THE DYNAMIC LIQUIDATION OF BANKS' BAD LOANS*

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Abstract

We consider the optimal dynamic liquidation of banks' bad loans. The banks' liquidation strategy affects the value of collateral and hence other firms' access to credit. In this framework we characterize the socially optimal liquidation path. We show that the 'liquidate immediately' strategy is optimal in a small banking crisis whereas the 'liquidate gradually' strategy is optimal in a large banking crisis. It is argued that liquidation is likely to be postponed at society's cost when the regulator has a short time horizon. We apply our analysis to the Japanese banking crisis.

Keywords: Bad loans; Liquidation; Value of collateral; Credit allocation; Japan

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1. Introduction

In this paper we consider the optimal dynamic liquidation of banks' bad loans. We have in mind situations where a devaluation or a collapse in asset prices have resulted in financially distressed economies and left banks as owners of a portfolio of bad loans.[1] Such situations have arisen e.g. in Japan following the burst of the asset price bubble in 1989, in Scandinavia in the early 1990's, in Asia following the Asian crisis in 1997, and most recently in Argentina following the devaluation in the beginning of 2002.

In a financial crisis the value of firms' assets will typically fall relative to their liabilities and banks will experience an increase in their portfolio of bad loans. The banks' liquidation strategy will affect the value of collateral as the collateral securing their bad loans is put up for sale. In an economy where collateral is necessary for obtaining credit, the banks' liquidation strategy will thus affect other firms' access to credit.

In this framework, we derive the socially optimal liquidation path. We show that in general it is optimal to apply a gradual liquidation strategy. The intuition for the gradual approach is that if all bad loans are liquidated at once this can push down collateral prices to such an extent that even the most profitable projects cannot be financed.

We apply our analysis to Japan where collateral in the form of real estate and land has played an essential role in the credit allocation process. It is generally acknowledged that the regulatory response in Japan to the bad loan problem has been delayed. It has been argued that it would be better for Japan if the bad loans were liquidated quickly as it was done in the US savings and loan crisis in the end of the 1980's. However, the bad loan problem in Japan is much larger than it was in the US (Hoshi and Kayshap, 1999). We argue that the 'liquidate immediately' strategy is optimal in a small banking crisis whereas the 'liquidate gradually' strategy is optimal in a large banking crisis. Hence, the successful quick resolution of the US savings and loan crisis, may not be the optimal strategy in the Japanese banking crisis.

Next, we analyze how the regulator's preferences may affect the chosen liquidation path. We argue that if the regulator has a short time

[1] There are many definitions of bad loans, also called non-performing loans. Two common characteristics are a) that repayments have been delayed and b) that the market value of collateral is low.

horizon it may lead to a postponement of liquidation. Considering the short life span of most Japanese administrations and the politicizing of Japan's financial authorities this constitutes a plausible reason why the regulatory response to the Japanese bad loan problem has been delayed and half-hearted.

We also consider the effect of the illiquidity in the market for collateral. In Japan the markets for real estate and land have been very illiquid. We argue that the illiquidity in these markets have lengthened the optimal liquidation path.

In addition to Japan our analysis is highly relevant for transition economies. Banks in many transition economies are burdened with an excessive amount of bad loans (see Anderson and Kegels, 1999 for some statistics on bad loans in transition economies). For example, in China official estimates put the level of bad loans in the 'big four' state banks at 23 percent of total assets, much higher than the corresponding number in Japan.[2] Our analysis therefore suggests a gradual approach to the liquidation of the bad loans in China. Also, the experiences in for example Hungary and the Check Republic where the authorities delayed action suggest that regulatory incentive conflicts have also been present in some transition economies.

Our analysis is related to different strands of literature. First and foremost, it is related to Kiyotaki and Moore (1997), with whom we share that entrepreneurs' credit constraints depend on forward looking asset prices. Our modelling of how collateral prices affect credit constraints is borrowed from their paper. Their paper and the related literature is, however, principally concerned with how balance sheet effects amplify business cycles (e.g. Bernanke and Gertler, 1989) and how fluctuations in collateral values generate financial cycles (Kiyotaki and Moore).

Our paper is also related to Mitchell (2001a), who considers the cost of different policies for cleaning up banks' balance sheets. She focuses on agency costs due to asymmetric information whereas we focus on the feedback effect on the real economy of banks liquidating their bad loans.

