

IDENTIFY TOO BIG TO FAIL BANKS AND CAPITAL INSURANCE: AN EQUILIBRIUM APPROACH

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Abstract

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The objective of this paper is develop a rational expectation equilibrium model of capital insurance to identify too big to fail banks. The main results of this model include (1) too big to fail banks can be identified explicitly by a systemic risk measure, loss betas, of all banks in the entire financial sector; (2) the too big to fail feature can be largely justified by a high level of loss beta; (3) the capital insurance proposal benefits market participants and reduces the systemic risk; (4) the implicit guarantee subsidy can be estimated endogenously; and lastly, (5) the capital insurance proposal can be used to resolve the moral hazard issue.

We implement this model and document that the too big to fail issue has been considerably reduced in the pro-crisis period. As a result, the capital insurance proposal could be a useful macro-regulation innovation policy tool.

Keywords: Systemic Risk, Too Big To Fail, Capital Insurance

1. INTRODUCTION

The objective of this paper is to develop a new methodology to identify too big to fail (TBTF) banks by a rational expectation equilibrium model¹. Since the too big to fail issue is virtually linked to the implicit guarantee subsidy², this model can be also useful on the assessment of the implicit subsidy endogenously. The model generates a new systemic risk measure, loss beta, and we demonstrate that this concept of loss beta captures some essential economic elements of the TBTF issue.

The financial crisis 2007-2009 sparks substantial research interests in measuring the systemic risk recently. Acharya et al (2012), Brownless and Engle (2011) show that time-varying correlation structure play a crucial role in their systemic risk measurements (see also v-lab webpage in New York University); and it is well documented that the time-varying correlation coefficients among big financial

institutions are broadly positive. Consequently, several approaches have been proposed to cast the connectivity and correlative features among top banks in studying the systemic risk, including Adrian and Brunnermeier (2016)'s CoVaR approach conditional on financial institutions being in a state of financial distress; the network approach by Acemoglu et al (2015) and Elliott et al (2014); the default probability of the whole financial system developed by Shin (2008); the marginal expected shortfall measure approach in Acharya (2009), Brownless and Engle (2011) and Acharya et al (2012), and the CDS premium approach in Zhou, Huang and Zhu (2009). Hansen (2012) explains the challenge to measure the systemic risk, and a comprehensive survey of systemic risk measures is presented by Bisias et al (2012)³. None of these approaches, however, explores an equilibrium mechanism in which banks and regulator interact with each other in their best interests.

At the same time, in response to the crisis, the Basel Committee on Banking Supervision (BCBS)⁴ has

¹ The term "too big to fail" is frequently interchanged with other terms such as "too important to fail" (TITF), "too interconnected to fail" (TITF) or "global systemically important banks" (G-SIBs) with might be slightly different contexts. A bank is deemed to be TBTF in this paper if the bank has implicit government guarantee during a crisis.

² The implicit (guarantee) subsidy, or alternatively, capital surcharge, is often estimated by funding costs with and without the guarantee. See, for instance, IMF (2014) and Green/EFA group report (2014). See also O'Hara and Shaw (1990) in the context of deposit insurance; and BCBS (2013) for assessment methodology.

³ Other notably papers include Allen and Gale (2000); Hellwig (2009); Lehar (2005); Battiston et al (2012); Billio et al (2012); and Rochet (2009).

⁴ Specifically, BCBS issues addition documentation for worldwide regulation of banks. First, The Basel Committee has issued consultative version of "Basel III's leverage ratio framework and disclosure requirements" published in June 2013. Basel III's leverage ratio is defined as the "capital measure" divided by the "exposure measure", where the capital measure is a Tier 1 capital and the exposure measure is defined as the sum of on-balance sheet

adopted a series of reforms to improve the resilience of banks and banking systems. They include forcing capital requirements for the banks, improving risk coverage, proposing a leverage ratio, introducing capital conservation and countercyclical buffers as well as a global standard for liquidity risk.⁵

BCBS describes the additional loss absorbency requirements that will apply to G-SIBs. The methodology is based on an indicator-based measurement approach, and weights equally each of the following five categories of systemic importance: "size", "cross-jurisdictional activity", "interconnectedness", "substitutability" and "complexity". Banks that have a score produced by the indicator-based measurement approach that exceeds a cutoff level set by the Committee are classified as G-SIBs.⁶ The advantage of the multiple indicator-based measurement approach for identifying TBTF banks is that it encompasses many dimensions of systemic importance; however, a cut-off number of G-SIBs is chosen exogenously by the Committee so an equilibrium approach is still missing.

