilises. However, there have been Barneys to and against MPT and its application in contemporary times. Maillard et al. (2010) argue that optimal portfolios incline towards a focussed subset of stocks and that the mean-variance strategy is sensitive to the inputs. Bielstein and Hanauer (2019) extracted data from Compustat, IBES, and CRSP of one thousand largest stocks based on market capitalisation and developed portfolios based on common stocks which traded on the NYSE, AMEX, and NASDAQ for the period 1985 to 2014. Their findings indicate that the implied cost of capital is a good proxy for expected returns. Despite the academic appeal of the Markowitz mean-variance model, it creates issues related to collating consistent predictions of the expected return of a security. Contemporary research focuses on minimum volatility portfolio optimisation, which implicitly assumes that expected returns for all assets are equal.

There is a lack of detailed and multiple documentation of annual returns of invested global multi-asset market portfolios. Doeswijk et al. (2020) constructed a global multi-asset market portfolio. They developed a yearly return index using distinctive data sets created by them, which covered the full spectrum of the market of financial investors. They studied the returns for the period 1960 to 2017. They created portfolios based on ten asset classes based on market cap, and the constituents of this portfolio were all assets in which financial investors have invested. Their findings indicate that Investors in the market portfolio earned 3.4 per cent per year more than savers.

The equally weighted portfolio (1/N) concept has not gained much traction among the investing community. It is another variation of construction portfolios by allocating equal funds across the portfolio's constituents. From the 1870s, investors from the United Kingdom were suggested to diversify portfolios based on equal weights by financial advisors using the 1/N rule based on the economic situations of the global market (Sotiropoulos & Rutterford, 2018). According to Kritzman et al. (2010), the 1/N methodology's greater performance could be connected to avoiding concentrated portfolios; on rebalancing days, it trades by buying at low prices and selling at higher prices rarely underperforms the poorest performing asset. According to Dew (2014), RenAssets Plamen Monovski in his article has suggested that owning an equally weighted portfolio is an effective way to invest in markets, especially in emerging markets.

Maillard et al. (2010) state that investors contemplate ease-of-use strategies. Two such strategies that can be considered are the global minimum variance portfolio and the equally weighted portfolio. Equity-weighted portfolios that are "1/N" are used in practice as well (Bernartzi & Thaler, 2001). Bolognesi et al. (2013) undertook a comparative study of two major equity index construction methodologies, the weighted market capitalisation methods and the 1/N (equal) weights method on the Dow Jones (DJ) Euro Stoxx index for the period 2002 to 2011. Their findings indicated that equally weighted portfolios gave better risk-adjusted returns than capitalisation-weighted indexes by rebalancing time frequencies (annual, semi-annual, quarterly, and monthly). They further validated their findings using the Fama and French (1993) regression analysis. Research by GlobalCapital (2018) indicates that the portfolios based on market capitalisation are popular; however, their equally weighted comrades have an edge with lesser concentration risk and good performance. Their research further adds that equally weighted portfolios normally share the same large number of stocks in the portfolio on similar lines to the market-capitalisation-based portfolio, which has more weight for the largest companies than the other constituents of the portfolio. This leads to concentration risk. Hlawitschka and Tucker (2008) examined one hundred closed-ended ETFs based on the weekly returns for the period 2002 to 2005 under different equity weights, mean-variance, and equally weighted portfolios. They found that the returns of equally weighted portfolios were more than the ETFs. Those investors who stayed invested in equally weighted portfolios were ready to pay higher fund fees as it provided them more utility than the other portfolios. Pae (2010) researched the conditions under which equally weighted portfolios have higher risk-reward trade-offs than value-weighted portfolios based on the Modigliani-Miller model and CAPM. The results of the sample study by Whited and Wu (2006) indicate that the market premium based on equal weights was greater than the market premium based on value weighting. Their study was based on a sample of all non-financial firms fulling debt criteria from the S&P500. It included a panel of 1390 firms' quarterly results for the sample period October 1975 to December 2001. DeMiguel et al. (2009) analysed portfolios based on seven datasets from the Kenneth-French datasets library from 1963 to 2004. They evaluated the optimised returns and volatility based on fourteen models, including advances in Bayesian estimation designed to reduce estimation errors. Their findings indicated that none of the optimised portfolios consistently outperformed the 1/N (equally weighted) portfolios. They found that the equally weighted portfolios produced Sharpe ratios that were 50 per cent higher than those based on the Markowitz mean-variance models (Markowitz, 1991). Rothery (2016), in this article, highlights that equal-weighted portfolios require some extra exertions, which may lead to outperforming other portfolios in the long run. He identified such a propensity in the race between the equally weighted style of investing and market-cap weighted portfolios; for example, the S&P500, which is market-weighted, gave an average annual return of 7.4 per cent as compared to the equal-weighted rebalanced portfolio of S&P500 which gave an average annual return of 8.7 per cent during 2006–2016.

Plykha et al. (2016) undertook a comparative study of the performance of equal and value-weighted portfolios constructed from large, medium, and small-cap indices from the S&P database. With monthly rebalancing, it was found that equal-weighted portfolios outperformed the value-weighted portfolios based on Sharpe ratio, alpha, and total average returns. They identified that the higher return of the equal-weighted portfolios could largely be attributed to the rebalancing of portfolios to constant weights as needed under equal-weighted portfolios, and a small portion of these higher returns relate to the contribution by systematic risk. Their study relates to one hundred stocks from the S&P500 index for February 1967 to December 2009.

In their research relating to portfolio constraints, Abate et al. (2022) undertook a study to identify which constraints were more effective for generating better returns for investors. Applying multiple popular techniques like classical weights, flexible weights, set on variance, risk of tracking error with beta limitations on the monthly returns of the panel data representation of all sector indices constituting the MSCI World Index all country indices from 1995 till 2020, their results indicated equal-weighted strategies were the best strategies. Their strategies were applied on a rolling window method with annual rebalancing.

Adosoglou et al. (2022) introduced a novel "Lazy Network: A Word Embedding-Based Temporal Financial Network to Avoid Economic Shocks in Asset Pricing Models", neural network-embedding model to develop equally weighted and value-weighted portfolios. The model analysed the impact of economic shocks on the companies that constitute the sampled portfolios, namely four-value weighted and four equally weighted portfolios. The portfolios were constructed on the three-factor model and five-factor alpha models. Their findings indicate that portfolios with 1/N weights returned larger gains in magnitude than the value-weighted results. Additionally, the centrality of the eigenvector was dominant for the equal-weighted portfolios. Raudys et al. (2022) analysed the association between the portfolio performance and the lag using the Sharpe ratio optimisation technique and the 1/N (equal-weighted) method for the historical portfolio returns. Their study was to identify the presence of recurring similar portions of chaotic historical data and whether such information can improve portfolio sustainability. Hedges (2004) dwelled upon different ticket sizes of portfolios based on three equally weighted portfolios of monthly returns of hedge funds. These hedge funds were divided into three categories, namely, small, medium, and large, but equally weighted from 1995 to 2002. His findings indicated that equally weighted small-cap funds were the outperformers as compared to the larger funds and mid-cap funds, thus indicating that equal weighting amongst different-sized portfolio generate unique returns specific to them