Finally, the issue of the optimal resolution of small versus large banking crises is also addressed by Mitchell (2001b). She argues that if many banks are discovered to be in distress, a situation labelled 'too many to fail' may happen in which high social costs of bank closures make rescues less costly. She argues that this situation may arise because of a convexity in intervention costs in the number of banks to which the intervention policy is applied. We obtain, endogenously, a convexity in the cost of liquidation and we show that this will lead to a postponement of liquidation in a large banking crisis.

[2] See Financial Times, Nov. 24th, 2003. Independent economists put the figure even higher, at 40 percent.

The remainder of the paper is organized as follows. In Section 2, we present our basic model and in Section 3, we formulate the dynamic problem and derive the socially optimal liquidation path. In Section 4, we study the qualitative differences between small and large banking crises. In Section 5, we study the impact of the regulator's preferences and obtain other comparative static results. In Section 6, we provide some concluding remarks.

2. The Model

In our model, liquidation affects the value of collateral and hereby credit constraints, production, and social welfare in a financially distressed economy. Our model hereof builds on Bernanke, Gertler, and Gilchrist (1996) who present a simplified version of the model from Kiyotaki and Moore (1997).

Consider a discrete time economy with a single agent, an entrepreneur, who has an arbitrarily large time horizon T . The entrepreneur has preferences over the consumption of a single good given by

$$\sum_{t=0}^{T} \beta^t v(c_t)$$

where ct is the date t consumption, v is twice differentiable, increasing, and concave, v' > 0 and v" < 0, and β is the entrepreneur's discount factor. The entrepreneur is endowed with a convex production technology and fixed assets of value Pt. We assume that the fixed assets held by the entrepreneur have a given, exogenously determined size. Production in period t is given by f (Xt) where Xt is the input and f (·) is assumed twice differentiable everywhere with f ' > 0 and f " < 0. Since we have only one good; input, production, and consumption are all given in units of the consumption good, whose price we normalize to one.

The entrepreneur has access to a credit market for production input. Following Hart and Moore (1994), we assume that the entrepreneur is essential to the project and cannot commit to not withdrawing his human capital. However, a contract that gives the creditor the right to seize the fixed assets if the entrepreneur does not fulfill his financial obligations, is enforceable and the fixed assets can thus serve as collateral. Therefore, the value of the fixed assets determines the amount of production input the entrepreneur can borrow in each period. We assume that the output is perishable such that the output in period t cannot be stored but can only be consumed by the entrepreneur in period t[3] .Therefore, in each

[3] The purpose of this assumption is to avoid the situation where the entrepreneur continually postpones consumption, thereby accumulating the good until he no longer needs external financing and the agency problem therefore ceases to exist. See Kiyotaki and Moore (1997) for a similar assumption.

period the entrepreneur maximizes net production subject to a collateral financing constraint

$$f(X_t) - X_t$$
$$st.\ X_t \leq P_t. \tag{1}$$

By a financially distressed economy we mean an economy where the value of collateral is at a level such that the financing constraint (1) always binds in optimum

$$X_t = P_t.$$

Substituting Xt into the production function yields the consumption in each period

$$c(P_t) = f(P_t) - P_t.$$

Since f is concave, the consumption c(Pt) is concave in the value of collateral Pt. Having assumed that the financing constraint (1) always binds, we know that

$$f'(P_t) > 1$$

and hence

$$c'(P_t) = f'(P_t) - 1 > 0.$$

We therefore have that consumption is increasing in the value of collateral. Letting

$$u(P_t) = v(c(P_t))$$

we can write the entrepreneur's utility as a function of the path of collateral prices

$$\sum_{t=0}^{T} \beta^t u(P_t).$$

Since c(Pt) is increasing, twice differentiable, and strictly concave, so is u(Pt).

2.1. The Pricing of Collateral

Following a financial crisis, we assume that banks are the owners of a portfolio of bad loans secured by an underlying amount of collateral Q0. We will treat the initial amount of collateral securing bad loans, Q0, as given and reflecting the size of the banking crisis.

Let Qt denote the amount of collateral pledged as security for outstanding bad loans at the beginning of period t and let qt denote the amount of collateral sold during period t. For simplicity, we will assume that the amount of collateral securing bad loans does not change, save by liquidation. Thus, Qt evolves as follows

$$Q_{t+1} = Q_t - q_t.$$

The value of the fixed assets owned by the entrepreneur is determined by

$$P_t = L - \varepsilon_S q_t - \varepsilon_L Q_{t+1}, \tag{2}$$

where L is the long term value (when all the collateral securing bad loans has been sold) and εS and εL are the elasticities of the price of collateral with respect to collateral sold in period t and the remaining, unsold collateral respectively. This price specification captures that we are considering a financial crisis where the price of collateral is depressed as long as there is an overhang of collateral waiting to be sold. We assume that εS > εL reflecting that current liquidations have a larger effect on the price than future, anticipated liquidations.