This paper develops an equilibrium approach to link the systemic risk measure study and recent regulatory proposals. The mechanism behind the model is as follows. We suggest that TBTF banks have to pay *insurance premium* up front to exchange for its *implicit guarantee subsidy*. The agreement between the bank and the regulator (the issuer of the contract), which injects the guaranteed capital contingent upon a stressed time period, is treated as an insurance contract (a *capital insurance* contract). Each bank predicts the best insured amount whenever the pricing structure of the capital insurance is given by the issuer. The issuer of the capital insurance fully predicts each bank's optimal insured amount, determines the optimal pricing structure, and simultaneously identifies those banks endogenously which are willing to purchase this kind of capital protection. Those banks to purchase the capital insurance are identified as *too big to fail* banks under this approach. The idea of capital insurance to study the systemic risk is first briefly proposed by Kashyap et al (2008). It is also resemble to the special tax program proposed in Acharya et al (2010) in which the insurance premium is viewed as special tax for too big to fail.⁷ This model has four important implications. First, the model generates a new equilibrium systemic risk measure, loss beta,

which is defined as a ratio of the covariance between a bank's loss portfolio with the aggregate loss portfolio in the entire bank sector to the variance of the aggregate loss portfolio. Given its concentration on loss portfolios, this approach to the systemic risk leads to starkly difference between our systemic risk measure with other systemic measures that based on classical beta, downside beta or tail beta (Bawa and Lindenberg, 1977; Hogan and Warren, 1974; Van Oordt and Zhou, 2014). For instance, Benoit et al (2012) in a recent empirical study shows that from both theoretical and empirical perspective, the marginal expected shortfall measure introduced in Acharya (2009), Brownless and Engle (2011), Acharya et al (2012) is largely explained by the classical betas of banks; and the classical beta of financial institution captures to some extent the interconnectedness in the financial sector but adds little to rank too big to fail banks.

Second, we present an algorithm to identify too big to fail bank and this algorithm relies merely on the loss betas of all banks in the market. Not only banks with large loss betas are TBTF; we also show that TBTF banks must have large loss betas. Therefore, the too big to fail feature is largely captured by the loss beta measure.

Third, we demonstrate several positive effects of a capital insurance proposal. The social welfare for the regulator is shown to be positive and the total systemic risk is reduced with the implementation of the capital insurance market. TBTF banks are beneficial by purchasing the capital protection in the capital insurance market, and those banks with larger systemic risk components enjoy more expected utility enhancing. More importantly, the capital insurance market can be used by the regulator to reveal banks' *true* loss portfolios and identify TBTF banks correctly in the presence of asymmetric information between banks and the regulator. Overall, the capital insurance proposal is shown to be a useful macro-regulation policy tool to address the TBTF issue.⁸

At last, we calibrate this model by using several different capital insurance contracts. We find out that TBTF banks can be consistently identified with this equilibrium approach over the pre-crisis and pro-crisis period, and the too big to fail concern has been considerably reduced after the financial crisis.

This paper merges two important strands of previous research. By viewing capital insurance as an innovation in a capital market, we explore a similar framework examined in Allen and Gale (1994) to characterize the equilibrium among a group of buyers and a seller in the presence of one financial innovation. We follow Harris and Raviv (1995) to study the optimal payoff structure within a given specification form of the payoff structure of financial innovation. Moreover, we study the optimal portfolio risk at the presence of financial innovation as in Simsek (2013). On the other hand, treating the capital insurance as an insurance contract between banks and regulator, we develop the model by drawn on some essential insights in Borch (1962), Arrow (1964), Ravi (1979) and Mace (1999). It is worth noting that the framework presented in this paper is

exposures, derivative exposures, securities financing transaction exposures, and other off-balance sheet exposures. Second, the version of "Global systemically important banks: updated assessment methodology and the higher loss absorbency requirement" published in July 2013 sets out the Basel Committee's methodology for assessing and identifying global systemically important banks (G-SIBs).