Kanuri et al. (2017) investigated the monthly performance of equally weighted dividend twenty-eight ETFs in the bull and bear phase from 2004 to 2014 and compared the same with the S&P500 ETF (as a proxy of US markets). They analysed the absolute and risk-adjusted returns for these funds. Their findings indicate that despite a high correlation between the dividend ETFs and S&P500 ETFs, the former had marginally outperformed the latter during the bull market period and underperformed during the bear period due to higher volatility. Abadi and Silva (2019) evaluated portfolios relating to all 13 countries of the middle east plus north African regions by comparing portfolios based on smoothed cap-weighted, fundamentally weighted portfolios, and equal-weighting portfolios to minimise concentration risk using monthly returns for the period December 2004 up to December 2015. The portfolios were constructed with diverse degrees, varied development schemes, and different areas and evaluated using a big set of performance channels, including Fama-French multi-factor models (Fama & French, 2014) and the seven-factor model. On the variance side, Shigeta (2017) found that the portfolios with equal weights were optimal when a high degree of risk aversion existed amongst the investors about variances of portfolios. Multiple empirical studies have indicated that equally weighted portfolios have performed better than value-weighted portfolios. Studies by Breen et al. (1989), Grinblatt and Titman (1989), Korajczyk and Sadka (2004), Cai and Schmidt (2020), Lessard (1974), Ohlsom and Rosenberg (1982), and Roll (1981) indicate that equally weighted portfolios have generated more and have generated higher returns than value-weighted portfolios.

Forecasting stock prices, indices, fund net asset values, etc., has been one of the most domineering financial speculations market stakeholders are confronted with. Among various forecasting models, the ARIMA is widely applied in various industries, including economics and banking. It is found to be efficient in predicting short-term movements of time series data (Analytics Vidhya, n.d., https://www.analyticsvidhya.com/). Afeef et al. (2018) deployed the Box and Jenkins (1970) ARIMA model to predict the stock process of the oil and gas development company limited (OGDCL) of Pakistan using 3632 observations of the daily prices from 2004 to 2018. Their findings indicate that some of the ARIMA prototypes reflected a strong prospect of predicting short-run prices, thus adding to the existing literature on ARIMA that is effective for short-term prediction. Malik et al. (2017) also found in their study that ARIMA (1, 1, and 0) was apposite for stock prediction of the sample companies, namely, ABL, HBL, and MCB, which are listed on the Karachi Stock Exchange in Pakistan. Pathak and Kapadia (2021) applied the ARIMA model for time series prediction of the stock indices from the National Stock Exchange of India (NSE). Their findings indicate that ARIMA (1, 1, and 2) model reflected a strong potential for predictions in the short term. Ho et al. (2021) undertook a study to predict the stock price using machine learning models of ARIMA, neural network, and long short-term memory. They predicted the prices of FTSE Bursa Malaysia's daily closing prices from January 2020 to January 2021 (COVID-19 period) using the root mean square errors (RMSE) and mean absolute percentage errors (MAPE). Their findings indicated that these models were able to provide significant accuracy for predicting price movement during the pandemic period.

In recent times, the discussion of an equally weighted portfolio outperforming other weighted models has come in for evaluation among industry participants and academicians. Literature indicates that the "1/N" approach seems to do better than the MPT approach or any other approach such as value investing. A key aspect of the equal-weighted approach is that it avoids concentrated portfolios and therefore has lower concentration risk and additionally it reduces the cost of periodic balancing.

Even while replicating an index; it was found that the equally weighted approach gave better returns than other combinations. In a few cases, the equally weighted portfolios were found to outperform the ETFs as well. Past research has indicated that even using temporal financial neural networks, equally weighted portfolios outperformed other portfolios during economic shocks. The research done so far on equally weighted portfolios has been confined to an index or an asset class or geography. Therefore, the current study aims to contribute to the body of knowledge by evaluating top global funds as it would then indicate that the "1/N" approach works at a global level as well.

3. RESEARCH METHODOLOGY

3.1. Sample

3.1.1. For the construction of the global portfolio benchmark

The primary objective is to make the benchmark inclusive and representative of all investible asset classes across the globe, as far as possible. The index representing an asset class is chosen based on its popularity among the ETF providers to set the benchmark for investment performance. This is to ensure sufficient liquidity in investing based on that asset class. Index funds representing the selected indices were chosen based on size, expense ratio, and vintage. As these funds closely track the indices for the global asset classes, any alternative index fund with a similar expense ratio would not alter the research conclusions. The names of the representative index funds and their corresponding indices are enlisted in Table 1 (see the Appendix). These funds cover the following asset classes with securities across the globe:

1. Equities
2. Fixed income securities
3. Commodities
4. Real estate
5. Currency basket
6. Cryptocurrencies and digital assets.

3.1.2. For evaluation of actively managed equity funds

For evaluating various alternatives available to the investors, two hundred and seven globally diversified and actively managed mutual funds inclusive of major equity funds, top twenty fixed-income funds, and top twenty real estate funds were chosen for the study. The net assets of the individual funds ranged from one billion USD to two hundred and fifty-eight billion USD, totalling up to over five trillion USD, which is about thirty-three per cent of the assets of the globally actively managed funds. The data of the net asset values (NAVs) for the sample funds and related benchmarks were collated using Bloomberg (https://bba.bloomberg.net/), Yahoo Finance (https://finance.yahoo.com/), and Investing.com (https://www.investing.com/) as the data sources. The names of the funds and their ticker are tabulated in Table 2 (see the Appendix) the quoted prices for all assets are taken in USD to ensure consistency.

3.2. Data and computation platform

The daily price data for all the ETFs and mutual fund schemes (MFS) considered in the research paper have been taken from Yahoo Finance. The earliest data point for the cryptocurrency is in September 2014. To ensure uniformity in comparison, the daily price data for all assets have been taken from September 2014 to October 2021. The daily data is processed, and monthly Sharpe ratios are computed from October 2014 onwards using Python.

3.3. Methodology

The methodology of the data analysis is discussed in this section. The process covers setting of the global benchmark and evaluating and selecting a method to build an investment portfolio. For, the purpose of, this analysis, monthly return is calculated as:

$$R_i = [P_1 - P_0]/P_0 \quad (1)$$

where, R_i = return on individual security, P_1 = current month's close price, P_0 = previous months close price.

The Sharpe ratio is used as the investment performance metric and hence is sought to be maximised to achieve the best investment performance.

$$Sharpe\ ratio = [R_p - R_f]/SD_p \quad (2)$$

where, R_p = average monthly return, R_f = risk-free rate, SD_p = standard deviation of the respective month returns for the securities.

The maximisation process for the Sharpe, wherever needed, is done using the sequential least-squares quadratic programming (SQLSP) method in the optimisation function of the Scipy library in Python. Further, since we are taking daily data, the change in the risk-free asset on average is generally negligible and hence considered as nil in the relevant calculations throughout the study.

3.3.1. Setting the benchmark

The easiest way to get investment access to an asset class is through ETFs, for example, an investor can replicate the MSCI World or the MSCI All Country World Index (ACWI) (Ziggel & Armbruester, 2016) for global equities. The study aims at setting a benchmark for any international investor. For a global financial investor, securities are available across all asset classes and across the world. Hence, the investment spectrum needs to be inclusive and representative of these variations to set a meaningful benchmark for any global investor.