Our pricing function (2) is, we believe, the simplest possible way to capture the trade off between the price depressing effect of liquidating now versus the price depressing effect of postponing liquidations. Alternatively, we could have chosen to use a rational expectations relationship between prices today and prices tomorrow

$$P_t = L_1 - \varepsilon_S q_t + \frac{1}{1+r} E_t(P_{t+1}), \tag{3}$$

where E(·) is the expectation operator and r is the risk free real interest rate. For such a pricing function the 'postpone liquidation' strategy is always optimal. We take the view that in real life banks do not live forever and hence the market expects that the collateral securing bad loans eventually will come on to the market. Therefore, the market is depressed as long as there is an overhang of outstanding bad loans. Rather than modelling a latest day of liquidation ˆ T , where Q ˆ T = 0, as a stochastic variable or assuming an artificial fixed day, we impose the depressing effect of unsold collateral directly through the pricing function.

Moreover, one can argue that, in practice, the market cannot perfectly foresee all the banks' future liquidations, given the complexity involved in the liquidation decisions. By successively substituting into (3) and let T → ∞, it can be seen that the only difference between (3) and (2) is the last term where in (3) the liquidations in each period are weighted by their discounting. Hence, our pricing function may be seen as an approximation to the rational expectations relationship taking into account the market's limited capacity to forecast all the banks' future liquidations.

3. Optimal Liquidation

In this section, we analyze the socially optimal liquidation path. Since the entrepreneur is the only agent, we maximize social welfare by maximizing the entrepreneur's welfare. For our analysis, we will use a time horizon of T where T could be infinity. We thus have the following social optimization problem:

$$\max_{(q_t)_{t=0}^{T}} \sum_{t=0}^{T} \beta^t u(L - \varepsilon_S q_t - \varepsilon_L Q_{t+1})$$

$$s.t \quad \begin{array}{l} Q_{t+1} = Q_t - q_t \\ q_t \geq 0 \\ Q_{T+1} \geq 0 \\ Q_0 > 0 \quad \text{given} \end{array} \qquad t = 0, ..., T$$

The following proposition characterizes the optimal liquidation path.

Proposition 1. For T sufficiently large and β > εS−εL/εS, the solution for qt has the following form

$$
\begin{aligned}
q_t &> 0 \quad for \quad 0 \le t \le t^* \quad and \quad \frac{u'(P_t)}{u'(P_{t-1})} = \frac{1}{\beta}\frac{\varepsilon_S - \varepsilon_L}{\varepsilon_S} \quad for \quad 1 \le t \le t^* \qquad (4)\\
q_{t^*} &= Q_{t^*}\\
q_t &= 0 \quad for \quad t > t^*
\end{aligned}
$$

For $\beta \le \varepsilon S - \varepsilon L/\varepsilon S$, the solution for qt has the following form

$$q_t = 0 \qquad \forall t.$$

Proof. See the Appendix.
Rewriting the first order condition from (4), we have

$$(\varepsilon_S - \varepsilon_L)u'(P_{t-1}) = \beta u'(P_t)\varepsilon_S. \qquad (5)$$

In any period, the marginal cost of liquidating bad loans minus the saved cost of not having to carry bad loans into the next period should be equal to the discounted marginal cost of not having to liquidate in the next period. A necessary and sufficient condition for having positive liquidations is

$$\beta > \frac{\varepsilon_S - \varepsilon_L}{\varepsilon_S}. \qquad (6)$$

The intuition herefore is that the weight put on future periods has to be larger than the ratio of 'cost of liquidation in this period' to 'cost of liquidation tomorrow' for the zero liquidation path not to be socially optimal. 'Cost of liquidation in this period' is the net impact of liquidation on the value of collateral $\varepsilon S - \varepsilon L$, whereas 'cost of liquidation tomorrow' is εS . In the remainder of the paper, we assume that (6) is fulfilled.