⁵ Specifically, BCBS issues addition documentation for worldwide regulation of banks. First, The Basel Committee has issued consultative version of "Basel III's leverage ratio framework and disclosure requirements" published in June 2013. Basel III's leverage ratio is defined as the "capital measure" divided by the "exposure measure", where the capital measure is a Tier 1 capital and the exposure measure is defined as the sum of on-balance sheet exposures, derivative exposures, securities financing transaction exposures, and other off-balance sheet exposures. Second, the version of "Global systemically important banks: updated assessment methodology and the higher loss absorbency requirement" published in July 2013 sets out the Basel Committee's methodology for assessing and identifying global systemically important banks (G-SIBs).

⁶ Alternatively, bucketing approach is used to determine loss-absorbing capacity of G-SIBs, while the principles on loss absorbing and recapitalization capacity of G-SIBs in resolution are discussed in the consultative document of November 2014.

⁷ Therefore, the developed equilibrium in this article is also an equilibrium of a special tax program.

⁸ Classical prudential regulation theory of banks is explained in Dewatripoint and Tirole (1994); Hanson, Kashyap and Stein (2011). See also Aiyar, Calomiris and Wieladek (2014) for a comprehensive discussion on bank capital regulation.

different from the classical insurance setting in which the law of large numbers (risk-pooling principle) holds under an independent assumption of the individual risk across a group of insureds. Indeed, the failed risk-pooling principle with correlated underlying risks proposes a challenge in measuring the systemic risk. The capital insurance innovation as shown in this paper shed lights to resolve the correlated risk management problem.⁹

The paper proceeds as follows. In Section 2 we present a theory of capital insurance. In Section 3 we report our empirical analysis and illustrate some implementation issues. Section 4 concludes and all proofs are given in Appendix A.

2. THEORY OF CAPITAL INSURANCE

2.1. Model Setup

There are N financial institutions, namely banks, indexed by $i = 1, \dots, N$, in a financial sector. Each bank is endowed with a loss portfolio (or exposure, and we do not distinguish these two concepts in this paper), X_1, \dots, X_N , respectively. These loss portfolios are given exogenously, model-free, with systemic risk components. There is a capital insurance market in which each bank decides to purchase or not a capital insurance contract to hedge the systemic risk. The prototype capital insurance contract's payoff structure (or indemnity in insurance terminology) is $I_i(X, X)$ for bank i where X represents the aggregate loss, $X = \sum_{i=1}^N X_i$, of the financial sector.

We follow standard insurance literature (Arrow, 1963; and Raviv, 1979) to apply a classical linear insurance premium principal. Specifically, the insurance premium P_i for bank i to pay for is, $P_i = (1 + \rho)E[X, X]$ where ρ is a load factor that is determined by the issuer. It is convenient for now to assume a constant load factor across the financial sector, and its extension to a bank-specific load factor is presented in Section 3. In this paper, we focus on the following capital insurance contract, $I_i(X, X) = a_i Z$ for each bank i , where a_i is a nonnegative coinsurance coefficient and $Z = I(X)$ is an arbitrarily specification of indemnity that relies on the aggregate loss in the financial sector. Bank i chooses the best coinsurance coefficient a_i , and the optimal coinsurance coefficient is written as $a_i(\rho)$, to highlight its dependence on the load factor ρ .