Albulescu et al. (2021) tested several mature European stock market indices for the existence of nonlinearity. Their findings indicated the presence of nonlinear and chaotic behaviour of stock markets, which require adequate forecasting techniques for the prediction of future prices. This seems to be further support by the research undertaken by Abhyankar et al. (1997), wherein they found the presence of nonlinearity and chaos in stock market returns on the world's six most important stock-markets, FTSE-100, S&P500, DAX, and Nikkei, Futures of FTSE-100 and S&P500, according to their study the presence of volatility clustering is a contributing factor to the nonlinearity.

Forecasting based on machine learning models and algorithms also indicates the same. Guo, Yao, and Yao (2022), built an optimum portfolio using numerical algorithms in the context of market uncertainty with a view to optimally satisfying investors' utility intentions. They adopted a new artificial bee colony algorithm for the same. However, the paper did not explore the best possible portfolio achievable given the actual market data of the global assets. Chen and Zhou (2022) also attempted to build an optimum portfolio using a new algorithm named F-MOEA/D, which is based on a Pareto front evolution strategy and the decomposition-based multi-objective evolutionary algorithm. This paper has again not explored the best possible portfolio achievable on an ex-post basis. Further, Guidolin and Nicodano (2010) attempted to create an optimal portfolio using third and fourth moments applied to the returns of the US industry portfolios (IND) and eleven book-to-market (BM) international portfolios, along with eight international and emerging (IE) market stock indexes. This paper is not applied to all asset classes across the globe. It also does not look at the best possible portfolio on an "ex-post" basis.

Literature supports the fact that since markets are nonlinear it is difficult to predict the ex-ante "ideal" portfolio and hence the "ex-post" returns analysis helps us to identify the "ideal" portfolio.

An "ideal" portfolio can only be known ex-post. Accordingly, that combination of the securities that maximises the Sharpe ratio for a month on an ex-post basis is used to set the benchmark for any investor's global performance.

Using the ETFs as enlisted in Table 1 (see the Appendix), a portfolio was constructed on a monthly basis such that the realised Sharpe ratio was the maximum for that combination of the portfolio constituents.

In this method, it is unlikely that any portfolio built based on these asset classes would consistently outperform the benchmark. Therefore, the benchmark is the "ideal" performance, which every investor should aim to achieve. An investor would know ex-post how close their actual performance was to the benchmark, which would also get released ex-post. The time series of such optimally experienced Sharpe ratios computed for every month from October 2014 onwards are provided in Table 3 (see the Appendix). Continuation of this monthly rebalancing would provide an ex-post framework to the investment community to compare the actual investment performance of any portfolio to what could have been ideally achieved.

The choice of monthly frequency for rebalancing is guided by the fact that a month provides the minimum time for a significant number of data points (about 21) in a cycle to compute the Sharpe ratio. Also, practically rebalancing more than once a month may lead to higher transaction costs. Cycle time longer than a month (a quarter or six months) may be too long to capture any sudden structural changes, and hence relative asset weights may be altered significantly. In view of this, a monthly cycle for rebalancing is chosen.

3.3.2. Evaluating and selecting a method to build an investment portfolio

Part 1

Using the five approaches as outlined below, the portfolio of the chosen index funds was constructed on a monthly basis.

1) Relative weights of the chosen securities based on the maximum Sharpe ratio for the previous month. This is the simplest method as prescribed using Markowitz portfolio optimisation, wherein it is difficult for an investor to form a view on the next month's return and volatility for security.

2) A modified version of the method in one above with returns predicted based on ARIMA and volatility based on the previous month. Relative weights of the selected securities computed to optimise Sharpe with these assumptions would make the coming month's portfolio.

3) A modified version of the method in 1 above wherein the relative weights of the chosen securities for the portfolio for the coming month are based on the minimum volatility in the previous month (global minimum variance portfolio).

4) Relative weights based on the size (market cap) of the index representing the asset class (global index).

5) Equal weight portfolio of the selected securities.

Weights based on the maximum Sharpe ratio in the previous month: In this approach, weights of the constituent assets are derived by maximising realised Sharpe ratio for a particular month. These weights are used to construct the portfolio for the next month. This exercise is repeated every month during the study period. For example, price and return data of the constituent ETFs for October 2014 is used (with the help of the solver algorithm in Python) to derive the weights of these constituents that would maximise the Sharpe ratio. This would be the maximum possible Sharpe ratio, known ex-post; for October 2014. The same weights then are used to construct the portfolio for November 2014 and thus the Sharpe ratio for November 2014 is computed based on the weights derived from October 2014 data. Once again, the same exercise is repeated using the November 2014 price data of the constituents to derive the weights for the portfolio constructed for December 2014. This rebalancing is repeated month on month and the time series of Sharpe ratios are computed (see Table 4 in the Appendix).

Weights are computed by maximising the Sharpe ratio based on returns predicted using ARIMA and volatility observed in the previous month: This approach is a modification of the approach discussed as above wherein the historical returns (i.e., the returns in the previous month) of the constituent assets are replaced by the returns predicted using ARIMA (1, 0, and 1). ARIMA is a method to predict future values of a stationary time series using its past values. The user does not need any additional data and the computation is made simple using the available software tools. As is common for financial time series, the AR component of order 1 and the MA component of order 1 were found to be appropriate (Box & Jenkins, 1976) for predicting the next period return using historical data for daily return in the previous month. Historical volatility for the constituent assets is assumed to persist and hence built into the optimisation in the same manner as in 1 above. The time series of the realised Sharpe ratios are shown in Table 5 (see the Appendix).

Weights based on minimising the previous month's volatility: Global minimum volatility (GMV) portfolio is actively studied in the finance and investment literature and aims to minimise the risk through a process of investing keeping an eye on the volatility and correlation of the constituent of the portfolio ("Natixis Global Asset Management", 2016). In this method, the weights of the constituent assets for a month are derived by minimising the portfolio standard deviation for the previous month. For example, in October 2014, the Solver Add-in of MS Excel was used to compute the weights that minimise the risk. The GMV portfolio for November 2014 was constructed using weights based on the minimum volatility portfolio obtained using October 2014 daily returns. The time series of the realized Sharpe ratios are shown in Table 6 (see the Appendix).

Weights based on the market capitalisation of the indices tracked by the constituents: Doeswijk et al. (2020) constructed a global portfolio from 1959 to 2017 and identified weights based on the market capitalisation of the respective index. The same weights (up to 2017, suitably matched with the asset classes created for this study) were used and further analysis extending the weights from 2017 to 2021 was done using the price data for the respective indices. In this method, the relative weights based on the size (market cap) of the index representing the asset class (global index) are computed on daily basis. The monthly averages of these daily weights of the constituent assets for each month are computed and used to construct the portfolio. For example, for the first month, that is, October 2014, the average global weights are computed as per the dataset developed by Doeswijk et al. (2020), and the same is used to compute the Sharpe ratio time series for this approach. The time series of the realised Sharpe ratios are shown in Table 7 (see the Appendix).