4. The Size of a Banking Crisis

Of particular interest in the case of Japan is whether a large banking crisis is different in nature than a small banking crisis and in particular whether a large banking crisis has a different optimal liquidation path. A number of authors, including Friedman (2000) and Glauber (2000), have argued that it would be better for Japan if all bad loans were liquidated quickly as it was done in the US savings and loans crisis. One noteworthy difference between the US savings and loan crisis and the Japanese banking crisis is that the bad loan problem has been much larger in Japan (see for instance Hoshi and Kayshap, 1999).[4] We now argue that small and large banking crises are different in the sense that the optimal strategy may well be 'liquidate immediately' in a small crisis, but 'liquidate gradually' in a larger one.

Definition 1. A banking crisis is small if Q0 satisfies

$$(\varepsilon_S - \varepsilon_L)\, u'(L - \varepsilon_S Q_0) \le \beta u'(L)\varepsilon_S. \qquad (7)$$

The interpretation of this definition is that a banking crisis is small if liquidating all bad loans, Q0, does not satisfy the first order condition (5). That is, we are at a corner solution.

For such a banking crisis $t* = 0$ and the 'liquidate immediately' strategy will in fact be optimal. As opposed hereto, when (7) is not fulfilled there will be an optimal gradual liquidation strategy characterized by our first order condition (5). The economic intuition for the 'liquidate gradually' strategy is that if all bad loans are liquidated simultaneously, this will push down collateral prices to such an extent that even the most profitable projects cannot be financed. On the other hand, if Q0 is small, the (net) marginal cost of liquidating today will be less than the marginal cost of liquidating tomorrow, and the 'liquidate immediately' strategy will therefore be optimal.

Our analysis provides an argument why it may be appropriate in the Japanese case to apply a 'liquidate gradually' strategy even though authorities were successful in disposing of bad loans quickly in the US savings and loan crisis.[5] In the following section we turn to other issues which may have affected the timing of liquidation in Japan.

5. ComparativeStatics

In this section we will consider the comparative statics with respect to β, εS, L and εL and provide some economic interpretations. We will say that a change in one of the parameters delays liquidation if it leads to a higher $t*$.

5.1. The Regulator's Time Preferences and Liquidation

In this section, we will analyze how the regulator's preferences may affect the chosen liquidation path. Following Aghion et at. (1999) and Mitchell (2001) we have in mind a situation where banks have private information about their amount of bad loans. In this case, the banks have the capacity and may also have the incentives to hide bad loans.[6]

To deal with the asymmetric information problem the regulator performs on-site inspections. However, with banks having incentives to roll over bad loans, the regulator can induce banks to do so,

[4] According to statistics from the Financial Services Agency in Japan the amount of non-performing loans to total loans peaked in March 2002 with 8.4% (Financial Services Agency, 2004). As a comparison at the peak of the US savings and loan crisis in 1991 the figure was 3.7% (Corbett, 1999).

[5] Koo (2001) also argues (albeit informally) that the optimal response in the Japanese banking crisis should differ from that of the US savings and loan crisis due to the difference in the size of the problems.

[6] Harr (2004) argues that banks may have incentives to hide bad loans because they have to fulfill accounting-based capital requirements. Peek and Rosengren (2003) provide empirical evidence for the fact that Japanese banks during the 1990's have rolled over bad loans to satisfy the Basel capital rules.

simply by performing 'soft' inspections.[7] Because of asymmetric information regarding the quality of the banks' loan portfolio, the regulator can claim to the public that banks deal with their bad loans properly.

To sum, it may be rational for banks to underestimate their bad loan portfolio and possible for the regulator to accept this estimate if the regulator wishes to do so.[8] This explanation justifies the fact that we, in the following, assume that the regulator can influence the chosen liquidation path without being punishable for not serving the entrepreneur's interests.

Specifically, we assume that the regulator has the same preferences as the entrepreneur except that the regulator does not care about the future should he not be re-appointed. Given that the regulator is uncertain of re-appointment by the government and let $\hat{\beta}$ be the discount factor of the regulator, we have that $\hat{\beta} < \beta$. We obtain the following result:

Corollary 1. A decrease in the discount factor, β, delays liquidation.