Each bank i , $i = 1, \dots, N$, is risk-averse, and its risk preference is represented entirely by the mean and the variance of the wealth with the reciprocal of risk aversion parameter $\gamma_i > 0$.¹⁰ Given a load factor ρ , bank i solves an optimal portfolio problem by choosing the best coinsurance coefficient:

$$\max_{\{a_i \geq 0\}} \left\{ E[\tilde{W}^i] - \frac{1}{2\gamma} Var(\tilde{W}^i) \right\}, \quad (1)$$

where:

$$\tilde{W}^i = W_0^i - X_i + a_i Z - (1 + \rho)E[a_i Z]$$

is the *ex post* terminal wealth for the bank i after

purchasing the capital insurance and W_0^i is the initial wealth of bank i . Similarly, $W^i = W_0^i - X_i$ represents the *ex ante* wealth of bank i before buying capital insurance. Moreover, without loss of generality we assume that each $\gamma_i = \gamma$ for $i = 1, \dots, N$, so these banks are distinguished from each other due to in essence their different loss exposures instead of risk preference. By the first order condition in (1), the optimal coinsurance coefficient for bank i is given by

$$a_i(\rho) = \max \left\{ \frac{Cov(X_i, Z) - \rho E(Z)\gamma}{Var(Z)}, 0 \right\} \quad (2)$$

The issuer of capital insurance contracts can be a private-sector, reinsurance company, a central bank or a government entity such as Financial Stability Oversight Council (FSOC) in Dodd-Frank Act, which is universally named as a *regulator*. The regulator is assumed to be risk-neutral and receives the insurance premium from each capital insurance contract. Therefore, the terminal wealth of the regulator is

$$W^r = \sum_{i=1}^N (1 + \rho)E[a_i Z] - \sum_{i=1}^N a_i Z - \sum_{i=1}^N c(a_i Z), \quad (3)$$

where $c(a_i Z)$ denotes the cost for the regulator to issue the contract $a_i Z$. This regulatory cost $c(\cdot)$ can be a fixed cost, a constant percentage of the indemnity or a general function of the indemnity. To focus on the equilibrium analysis of TBTF, we assume that the regulatory cost is a constant for each bank.¹¹

Given the optimal demand for each bank (with a load factor ρ) in (2), the regulator is presumed to maximize the expected welfare $E[W^r]$ by determining the best load factor ρ and the optimal insurance premium in (2). Specifically, by plugging equation (2) into equation (3), the regulator's optimal load factor is derived from the following optimization problem:

$$\max_{\{\rho > 0\}} \rho \sum_{i=1}^N \max \left(\frac{Cov(X_i, Z) - \rho \gamma E[Z]}{Var(Z)}, 0 \right) \quad (4)$$

and the optimal coinsurance coefficient for each bank $i = 1, \dots, N$, is given by $a_i(\rho^*)$, where ρ^* is the optimal load factor in (4). In the end, the capital insurance's payoff for each bank i , $a_i(\rho^*)Z$, relies on both demand (from all banks) and supply (from the regulator) in a rational expectation equilibrium. Both the optimal $a_i(\rho^*)$, and ρ^* are determined endogenously.

In light of the non-concavity feature of its objective function, the regulator's optimization problem (4) is non-standard; thus, its solution cannot be easily characterized by virtue of the first order condition. In Appendix A, we elaborately reduce the optimization problem (4) to a set of standard optimization problems; and as a consequence, solve the existence of the equilibrium.

The following definition captures the main insights of the equilibrium model on TBTF.

Definition 1. *With a capital insurance $Z = I(X)$, the loss*

⁹ Panttser and Tian (2013) conducts a comparative analysis between capital insurance and classical coinsurance contract in a correlated risk environment.

¹⁰ Mace (1991) addresses the aggregate uncertainty insurance under the same assumption.

¹¹ We refer to Huberman, Mayers and Mayers (1982) for a discussion of the cost structure in insurance market.

beta of bank i is $\frac{Cov(X_i, Z)}{Var(Z)}$.

Bank i is deemed to be TBTF, from the capital insurance $Z=I(X)$ perspective, if its optimal coinsurance coefficient $a(\rho^*)$, is positive.

The capital insurance premium, $(1 + \rho^*)a(\rho^*)E[Z]$, is an insurance capital for bank i .

Clearly, the capital insurance premium offers an assessment of the implicit subsidy from an insurance perspective.