Equal weights: This approach assumes equal weights (1/N) of each security for every month. In the present study, the benchmark is being constructed based on eight asset classes and hence $1/8 = 0.125$ is the weight for each asset class, as represented by the relevant ETF. This process is carried on for each month and the Sharpe ratio is computed. For example, for October 2014, the portfolio return would be the weighted average return, where the weight for each ETF would be 0.125. The time series of the realised Sharpe ratios are shown in Table 8 (see the Appendix).

Part 2

Likely, these methods may not capture all the techniques used by professional fund managers based on several types of intuitive and quantitative investing, factor models, models based on machine learning and artificial intelligence, and various hybrid models. View-based approaches used in practice and even some of the methods presented in the academic literature such as the Black-Litterman model may require subjective evaluation by expert fund managers. Many of these methods are not comprehensively articulated in the public domain. Hence an indirect method for evaluating such approaches was adopted.

For evaluating the actively managed funds' investment performance; two hundred and seven global actively managed funds were selected, representing a large proportion of the actively managed investment universe, and their ex-post Sharpe ratios were compared with the ex-post Sharpe ratios of the portfolios built based on the five exploratory methods as above. Such a comparison was conducted using the one-way analysis of variance (ANOVA) technique with Sharpe ratios as the dependent variable analysed as input categories of the two hundred and seven sampled funds and the five approaches considered for portfolio construction (see Table 9 in the Appendix).

4. RESEARCH FINDINGS

With respect to the first objective of the study, the global benchmark series is shown in Table 3 (see the Appendix) up to October 2021. This series needs to be updated regularly on monthly basis.

The analysis of variance (ANOVA) has shown a significant p-value (F-value: 1.74 for 219 degrees of freedom with a p-value of 0.00). The equal weight portfolio has the highest mean Sharpe ratio over the study time frame of eighty-four months of 0.132. This approach has resulted in an average Sharpe ratio which is higher than all the actively managed mutual funds. Actively managed funds focus on a specific asset class. While they may build "locally" optimum portfolios, a global optimum portfolio over a seven-year period is better achieved through diversification across the asset classes with a simple method of building the portfolio using equal weights and rebalancing on monthly basis.

5. DISCUSSION

The observed superior performance of a simply constructed equally weighted investment portfolio needs further discussion. Among the methods which are documented in academic research, such as the Markowitz model using historical returns and variance or the ARIMA model based on time series concepts; suffer from the fact that the security returns tend to fluctuate widely from one period to the other. Therefore, the historical returns data or even time series techniques such as ARIMA improvising upon the historical returns data would not be meaningful in predicting the weights of securities in a portfolio for achieving optimum Sharpe ratios in future periods.

It may be possible for an expert with superior insights to construct such an optimum portfolio. However, there are three challenges with this approach: 1) expertise, in all likelihood, would be restricted to geography or an asset class and not all-pervasive across all global asset classes; 2) insights and opinions are subjective, and an expert may not be able to retain that edge consistently over a long period of time; and 3) even if such expertise exists, it may not be readily accessible to every investor across the globe.

Therefore, constructing an equally weighted portfolio out of the ETFs which are based on the various global assets may be an "easy-to-use" and at the same time "conceptually sound" method to build a high-performance investment portfolio for any investor across the globe having access to these ETFs.

6. CONCLUSION

All investors aim to construct and maintain an optimum portfolio. The optimisation is aimed at maximising the reward-to-risk ratio. Sharpe ratio is the commonly used metric for the same.

In this context, the paper covers two significant aspects. Firstly, the paper develops a benchmark portfolio with constituents covering all investible asset classes including digital assets and cryptos. The weights of the constituents for the benchmark are derived based on ex-post maximisation of the Sharpe ratio. This benchmark can be readily used by any investor or fund manager to compare ex-post investment performance.

The second part of the paper explores building an ex-ante portfolio with the aim of obtaining optimal performance. Five approaches are explored using the same constituents as in the benchmark developed in the first part of the paper. The five approaches are using 1) relative weights derived based on the optimisation of the Sharpe ratio in the previous month, i.e., classical Markowitz optimization; 2) relative weights derived with the returns predicted using ARIMA and historical volatility used to optimise Sharpe ratio; 3) relative weights derived by minimising volatility in the previous month; 4) relative weights based on the market capitalisation of the indices which are underlying the portfolio constituent ETFs; and 5) equal weights of the constituents.

The ex-post performance of the portfolios using these five approaches is compared with the ex-post investment performance of two hundred and seven global funds using the technique of ANOVA. This comparison leads to the following conclusion.

The limitations of the study are as follow. They are related to the top 207 globally diversified and actively managed funds having net assets up to five trillion USD, which is approximately thirty-three per cent of the actively managed global funds, the sample can be increased depending upon the scope of research. The findings are to be read in the context of the period under study and comparative studies with models like Fama-French and Carhart have not been considered in the present study.

Among all the various models, approaches, methods, techniques, and algorithms prescribed by academicians and followed by practitioners (as represented by the investment performance of the sampled two hundred and seven funds); building an equal-weighted portfolio using ETFs from all investible asset classes investing in global securities and rebalancing the portfolio on monthly basis would be an easy-to-execute as well as an effective method for an average investor to construct the portfolio.

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APPENDIX

Table 1. Fund name, ticker of the fund, and the corresponding index

No.	*Asset class*	*Index representing the asset class*	*Index fund chosen tracking the asset class*
1	Global equity	MSCI All Country World Index (ACWI)	iShares MSCI ACWI Index Fund
2	Real estate_U.S.	MSCI U.S. Investable Market Real Estate 25/50 Index	Vanguard Real Estate ETF (VNQ)
3	Real estate_outside of the U.S.	S&P Global ex-U.S. Property Index	VNQI
4	Bonds_U.S. (Government as well as corporate)	Bloomberg U.S. Aggregate Float Adjusted Bond Index	BND
5	Bonds_Ex U.S.	Bloomberg Global Aggregate ex-USD Float Adjusted RIC Capped Index (USD Hedged)	BNDX
6	Commodities	Deutsche Bank Index Quant (DBIQ) Optimum Yield Diversified Commodity Index	DBC
7	Currencies other than crypto	Deutsche Bank (DB) Long USD Currency Portfolio Index Excess Return	UUP
8	Crypto	No index with more than one year vintage. Oldest quotes available are those of Bitcoin. Hence BTC used as proxy.	BTC

Table 2. Names and tickers of the active and passive funds used for performance comparison (Part 1)