Proof. Pick a $\hat{\beta} < \beta$ and let $\hat{P}_t$, $(\hat{q}_t)$, and $\hat{t}*$ be the resulting optimal price path,
the optimal liquidation path, and the final period of liquidation respectively. In order to compare price paths, we will write $\hat{P} > P$, when $\hat{P}_t > P_t \quad t \leq t*$ and $\hat{P}_t \geq P_t \quad t > t*$. In order to show that $\hat{t}* \geq t*$, we will assume that this is not true and show a contradiction. Assume $\hat{t}* < t*$, then the new resulting price path must be higher, $\hat{P} > P$, since if this was not true, there would exist some $\bar{t} < t*$ such that $\hat{P}_{\bar{t}} \leq P_{\bar{t}}$. But by successive substitution of (5), we know that

$$u'(P_t) = \left(\frac{1}{\beta}\frac{\varepsilon_S - \varepsilon_L}{\varepsilon_S}\right)^{t-\bar{t}} u'(P_{\bar{t}}) \quad \text{and}$$
$$u'(\hat{P}_t) = \left(\frac{1}{\hat{\beta}}\frac{\varepsilon_S - \varepsilon_L}{\varepsilon_S}\right)^{t-\bar{t}} u'(\hat{P}_{\bar{t}})$$

Hence $\hat{P}_t < P_t$ for all $t \geq \bar{t}$ and hence $\hat{t}* \geq t*$ which gives us a contradiction.

On the other hand, we cannot have $\hat{P} > P$ either. Suppose this was true, then, $\hat{q}_0 < q_0$ in order to have $\hat{P}_0 > P_0$. But then $\hat{Q}_1 > Q_1$and we must have $\hat{q}_1 < q_1$ in order to have $\hat{P}_1 > P_1$. Continuing this line of reasoning, we would have that $\hat{q}_t < q_t \quad t \leq \max(\hat{t}*, t*)$. But this is not feasible since (for T sufficiently large)

$$\sum_{t=0}^{T} \hat{q}_t = \sum_{t=0}^{T} q_t = Q_0$$

Hence, we must have that $\hat{t}* \geq t*$. On the other hand, due to the discrete nature of t, we cannot hope for a stronger conclusion than this; an infinitesimal change in β would generally leave t∗ unchanged.

It should be noted that in the case where $\hat{\beta} \leq (\varepsilon_S - \varepsilon_L)/\varepsilon_S \leq \beta$, it is socially optimal that the banks liquidate their bad loans over time but due to the regulator's short time horizon there will be no liquidation.

It is generally acknowledged that the regulatory response in Japan has been delayed and hence the bad loan problem prolonged at society's costs (see for example Hutchison and McDill, 1998; Ueda 1999 and Kanaya and Woo, 2000). Corollary 1 suggests that this may be due to the fact that the regulatory authorities have had a short time horizon. To illustrate the effect of the regulatory authorities' time horizon we use as a proxy for the regulator's probability of re-appointment the average time a prime minister has been in office since the burst of the asset bubble in 1989.[9] In the period from August 1989 to June 2004, there has been 9 different prime ministers equalizing an average time in office of less than 20 months.

This corresponds to a yearly re-appointment probability, p, of 0.39, significantly decreasing the regulator's effective discount factor, $\hat{\beta}$.[10]

The present government administration of Junichiro Koizumi has by Japanese standards been an exceptionally long lived one (running at its fourth year at the time of writing). Perhaps not

[7] For instance in Japan, one head of the Financial Supervisory Agency suggested that "bankers should come to him if they were faced with an unduly harsh inspection" (quote from the The Economist, Jan. 6th, 2001).

[8] Peek and Rosengren (2003) argue in their empirical study of the misallocation of credit in Japan that "...a lack of transparency and the use of financial gimmicks allowed bank supervisors to implement forbearance policies that allowed banks to understate their problem loans..."(Peek and Rosengren, 2003 p. 8).

[9] One can argue that the prime minister is seldom responsible for bank closures or bank bailouts. However, we would argue that in the Japanese case politics has played an essential role in the financial authorities that have been in charge of overseeing banks. Until 1998 the Ministry of Finance had the responsibility of supervising and regulating banks. From 1998 this task has been performed by a new agency, the Financial Supervisory Agency (FSA) which in the meantime has changed name to the Financial Services Agency. Suzuki (2000) documents that these financial authorities have been heavily politicized and for example have been very weak in carrying out proposals about bad loan management. Indeed, the financial authorities' suggestions have often been overruled by the prime minister. As an example, in September 2002 Hakuo Yanagisawa, Japan's financial services minister and head of the FSA, was sacked mainly because the FSA had been opposed to use public funds to bail out the banking sector (see Financial Times, Sept. 25th 2002, Sept. 30th 2002). The FSA has had seven chiefs since its creation in late 1998.