2.2. Identifying TBTF Banks

By virtue of equation (2), bank i is too big to fail as long as its loss beta, $Cov(X_i, Z)/Var(Z)$, is large enough such that:

$$\frac{Cov(X_i, Z)}{Var(Z)} > \rho^* \left(\gamma \frac{E[Z]}{Var(Z)} \right). \tag{5}$$

But the optimal load factor ρ^* in (5) is subject to determined endogenously. The optimal load factor is solved by (4), and it depends on all loss portfolios information, in particular, all banks' loss betas. Therefore, an individual bank's loss beta is not sufficient yet to recognize whether it is too big to fail or not; rather, we have to study the entire financial sector as a whole to identify all TBTF banks simultaneously. Briefly speaking, a bank is TBTF only when its loss beta is relatively large compared with other banks' loss betas in the same financial sector.

Again, because of its non-standard feature, it is plausible to have multiple optimal solutions in (4) and thus multiple equilibria in the capital insurance market. We argue that this plausible multiple equilibria issue is not serious though.¹² The higher the load factor is, the less banks are identified as TBTF and those identified TBTF banks have to pay higher insurance premiums. In contrast, a smaller load factor ensures a larger number of TBTF banks whereas each TBTF bank pays a smaller insurance premium. Evidently, the regulator is willing to choose the smallest load factor, among all plausible solutions of ρ^* , to enlarge the number of TBTF banks under monitoring even though the expected welfare for the regular is indifferent. Those banks with higher systemic risk components also desire a smaller load factor because of smaller insurance premiums. Only banks with relatively small loss betas have benefited from a higher load factor, because these banks are otherwise characterized as TBTF and forced to pay insurance premiums. For these reasons, it is reasonable to choose the *smallest* load factor for the regulator in the presence of possible multiple optimal solutions in problem (4).

As shown in Appendix A, the following simple algorithm identifies TBTF banks by merely using of loss betas.

Step 1. Let

$\beta_i = \frac{Cov(X_i, Z)}{Var(Z)}$, and reorder $\{\beta_1, \dots, \beta_N\}$, such that

$\beta_1 \geq \dots \geq \beta_N > 0$. We omit those banks with negative or zero loss betas.

Step 2. Let

$$\tau_m = \frac{1}{2m} \sum_{i=1}^m \beta_i \text{ for } m=1, \dots, N.$$

Define:

$$\bar{\tau}_m = \min\{\beta_m, \max(\beta_{m+1}, \tau_m)\} \text{ for } m=1, \dots, N-1 \text{ and } \bar{\tau}_N = \tau_N.$$

Step 3. Compute $B_m = h_m(\bar{\tau}_m)$ for each $m=1, \dots, N$,

where $h_m(\tau) = \sum_{i=1}^m (\beta_i \tau - \tau^2)$.

Step 4. Compute m^* as $\text{argmax}_{1 \leq m \leq N} B_m$, and choose the smallest m^* if there exist multiple solutions of m^* .

Step 5. Bank i is TBTF if and only if $\beta_i > \bar{\tau}_{m^*}$,

for $i = 1, \dots, N$.

The next proposition shows that the bank with the highest loss beta must be a TBTF bank.

Proposition 1. *Among all banks in a financial sector, the bank with the highest loss beta must be too big to fail.*

By Proposition 1, there do exist TBTF banks in any financial sector. Therefore, the capital insurance is of necessary from the regulatory perspective. Several examples are presented below to illustrate the algorithm.

Example 1. If each bank contributes equivalently to the systemic risk in the sense that:

$$\frac{Cov(X_i, Z)}{Var(Z)} = c \text{ for any } i = 1, \dots, N,$$

and a positive number c , then each bank is TBTF and the optimal load factor is:

$$\frac{c}{2\gamma} \frac{Var(Z)}{E[Z]}$$

Moreover, the optimal coinsurance coefficient for each bank is its half loss beta.

Example 1 follows easily from Proposition 1, in which each bank has the same loss beta; therefore, each bank is too big to fail. The optimal load factor and the corresponding coinsurance coefficient can be calculated easily.