No.	*Ticker*	*Name*	*No.*	*Ticker*	*Name*
1	AEPGX	American Funds Europacific Growth-A	31	CNGCX	American Funds The New Economy Fund® Class 529-C
2	AGTHX	American Funds Growth Fund of Amer-A	32	CNGEX	American Funds The New Economy Fund® Class 529-E
3	AIVSX	American Funds Investment Company of Amer-A	33	CNGFX	American Funds The New Economy Fund® Class 529-F
4	AMCFX	American Funds AMCAP Fund® Class F-2	34	CWGIX	American Funds Capital World Growth & Income-A
5	AMCPX	AMCAP Fund-A	35	CWMAX	American Funds Washington Mutual Investors Fund
6	AMECX	American Funds Income of Amer-A	36	CWMCX	American Funds Washington Mutual Investors Fund
7	AMRFX	American Funds American Mutual Fund®	37	CWMEX	American Funds Washington Mutual Investors Fund
8	AMRMX	American Funds Mutual Fund-A	38	CWMFX	American Funds Washington Mutual Investors Fund
9	ANCFX	American Fund Fundamental Investor-A	39	DDFIX	Invesco Diversified Dividend Fund R5 Class
10	ANEFX	American Funds New Economy-A	40	DDFRX	Invesco Diversified Dividend Fund Class R
11	ANWPX	American Funds New Perspective-A	41	DFCEX	DFA Emerging Markets Core Equity-I
12	APGAX	Ab Large Cap Growth-A	42	DFEOX	DFA U.S. Core Equity 1 Portfolio
13	ARTIX	Artisan International-Inv	43	DFLVX	DFA U.S. Large Cap Value Portfolio Institutional Class
14	ARTKX	Artisan International Value-Inv	44	DFQTX	DFA U.S. Core Equity 2 Portfolio
15	AULRX	American Century Ultra® Fund R Class	45	DFSVX	DFA U.S. Small Cap Value Portfolio Institutional Class
16	AWSHX	American Fund Washington Mutual Inv-A	46	DFUSX	DFA U.S. Large Company Portfolio
17	BIV	Vanguard Intermediate-Term Bond	47	DLTNX	Doubleline Total Return Bond Fund Class N
18	BOND	Pimco Active Bond Exchange-Traded Fund	48	DODBX	Dodge & Cox Balanced
19	BSV	Vanguard Short-Term Bond Index Fund ETF	49	DODFX	Dodge & Cox International Stock
20	CAFAX	American Funds AMCAP Fund® Class 529-A	50	DODGX	Dodge & Cox Stock
21	CAFCX	American Funds AMCAP Fund® Class 529-C	51	DODWX	Dodge & Cox Global Stock
22	CAFEX	American Funds AMCAP Fund® Class 529-E	52	EGFFX	Edgewood Growth Fund Retail Class
23	CAFFX	American Funds AMCAP Fund® Class 529-F-1	53	EGFIX	Edgewood Growth Fund-Institutional
24	CAIBX	American Funds Cap Inc Bldr-A	54	ELD	Eldorado Gold Corp
25	CDIRX	Columbia Dividend Income Fund Class R	55	EMCB	Wisdomtree Emerging Markets Corporate Bond Fund
26	CMLAX	American Funds American Mutual Fund®	56	FAGOX	Fidelity Advisor Growth Opportunities Fund-M
27	CMLCX	American Funds American Mutual Fund®	57	FASMX	Fidelity Asset Manager 50%
28	CMLEX	American Funds American Mutual Fund®	58	FBGRX	Fidelity Blue Chip Growth
29	CMLFX	American Funds American Mutual Fund®	59	FCBFX	Fidelity® Corporate Bond Fund
30	CNGAX	American Funds The New Economy Fund®	60	FCGAX	Franklin Growth Fund Advisor Class

Table 2. Names and tickers of the active and passive funds used for performance comparison (Part 2)