[10] When calculating the yearly probability of remaining in office, we assume that the time in office is geometrically distributed with a constant and independently distributed yearly probability of re-appointment, p.

coincidentally, this administration has also shown the largest commitment to tackle the bad loan problems of the Japanese banks by for example replacing the heads of the Financial Service Agency and the Bank of Japan with more reform minded people. While outside the scope of our model, this recent evidence does seem to support that long lived administrations are more willing to take on banks' bad loan problem.

5.2. Liquidity and Liquidation

In this section, we will study how a change in εS will affect the optimal liquidation path. We interpret the size of εS as reflecting the illiquidity in the market for collateral where a high εS refers to an illiquid market. The reason is, that if the market is very illiquid there will be a high price impact of selling a given amount of collateral, all at the same time.

We obtain the following result.

Corollary 2. A decrease in liquidity, that is a higher εS , delays liquidation.

Proof. From the proof of Corollary 1 it can be seen that an increase in εS has the same effect as a decrease in β. Thus, an increase in εS increases t .

It may be noted, that if an increase in liquidity increases the long term value, L, this will tend to strengthen the above effect. This will happen to the extent that the long term value incorporates a liquidity premium. Since we would rather err on the conservative side, we have assumed that the long term value, L, remains constant in the derivation of the above comparative static result.

In Japan, the markets for real estate and land are very illiquid. Corollary 2 provides a reason for why liquidation of banks' bad loans has been postponed. To reduce the costs of liquidation, it has been argued that measures to improve liquidity in the real estate such as tax reforms, securitization of real estate and removal of building restrictions would be beneficial (see for example Shimizu, 2000). Corollary 2 shows that such initiatives will lead to a faster optimal liquidation path.

5.3. The Long Term Value and Liquidation

Consider now the effect of a change in the long term value, L, on the optimal liquidation path. We have the following result.

Corollary 3. A lowering in the long term value of the fixed asset, that is a lower L, delays liquidation.

Proof. Follows from the proof of Corollary 1.

In our model the long term value of the fixed asset, L, can be interpreted as reflecting the present value of the expected yields as well as a possible capital gain/loss, abstracting from the depressing effect of current and future liquidations. In Japan the value of real estate and land have fallen steadily up through the 1990's.[11] As argued by Shimizu (2000), a part of this trend may be due to a general lowering of asset returns because of deregulation in the financial market. Corollary 3 shows that a fall in the long term value of real estate and land will delay liquidation.

5.4. Future Liquidity and Liquidation

Finally, we consider the effect of a change in εL. We may interpret a lowering of εL as
reflecting that the market expects the economy to improve, increasing later liquidity. We have the following result.

Corollary 4. A decrease in εL delays liquidation.

Proof. Follows from the proof of Corollary 1.

Some observers have argued that one reason why the Japanese authorities postponed action was that they expected the economy would soon recover (Kanaya and Woo, (2000)).

If the economy recovered it would ease the process of liquidating the banks' bad loans as liquidity in the real estate and land markets would improve. Corollary 4 shows that if future liquidity is expected to increase it is optimal to delay liquidation.

6. Concluding remarks

In this paper we have analyzed the optimal liquidation of banks' bad loans. We found that small and large banking crises are different with respect to the optimal length of the liquidation path. It was argued that liquidation is likely to be postponed if the regulator has a short time horizon and future liquidity in the market for collateral is expected to increase. Moreover, we showed that the optimal liquidation path should be lengthened when the market for collateral is illiquid and when the long term value of the fixed asset is low. We have argued that our results are of relevance for the Japanese banking crisis. We want to emphasize that our analysis should only be considered as a first step, whatever small, towards a more fully understanding of the optimal liquidation of banks' bad loans. A next major step would be to incorporate the framework into a general equilibrium setting. This would, among other things, involve modelling the demand side of the market for the fixed asset. Such an analysis will yield further insights into the optimal resolution of a banking crisis.

7. Appendix

Consider the following problem for some arbitrarily large time horizon T

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[11] Land prices in Japan have in average fallen 3.9 percent annually in the period 1992-2002. (Bank of
Japan, Statistics 2003).

$$\max_{(q_t)_{t=0}^T} \sum_{t=0}^{T} \beta^t u(L - \varepsilon_S q_t - \varepsilon_L Q_{t+1})$$

$$s.t \quad \begin{array}{l} Q_{t+1} = Q_t - q_t \\ q_t \geq 0 \\ Q_{T+1} \geq 0 \\ Q_0 > 0 \quad \text{given} \end{array} \qquad t = 0, ..., T$$