Example 2. Consider a financial sector with two banks, $i=1, 2$, and assume that $Cov(X_1, Z) \geq Cov(X_2, Z)$. Then each bank is TBTF if $Cov(X_1, Z) = Cov(X_2, Z)$; and only bank 1 is TBTF if, and only if the following condition holds:

$$1 < \frac{Cov(X_1, Z)}{Cov(X_2, Z)} \leq \frac{1}{\sqrt{2}-1}$$

The first case in Example 2 follows from Example 1. Assume that $Cov(X_2, Z) < Cov(X_1, Z)$. Then only the first bank is TBTF, by using the algorithm, if

¹² However, the multiple equilibrium issue might be very severe in some economic contexts. See, for instance, Diamond and Dybvig (1983), Sundaresan and Wang (2013).

and only if $h_2(\bar{\tau}_2) \geq h_1(\bar{\tau}_1)$. It is easy to verify that, the last inequality holds if and only if:

$$\frac{\text{Cov}(X_1, Z)}{\text{Cov}(X_2, Z)} \leq \frac{1}{\sqrt{2}-1}$$

The next example is concerned with a financial system with more than three banks, in which only one bank is TBTF if this bank's loss beta significantly dominates all other banks' loss betas.

Example 3. Given a loss beta structure such that:

$$\frac{\text{Cov}(X_i, Z)}{\text{Var}(Z)} = c\tau^{i-1} \text{ for each } i = 1, \dots, N,$$

a positive number c and a positive number $\tau \in (0,1)$, only the first bank is TBTF when τ is small enough. Moreover, the optimal load factor is

$$\rho^* = \frac{c}{2\gamma} \frac{\text{Var}(Z)}{E[Z]}$$

Example 3 is interesting in its own right. Even though some banks contribute positively to the systemic risk and banks are heavily correlated, those banks might still not be TBTF banks, given the fact that by insuring the bank with the most significant systemic risk exposure, other banks' systemic risks can be insured to some extent. Example 3 illustrates an essential insight of the capital insurance proposal, which in contrast with the network approach (Acemoglu et al, 2015; Elliott et al 2014) to the systemic risk that connectedness amongst the banks play a key role.

2.3. Positive Social Values

The following result affirms a positive social value of the capital insurance market.

Proposition 2. *With an immaterial regulatory cost, the expected welfare of the capital insurance market for the regulator, $E[W^r]$, is always positive.*

The expected welfare for the regulator depends on a number of market factors such as all banks' loss betas in a financial sector. Under what circumstance the social value is positively related to loss betas or negatively affected by the loss betas? There is no clear-cut on a comparative analysis given the complexity of the equilibrium. Remarkably, Proposition 2 demonstrates a positive effect of the capital insurance market for all possible loss exposures.

We next study the effect of the capital insurance market to TBTF banks. While TBTF banks are identified by the regulator, an important question arises. Whether these TBTF banks are willing to purchase capital insurance contracts on their interests? What happens if these TBTF banks do not purchase the capital insurance? or even if they are enforced to purchase the capital insurance by a regulator, are they intend to manipulate the loss exposure because the purchase decisions are against their willingness? The next result resolves

this potential conflict interest between the regulator and TBTF banks.

Proposition 3. *The expected utility of a TBTF bank is strictly increased after purchasing the capital insurance. Moreover, the higher the loss beta of a TBTF bank, the higher the improved expected utility of the bank.*

Not only are TBTF banks willing to purchase the capital insurance contracts, but also the banks with higher loss betas have more ex post benefits, so those banks are more motivated to participate in this capital insurance market. Both Proposition 2 and Proposition 3 together ensure Pareto improvement by implementing a capital insurance market.