No.	Ticker	Name	No.	Ticker	Name
61	FCNTX	Fidelity Contrafund	135	OAKMX	Oakmark Fund-Inv
62	FCOR	Fidelity Corporate Bond ETF	136	ODMAX	Invesco Opp Developing Markets-A
63	FDGFX	Fidelity Dividend Growth Fund	137	OIGAX	Invesco Oppenheimer International Growth-A
64	FDGRX	Fidelity Growth Company Fund	138	OPGIX	Invesco Global Opportunities-A
65	FDIVX	Fidelity Diversified International Fund	139	OPPAX	Invesco Opp Global Fund-A
66	FDNRX	Franklin Dynatech Fund Class R	140	OSMAX	Invesco Opp Intl Small-Mid Co-A
67	FDSSX	Fidelity Stock Selector All Cap	141	OTCFX	T. Rowe Price Small-Cap Stock-Rtl
68	FDYZX	Franklin Dynatech Fund Advisor Class	142	OWLSX	Old Westbury Large Cap Strategies Fund
69	FEMSX	Fidelity Series Emerging Markets Opps	143	PCBIX	Principal Midcap Fund Institutional Class
70	FGSRX	Franklin Growth Series Class R	144	PEYAX	Putnam Equity Income-A
71	FIGRX	Fidelity International Discovery Fund	145	PGBAX	Principal Global Diversified Inc-A
72	FKDNX	Franklin Dynatech Fund-A	146	PIIIX	Principal Diversified International-Inst
73	FKGRX	Franklin Growth Fund-A	147	PLGJX	Principal Large Cap Growth I-J
74	FLPKX	Fidelity® Low-Priced Stock Fund Class K	148	PMBMX	Principal Midcap Fund-R3
75	FLPSX	Fidelity® Low-Priced Stock Fund	149	PMEGX	T. Rowe Price Inst Mid-Cap Growth
76	FLTB	Fidelity Limited Term Bond ETF	150	POAGX	Primecap Odyssey Aggressive Growth
77	FMAGX	Fidelity Magellan Fund	151	POGRX	Primecap Odyssey Growth
78	FNIAX	Fidelity Advisor New Insights-A	152	PONAX	Pimco Income Fund-A
79	FOCKX	Fidelity® OTC Portfolio Class K	153	POSKX	Primecap Odyssey Stock Fund
80	FOCPX	Fidelity OTC Portfolio	154	PRBLX	Parnassus Core Equity-Inv
81	FOSFX	Fidelity Overseas Fund	155	PRDGX	T. Rowe Price Dividend Growth
82	FRDAX	Franklin Rising Dividends Fund Advisor Class	156	PRDSX	T. Rowe Price QM U.S. Small-Cap Growth Equity-Inv
83	FRDPX	Franklin Rising Dividend-A	157	PRFDX	T. Rowe Price Equity Inc-Inv
84	FRDRX	Franklin Rising Dividends Fund Class R	158	PRGFX	T. Rowe Price Growth Stock
85	FSCSX	Fidelity Select Software & IT	159	PRHSX	T. Rowe Price Health Sciences
86	FTLS	First Trust Long Short Equity ETF	160	PRILX	Parnassus Core Equity Fund-Institutional Shares
87	FWDB	Foliobeyond Smart Core Bond Fund ETF	161	PRITX	T. Rowe Price International Stock
88	GATEX	Gateway Fund-A	162	PRMSX	T. Rowe Price Emerging Markets Stock
89	GOIOX	John Hancock III International Growth-1	163	PRMTX	T. Rowe Price Communications & Technology Fund Investor Class
90	GQETX	GMO Quality Fund-III	164	PRNHX	T Rowe Price New Horizons
91	GSY	Invesco Ultra Short Duration ETF	165	PRSCX	T. Rowe Price Science And Technology Fund
92	HACAX	Harbor Capital Apprecia-Inst	166	RAVI	Flexshares Readyaccess Variable Income Fund
93	HBLAX	Hartford Balanced Inc-A	167	RIGS	Riverfront Strategic Income Fund
94	HFMCX	Hartford Midcap Fund-A	168	RPMGX	T. Rowe Price Mid-Cap Growth-Inv
95	HLIEX	JPMorgan Equity Income-I	169	SBLGX	Clearbridge Lrg Cap Grow-A
96	HOLD	Immutable Holdings	170	SBLYX	Clearbridge Large Cap Growth Fund Class I
97	HYLD	High Yield ETF Frund	171	SEEGX	JPMorgan Large Cap Grow-I
98	HYLS	First Trust Tactical High Yield ETF	172	SGENX	First Eagle Global-A
99	ICSH	iShares Ultra Short Term Bond ETF	173	SMCWX	American Fund Smallcap World-A
100	IGIFX	American Fund Intl Gr & Inc-F1	174	SMGIX	Columbia Contrarian Core-I
101	IHGIX	Hartford Dividend & Growth-A	175	SMMU	Pimco Short Term Municipal Bond Active Exch Tr Fd
102	JAMRX	Janus Henderson Research-T	176	SRLN	SPDR Blackstone Senior Loan ETF
103	JARTX	Janus Henderson Forty-S	177	SVAAX	Federated Hermes Strategic Value Dividend Fund-A
104	JEMSX	JPMorgan Emerging Markets Equity-I	178	SWTSX	Schwab Total Stock Market Index-Sel
105	JENSX	Jensen Quality Growth-J	179	TBGVX	Tweedy Browne Global Value
106	JGMAX	Janus Hndrsn Triton-A	180	TCIEX	TIAA-CREF International Equity Index-Inst
107	JGRTX	Janus Hndrsn Enterprise-S	181	TEDIX	Franklin Mutual Global Discovery-A
108	JUEAX	JPMorgan U.S. Equity-A	182	TEMTX	Franklin Mutual Shares-C
109	JVLAX	John Hancock III-Disciplined Value-A	183	TEPLX	Templeton Growth Fund-A
110	JVMIX	John Hancock III Disciplined Mid-Cap Val-I	184	TIBAX	Thornburg Invest Inc Bld-A
111	LBSAX	Columbia Dividend Income-A	185	TRBCX	T. Rowe Price Blue Chip Grow
112	LSGRX	Loomis Sayles Growth Fund	186	TRIGX	T. Rowe Price International Value Equity
113	LZEMX	Lazard Emerging Markets Equity-Inst	187	TRLGX	T. Rowe Price Inst Large-Cap Growth
114	MADVX	Blackrock Eqty Dvdnd-I	188	TROSX	T. Rowe Price Overseas Stock-Inv
115	MAIIX	iShares-MSCI EAFE Intl-Inst	189	TRVLX	T. Rowe Price Value Fund
116	MFEBX	MFS Value Fund-B	190	TWCGX	American Cent Growth-Inv
117	MFEGX	MFS Growth Fund-A	191	TWCUX	Americanican Century Ultra-Inv
118	MGIAX	MFS International Intrinsic Value-A	192	TWEIX	American Cent Equity Inc-Inv
119	MGRAX	MFS International Growth-A	193	ULST	SPDR SSGA Ultra Short Term Bond ETF
120	MGRFX	Massmutl Sel M/C Gr E Ii-R5	194	VADAX	Invesco Equally-Weighted S&P 500-A
121	MIEIX	MFS Instl Intl Equity Fund-I	195	VAFAX	Invesco American Franchise-A
122	MINC	AdvisorShares Newfleet Multi-Sector Income ETF	196	VEXPX	Vanguard Explorer Fund-Inv
123	MINT	Pimco Enhanced Short Maturity Active ETF	197	VHCOX	Vanguard Capital Opport-Inv
124	MLAAX	MainStay Large Cap Growth-A	198	VHIAX	JPMorgan Growth Advantage-A
125	MRIBX	MFS Research International-B	199	VIPSX	Vanguard Inflation-Protected Securities Fund Investor Shares
126	MSFRX	MFS Total Return Fund-A	200	VLCAX	Vanguard Large Cap Index-Adm
127	MSIGX	Invesco Opp Main Street-A	201	VQNPX	Vanguard Growth & Income-Inv
128	MUNI	Pimco Intermediate Municipal Bond Active ETF	202	VTCIX	Vanguard Tax-Managed Cap Appreciation-Inst
129	MWTRX	Metropolitan West Total Return Bond-M	203	VTRIX	Vanguard International Value
130	NBGNX	Neuberger Berman Genesis-Inv	204	VWEHX	Vanguard High-Yield Corp-Inv
131	NEAR	Blackrock Short Maturity Bond ETF	205	VWNDX	Vanguard Windsor-Inv
132	NEWFX	American Funds New World-A	206	VWNFX	Vanguard Windsor™ II Fund Investor Share
133	NOSIX	Northern Stock Index Fund	207	VWUSX	Vanguard U.S. Growth Fund Investor Shares
134	NYVTX	Davis New York Venture-A			

Table 3. Time series for the benchmark ("ideal") Sharpe ratios: To be compared on ex-post performance for evaluating the performance of any investment portfolio

No.	Month	Sharpe	No.	Month	Sharpe	No.	Month	Sharpe
1	Oct-14	0.62	29	Feb-17	0.94	57	Jun-19	0.96
2	Nov-14	0.71	30	Mar-17	0.12	58	Jul-19	0.59
3	Dec-14	0.62	31	Apr-17	0.54	59	Aug-19	0.72
4	Jan-15	0.93	32	May-17	1.12	60	Sep-19	0.32
5	Feb-15	0.82	33	Jun-17	0.26	61	Oct-19	0.48
6	Mar-15	0.27	34	Jul-17	0.52	62	Nov-19	0.57
7	Apr-15	0.52	35	Aug-17	0.66	63	Dec-19	0.73
8	May-15	0.18	36	Sep-17	0.30	64	Jan-20	1.01
9	Jun-15	0.21	37	Oct-17	0.71	65	Feb-20	0.52
10	Jul-15	0.65	38	Nov-17	0.59	66	Mar-20	0.04
11	Aug-15	0.00	39	Dec-17	0.53	67	Apr-20	0.53
12	Sep-15	0.29	40	Jan-18	0.49	68	May-20	0.32
13	Oct-15	1.29	41	Feb-18	0.25	69	Jun-20	0.30
14	Nov-15	0.58	42	Mar-18	0.76	70	Jul-20	1.09
15	Dec-15	0.20	43	Apr-18	0.46	71	Aug-20	0.47
16	Jan-16	0.39	44	May-18	0.62	72	Sep-20	0.43
17	Feb-16	0.56	45	Jun-18	0.36	73	Oct-20	0.56
18	Mar-16	0.53	46	Jul-18	0.45	74	Nov-20	0.93
19	Apr-16	0.72	47	Aug-18	0.37	75	Dec-20	0.70
20	May-16	0.90	48	Sep-18	0.27	76	Jan-21	0.33
21	Jun-16	0.72	49	Oct-18	0.34	77	Feb-21	0.59
22	Jul-16	0.45	50	Nov-18	0.62	78	Mar-21	0.54
23	Aug-16	0.08	51	Dec-18	0.74	79	Apr-21	0.73
24	Sep-16	0.25	52	Jan-19	1.13	80	May-21	0.24
25	Oct-16	0.65	53	Feb-19	0.52	81	Jun-21	1.04
26	Nov-16	0.38	54	Mar-19	1.35	82	Jul-21	1.12
27	Dec-16	0.91	55	Apr-19	0.57	83	Aug-21	0.46
28	Jan-17	0.36	56	May-19	0.93	84	Sep-21	0.48