Letting £ denote the Lagrangian, we can convert the problem to the following unconstrained problem

$$\max \pounds = \sum_{t=0}^{T} \beta^t u(L - \varepsilon_S q_t - \varepsilon_L Q_{t+1}) + \sum_{t=0}^{T} \lambda_t (Q_{t+1} - Q_t + q_t) + \sum_{t=0}^{T} \phi_t q_t + \mu Q_{T+1}.$$

The Kuhn-Tucker theorem gives that at the maximum, the Langrangian must be a stationary point with respect to each Qt and qt. Differentiating £ with respect to each variable gives the first order conditions (substituting away φt and its associated complementary slackness condition from the problem)

$$\begin{aligned} \frac{\partial \pounds}{\partial q_t} &= -\beta^t u'(P_t)\varepsilon_S + \lambda_t \begin{cases} = 0 & if \quad q_t > 0 \\ \leq 0 & if \quad q_t = 0 \end{cases} \quad t = 0, ..., T & (8) \\ \frac{\partial \pounds}{\partial Q_t} &= -\beta^{t-1} u'(P_{t-1})\varepsilon_L - \lambda_t + \lambda_{t-1} = 0 \quad t = 1, ..., T & (9) \\ \frac{\partial \pounds}{\partial Q_{T+1}} &= -\beta^T u'(P_T)\varepsilon_L + \lambda_T + \mu = 0 & (10) \\ \mu Q_{T+1} &= 0 & (11) \\ \mu &\geq 0. & (12) \end{aligned}$$

If β > εS−εL/εS and T is sufficiently large, the solution for qt has the following form

$$\begin{aligned} q_t &> 0 \quad for \quad 0 \leq t \leq t^* \quad and \quad \frac{u'(P_t)}{u'(P_{t-1})} = \frac{1}{\beta}\frac{\varepsilon_S - \varepsilon_L}{\varepsilon_S} \quad for \quad 1 \leq t \leq t^* \\ q_{t^*} &= Q_{t^*} \\ q_t &= 0 \quad for \quad t > t^* \end{aligned}$$

where 0 ≤ t < ∞. Furthermore, the solution does not change with an increase in T. In particular, the limit solution (as T → ∞) exists and has the above form.

If β ≤ εS−εL/εS, the solution has the following form

$$q_t = 0 \quad \forall t.$$

Below, we show that there is a solution of this form for the case A) β > εS−εL/εS and B) β ≤ εS−εL/εS. Uniqueness follows from the fact that we are maximizing a strictly concave function on a convex set.

A) From (9), we have that

$$\lambda_t = -\beta^{t-1} u'(P_{t-1})\varepsilon_L + \lambda_{t-1}.$$

For 1 ≤ t ≤ t , we obtain by inserting (8) twice

$$\begin{aligned} \beta^t u'(P_t)\varepsilon_S &= -\beta^{t-1} u'(P_{t-1})\varepsilon_L + \beta^{t-1} u'(P_{t-1})\varepsilon_S \\ \beta u'(P_t)\varepsilon_S &= -u'(P_{t-1})\varepsilon_L + u'(P_{t-1})\varepsilon_S \\ \beta u'(P_t)\varepsilon_S &= u'(P_{t-1})(\varepsilon_S - \varepsilon_L), \end{aligned}$$

or

$$\frac{u'(P_t)}{u'(P_{t-1})} = \frac{1}{\beta}\frac{\varepsilon_S - \varepsilon_L}{\varepsilon_S} < 1.$$

By induction

$$u'(P_t) = \left(\frac{1}{\beta}\frac{\varepsilon_S - \varepsilon_L}{\varepsilon_S}\right)^t u'(P_0).$$

Given that the utility function is strictly increasing and differentiable, we have that u0(P0) is positive and finite. We define t as the smallest t N such that

$$\left(\frac{1}{\beta}\frac{\varepsilon_S - \varepsilon_L}{\varepsilon_S}\right)^{t+1} u'(P_0) < u'(L). \qquad (13)$$

Thus, t + 1 is the first period where the price would have been above L had qt and Qt not been constrained to be non-negative. We observe that t is finite, since u0(P0) is finite and u0(L) is strictly positive.

We will now argue that for a sufficiently large time horizon T it cannot be optimal to have non-liquidated loans at the final date, i.e. we cannot have QT+1 > 0. Since, by assumption, we have that qt = 0 for t > t , we then know that all remaining debt is liquidated at t , i.e. qt = Qt .