Table 4. Time series for the ex-post ("realised") Sharpe ratios based on ideal weights for the previous month

No.	Month	Sharpe	No.	Month	Sharpe	No.	Month	Sharpe
1	Nov-14	0.31	29	Mar-17	-0.01	57	Jul-19	0.03
2	Dec-14	0.24	30	Apr-17	0.17	58	Aug-19	0.36
3	Jan-15	0.47	31	May-17	0.69	59	Sep-19	-0.01
4	Feb-15	-0.13	32	Jun-17	0.02	60	Oct-19	0.08
5	Mar-15	0.08	33	Jul-17	0.16	61	Nov-19	-0.13
6	Apr-15	-0.43	34	Aug-17	0.15	62	Dec-19	-0.08
7	May-15	-0.17	35	Sep-17	-0.10	63	Jan-20	-0.43
8	Jun-15	-0.14	36	Oct-17	0.37	64	Feb-20	0.10
9	Jul-15	-0.03	37	Nov-17	0.41	65	Mar-20	-0.01
10	Aug-15	-0.26	38	Dec-17	0.22	66	Apr-20	-0.01
11	Sep-15	-0.13	39	Jan-18	0.25	67	May-20	0.17
12	Oct-15	0.25	40	Feb-18	-0.14	68	Jun-20	0.15
13	Nov-15	0.13	41	Mar-18	0.09	69	Jul-20	0.59
14	Dec-15	-0.05	42	Apr-18	0.08	70	Aug-20	-0.14
15	Jan-16	-0.18	43	May-18	0.37	71	Sep-20	-0.09
16	Feb-16	0.25	44	Jun-18	0.02	72	Oct-20	0.09
17	Mar-16	0.13	45	Jul-18	0.05	73	Nov-20	0.38
18	Apr-16	0.11	46	Aug-18	0.05	74	Dec-20	0.62
19	May-16	0.28	47	Sep-18	-0.17	75	Jan-21	0.09
20	Jun-16	0.27	48	Oct-18	-0.26	76	Feb-21	0.56
21	Jul-16	0.07	49	Nov-18	0.07	77	Mar-21	0.18
22	Aug-16	-0.08	50	Dec-18	0.02	78	Apr-21	-0.07
23	Sep-16	0.07	51	Jan-19	0.37	79	May-21	0.14
24	Oct-16	0.50	52	Feb-19	0.10	80	Jun-21	0.06
25	Nov-16	0.23	53	Mar-19	0.37	81	Jul-21	0.13
26	Dec-16	0.24	54	Apr-19	0.04	82	Aug-21	-0.08
27	Jan-17	0.00	55	May-19	-0.13	83	Sep-21	-0.13
28	Feb-17	0.52	56	Jun-19	0.49	84	Oct-21	0.22

Table 5. Time series for the ex-post ("realised") Sharpe ratios based on weights computed by maximization with returns predicted using ARIMA and volatility observed in the previous month

No.	Month	Sharpe	No.	Month	Sharpe	No.	Month	Sharpe
1	Nov-14	0.16	29	Mar-17	-0.07	57	Jul-19	0.11
2	Dec-14	-0.25	30	Apr-17	0.17	58	Aug-19	0.42
3	Jan-15	0.45	31	May-17	0.74	59	Sep-19	-0.01
4	Feb-15	0.03	32	Jun-17	0.02	60	Oct-19	0.17
5	Mar-15	0.10	33	Jul-17	0.44	61	Nov-19	-0.14
6	Apr-15	-0.10	34	Aug-17	0.35	62	Dec-19	-0.08
7	May-15	-0.13	35	Sep-17	-0.12	63	Jan-20	-0.11
8	Jun-15	-0.25	36	Oct-17	0.26	64	Feb-20	-0.07
9	Jul-15	0.18	37	Nov-17	0.24	65	Mar-20	-0.02
10	Aug-15	-0.24	38	Dec-17	0.16	66	Apr-20	-0.01
11	Sep-15	-0.03	39	Jan-18	0.06	67	May-20	0.23
12	Oct-15	0.34	40	Feb-18	-0.18	68	Jun-20	0.10
13	Nov-15	0.13	41	Mar-18	-0.17	69	Jul-20	0.42
14	Dec-15	-0.08	42	Apr-18	-0.03	70	Aug-20	-0.05
15	Jan-16	-0.19	43	May-18	0.40	71	Sep-20	-0.04
16	Feb-16	0.00	44	Jun-18	0.10	72	Oct-20	0.15
17	Mar-16	0.19	45	Jul-18	0.04	73	Nov-20	0.58
18	Apr-16	0.11	46	Aug-18	0.12	74	Dec-20	0.35
19	May-16	0.22	47	Sep-18	-0.06	75	Jan-21	0.13
20	Jun-16	0.32	48	Oct-18	-0.21	76	Feb-21	0.32
21	Jul-16	0.16	49	Nov-18	-0.27	77	Mar-21	0.06
22	Aug-16	-0.01	50	Dec-18	0.04	78	Apr-21	-0.11
23	Sep-16	-0.02	51	Jan-19	0.26	79	May-21	0.14
24	Oct-16	0.61	52	Feb-19	0.09	80	Jun-21	-0.05
25	Nov-16	0.18	53	Mar-19	0.62	81	Jul-21	0.10
26	Dec-16	0.31	54	Apr-19	-0.01	82	Aug-21	-0.04
27	Jan-17	-0.01	55	May-19	-0.15	83	Sep-21	-0.03
28	Feb-17	0.42	56	Jun-19	0.45	84	Oct-21	0.29

Table 6. Time series for the ex-post ("realised") Sharpe ratios with weights based on minimising the previous month's volatility