In period t , we know from (8) that

$$\lambda_{t^*} = \beta^{t^*} u'(P_{t^*})\varepsilon_S.$$

Successively substituting into (9) yields

$$\lambda_T = \beta^{t^*} u'(P_{t^*})\varepsilon_S - \sum_{s=t^*}^{T-1} \beta^s u'(P_{t^*})\varepsilon_L.$$

Inserting into (10), we obtain

$$\begin{aligned} \mu &= \sum_{s=t^*}^{T} \beta^s u'(P_{t^*})\varepsilon_L - \beta^{t^*} u'(P_{t^*})\varepsilon_S \\ &= \beta^{t^*}\left(\sum_{s=0}^{T-t^*} \beta^s \varepsilon_L - \varepsilon_S\right) u'(P_{t^*}) \\ &= \beta^{t^*}\left(\frac{1-\beta^{T-t^*+1}}{1-\beta}\varepsilon_L - \varepsilon_S\right) u'(P_{t^*}) \\ &= \beta^{t^*}((1-\beta^{T-t^*+1})\varepsilon_L - (1-\beta)\varepsilon_S)\frac{1}{1-\beta}u'(P_{t^*}). \end{aligned}$$

From the assumption β > εS−εL/εS, we have that

$$\varepsilon_L > (1-\beta)\varepsilon_S.$$

Hence for sufficiently large T (since t is finite), we have that μ > 0. From the complementary slackness condition, we then have that QT+1 = 0. Therefore, qt = Qt

We can deduce a little more knowledge from our expression for μ. Since μ ≥ 0, we cannot have t = T, which would result in a negative μ. The intuition for this result, is that in period T it is always preferable to postpone liquidation.

Next, consider period t + 1. By assumption

$$\lambda_{t^*} = \beta^{t^*} u'(P_{t^*})\varepsilon_S. \qquad (14)$$

We wish to show that

$$\lambda_{t^*+1} \leq \beta^{t^*+1} u'(P_{t^*+1})\varepsilon_S. \qquad (15)$$

From (9) we have that

$$\lambda_{t^*+1} = -\beta^{t^*} u'(P_{t^*})\varepsilon_L + \lambda_{t^*}. \qquad (16)$$

Inserting (14) into (16) we obtain

$$\lambda_{t^*+1} = -\beta^{t^*} u'(P_{t^*})\varepsilon_L + \beta^{t^*} u'(P_{t^*})\varepsilon_S.$$

(15) will be satisfied if

$$\begin{aligned} -\beta^{t^*} u'(P_{t^*})\varepsilon_L + \beta^{t^*} u'(P_{t^*})\varepsilon_S &\leq \beta^{t^*+1} u'(P_{t^*+1})\varepsilon_S \Leftrightarrow \\ \beta^{t^*} u'(P_{t^*})(\varepsilon_S - \varepsilon_L) &\leq \beta^{t^*+1} u'(P_{t^*+1})\varepsilon_S \Leftrightarrow \\ u'(P_{t^*})(\varepsilon_S - \varepsilon_L) &\leq \beta u'(P_{t^*+1})\varepsilon_S. \qquad (17) \end{aligned}$$

Which holds, as u0(Pt +1) = u0(L) which by (13) is larger than

$\left(\frac{1}{\beta}\frac{\varepsilon_S-\varepsilon_L}{\varepsilon_S}\right)^{t^*+1} u'(P_0)$, the price that would satisfy (17) as an equality.

The argument for periods after t + 1 is very similar. By assumption for t ≥ t + 1

$$\lambda_t \leq \beta^t u'(L)\varepsilon_S. \tag{18}$$

We wish to show that

$$\lambda_{t+1} \leq \beta^{t+1} u'(L)\varepsilon_S. \tag{19}$$

From (9) we have that

$$\lambda_{t+1} = -\beta^t u'(L)\varepsilon_L + \lambda_t. \tag{20}$$

Inserting (18) into (20) we obtain

$$\lambda_{t+1} \leq -\beta^t u'(L)\varepsilon_L + \beta^t u'(L)\varepsilon_S.$$

(19) will be satisfied if

$$\begin{aligned} -\beta^t u'(L)\varepsilon_L + \beta^t u'(L)\varepsilon_S &\leq \beta^{t+1} u'(L)\varepsilon_S \Leftrightarrow \\ \beta^t u'(L)(\varepsilon_S - \varepsilon_L) &\leq \beta^{t+1} u'(L)\varepsilon_S \Leftrightarrow \\ (\varepsilon_S - \varepsilon_L) &\leq \beta\varepsilon_S \end{aligned}$$

which holds by assumption.

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