No.	Month	Sharpe	No.	Month	Sharpe	No.	Month	Sharpe
1	Nov-14	0.07	29	Mar-17	-0.01	57	Jul-19	0.34
2	Dec-14	0.29	30	Apr-17	0.13	58	Aug-19	0.55
3	Jan-15	0.59	31	May-17	0.24	59	Sep-19	0.00
4	Feb-15	-0.04	32	Jun-17	-0.02	60	Oct-19	0.07
5	Mar-15	0.13	33	Jul-17	0.19	61	Nov-19	0.04
6	Apr-15	-0.16	34	Aug-17	0.36	62	Dec-19	-0.15
7	May-15	-0.04	35	Sep-17	-0.04	63	Jan-20	0.19
8	Jun-15	-0.18	36	Oct-17	0.37	64	Feb-20	0.10
9	Jul-15	0.02	37	Nov-17	0.10	65	Mar-20	-0.13
10	Aug-15	-0.22	38	Dec-17	0.11	66	Apr-20	0.31
11	Sep-15	0.19	39	Jan-18	-0.24	67	May-20	0.14
12	Oct-15	0.16	40	Feb-18	0.05	68	Jun-20	0.05
13	Nov-15	0.07	41	Mar-18	0.39	69	Jul-20	0.43
14	Dec-15	-0.05	42	Apr-18	0.02	70	Aug-20	-0.16
15	Jan-16	0.03	43	May-18	0.28	71	Sep-20	0.14
16	Feb-16	0.28	44	Jun-18	0.09	72	Oct-20	0.08
17	Mar-16	0.24	45	Jul-18	0.08	73	Nov-20	0.44
18	Apr-16	0.25	46	Aug-18	0.08	74	Dec-20	0.05
19	May-16	0.31	47	Sep-18	-0.16	75	Jan-21	-0.11
20	Jun-16	0.32	48	Oct-18	-0.15	76	Feb-21	-0.11
21	Jul-16	0.12	49	Nov-18	0.12	77	Mar-21	0.33
22	Aug-16	0.00	50	Dec-18	0.30	78	Apr-21	-0.01
23	Sep-16	0.04	51	Jan-19	0.44	79	May-21	-0.01
24	Oct-16	0.39	52	Feb-19	0.13	80	Jun-21	0.51
25	Nov-16	-0.14	53	Mar-19	0.70	81	Jul-21	0.17
26	Dec-16	0.16	54	Apr-19	0.35	82	Aug-21	-0.07
27	Jan-17	-0.25	55	May-19	-0.06	83	Sep-21	-0.04
28	Feb-17	0.62	56	Jun-19	0.41	84	Oct-21	0.05

Table 7. Time series for the ex-post ("realised") Sharpe ratios with weights based on the market capitalisation of the indices tracked by the constituents

No.	Month	Sharpe	No.	Month	Sharpe	No.	Month	Sharpe
1	Nov-14	0.11	29	Mar-17	0.72	57	Jul-19	0.58
2	Dec-14	0.25	30	Apr-17	0.08	58	Aug-19	0.11
3	Jan-15	-0.07	31	May-17	0.25	59	Sep-19	0.02
4	Feb-15	0.12	32	Jun-17	0.30	60	Oct-19	0.21
5	Mar-15	0.36	33	Jul-17	0.02	61	Nov-19	0.14
6	Apr-15	-0.02	34	Aug-17	0.32	62	Dec-19	0.23
7	May-15	0.10	35	Sep-17	0.13	63	Jan-20	0.33
8	Jun-15	-0.03	36	Oct-17	0.21	64	Feb-20	0.03
9	Jul-15	-0.26	37	Nov-17	0.42	65	Mar-20	-0.26
10	Aug-15	0.11	38	Dec-17	0.23	66	Apr-20	-0.11
11	Sep-15	-0.23	39	Jan-18	0.28	67	May-20	0.23
12	Oct-15	-0.06	40	Feb-18	0.34	68	Jun-20	0.19
13	Nov-15	0.40	41	Mar-18	-0.15	69	Jul-20	0.10
14	Dec-15	-0.02	42	Apr-18	0.00	70	Aug-20	0.30
15	Jan-16	-0.10	43	May-18	0.04	71	Sep-20	0.35
16	Feb-16	-0.11	44	Jun-18	0.06	72	Oct-20	-0.07
17	Mar-16	0.01	45	Jul-18	0.00	73	Nov-20	-0.09
18	Apr-16	0.40	46	Aug-18	0.25	74	Dec-20	0.60
19	May-16	0.07	47	Sep-18	0.07	75	Jan-21	0.43
20	Jun-16	0.10	48	Oct-18	0.02	76	Feb-21	-0.03
21	Jul-16	0.11	49	Nov-18	-0.26	77	Mar-21	0.09
22	Aug-16	0.38	50	Dec-18	0.10	78	Apr-21	0.13
23	Sep-16	0.01	51	Jan-19	-0.21	79	May-21	0.32
24	Oct-16	0.04	52	Feb-19	0.48	80	Jun-21	0.06
25	Nov-16	-0.30	53	Mar-19	0.26	81	Jul-21	0.16
26	Dec-16	-0.04	54	Apr-19	0.31	82	Aug-21	0.16
27	Jan-17	0.19	55	May-19	0.42	83	Sep-21	0.18
28	Feb-17	0.18	56	Jun-19	-0.22	84	Oct-21	-0.30

Table 8. Time series for the ex-post ("realised") Sharpe ratios based on equal weights

No.	Month	Sharpe	No.	Month	Sharpe	No.	Month	Sharpe
1	Nov-14	0.11	29	Mar-17	-0.07	57	Jul-19	-0.08
2	Dec-14	-0.28	30	Apr-17	0.41	58	Aug-19	0.00
3	Jan-15	-0.07	31	May-17	0.55	59	Sep-19	-0.06
4	Feb-15	0.28	32	Jun-17	0.10	60	Oct-19	0.21
5	Mar-15	-0.05	33	Jul-17	0.22	61	Nov-19	-0.27
6	Apr-15	0.01	34	Aug-17	0.39	62	Dec-19	0.07
7	May-15	-0.05	35	Sep-17	-0.01	63	Jan-20	0.26
8	Jun-15	-0.03	36	Oct-17	0.55	64	Feb-20	-0.28
9	Jul-15	0.07	37	Nov-17	0.46	65	Mar-20	-0.14
10	Aug-15	-0.23	38	Dec-17	0.26	66	Apr-20	0.25
11	Sep-15	0.02	39	Jan-18	-0.17	67	May-20	0.19
12	Oct-15	0.78	40	Feb-18	-0.06	68	Jun-20	0.06
13	Nov-15	0.08	41	Mar-18	-0.26	69	Jul-20	0.32
14	Dec-15	0.06	42	Apr-18	0.30	70	Aug-20	0.19
15	Jan-16	-0.18	43	May-18	-0.15	71	Sep-20	-0.11
16	Feb-16	0.13	44	Jun-18	-0.19	72	Oct-20	0.12
17	Mar-16	0.32	45	Jul-18	0.32	73	Nov-20	0.77
18	Apr-16	0.32	46	Aug-18	-0.05	74	Dec-20	0.51
19	May-16	0.28	47	Sep-18	-0.10	75	Jan-21	0.14
20	Jun-16	0.30	48	Oct-18	-0.25	76	Feb-21	0.34
21	Jul-16	0.05	49	Nov-18	-0.26	77	Mar-21	0.29
22	Aug-16	-0.16	50	Dec-18	-0.24	78	Apr-21	0.21
23	Sep-16	0.12	51	Jan-19	0.39	79	May-21	-0.22
24	Oct-16	0.05	52	Feb-19	0.26	80	Jun-21	0.10
25	Nov-16	0.05	53	Mar-19	0.50	81	Jul-21	0.28
26	Dec-16	0.66	54	Apr-19	0.30	82	Aug-21	0.17
27	Jan-17	0.05	55	May-19	0.32	83	Sep-21	-0.12
28	Feb-17	0.60	56	Jun-19	0.54	84	Oct-21	0.53

Table 9. Analysis of variance (ANOVA)

Sources	SS	df	MS	F	P-value
Between groups	19.16043	219	0.087491	1.742774	0.00
Within groups	916.6869	18260	0.050202		
Total	935.8473	18479	0.050644		