2.4. Aggregate Capital Insurance

In this section, we specialize the capital insurance - aggregate capital insurance - by assuming that the indemnity, Z , is the aggregate loss. With the aggregate capital insurance, we show that TBTF banks must have large loss betas. Accordingly to equation (2), the optimal coinsurance coefficient of the aggregate insurance for a TBTF bank i is:

$$a_i(\rho^*) = \frac{\text{Cov}(X_i, X)}{\text{Var}(X)} - \rho^* \frac{\gamma E[X]}{\text{Var}(X)}, \quad (6)$$

in which the second component on the right side of (6) is the same for all banks. The first component is (by abuse of notation) its loss beta of the loss exposure,

$$\beta_i = \frac{\text{Cov}(X_i, X)}{\text{Var}(X)}. \quad (7)$$

We define concretely the systemic risk from both the market level and the individual bank perspective in an aggregate capital insurance market.

Definition 2. *The systemic risk ex ante in the bank sector is the variance, $\text{Var}(X)$, of the aggregate loss in the financial sector. The systemic risk component of bank i is its loss beta, $\frac{\text{Cov}(X_i, X)}{\text{Var}(X)}$.*

Proposition 4. *The loss beta of a TBTF bank in the aggregate capital insurance market must be greater than or equal to $1/2N$.*

In Example 1, each bank has the same loss beta and belongs to TBTF banks, so each loss beta $\beta_i = 1/N$ because the sum of all loss betas equals to one. In spite of all possible loss exposures, Proposition 4 shows that all TBTF banks's loss betas must be bounded below by $(1/2N)$, a remarkably tight *distribution-free* lower bound of loss betas for all TBTF banks regardless the distribution of loss exposure of each bank.

We turn next to the systemic risk. By using our systemic risk measurements, we demonstrate that the systemic risk is indeed reduced in the entire financial sector by the next result.

Proposition 5. *In a positive correlated risk environment in the sense that:*

$$Cov(X_i, X_j) \geq 0, \forall i, j = 1, \dots, N,$$

the total systemic risk in the financial sector is strictly reduced after implementing the aggregate capital insurance.

2.5. Moral Hazard

We have so far assumed that the regulator recognizes all banks' true loss exposures in the capital insurance market. However, the asymmetric information about loss distributions between banks and the regulator could distort significantly the insurance premium, the optimal indemnity, and probably affect entirely the major insights of the capital insurance market. The objective of this subsection is to examine the moral hazard issue between banks and the regulator. We show that the regulator is able to reveal each bank's true loss exposure in the capital insurance market and to identify TBTF banks correctly; thus, the true loss exposures have to be reported in the presence of the capital insurance market.

Precisely, each bank i 's true loss exposure is denoted by X_i , but this bank's reporting loss exposure to the regulator is \hat{X}_i . We write $\hat{X}_i = X_i + \varepsilon_i$, for $i = 1, \dots, N$, and each noise term ε_i has mean 0 and variance σ_i^2 . We assume that these noise terms, $\varepsilon_1, \dots, \varepsilon_N$, are independent from each other. Moreover, these noise terms are independent from banks' true loss exposures $\{X_1, \dots, X_N\}$. For regulator, the aggregate loss is $\hat{X} = \sum_{i=1}^N \hat{X}_i$, but it might be not the true aggregate loss of the market due to the asymmetric information on the loss distributions.

We consider two kinds of moral hazard. First, we assume that these banks know the true loss exposures each other but they collectively report "wrong" loss exposures to the regulator. This case is called a *collective moral hazard* (see Farhi and Tirole, 2012, in a similar context). Second, these banks do not know the true loss exposures each other. In other words, each bank misrepresents its loss exposures to anyone else to take information advantage in the capital insurance market. This case is termed as a *mutual moral hazard*. In what follows, we show that the regulator is able to reveal the true loss exposures and identify TBTF banks with the help of the aggregate capital insurance in these two cases, respectively.

2.5.1. Collective Moral Hazard

Since bank i knows all true loss exposures in this collective moral hazard situation, bank i 's optimal coinsurance coefficient, if being positive with a given load factor ρ , is determined by the same equation (6). Moreover, even though the true loss exposure X_i and the true aggregate loss exposure X might be unknown to the regulator, the regulator fully observes $a_i(\rho)$ for each $i = 1, \dots, N$ from the capital insurance market. The next proposition shows that, given the information set $\{a_i(\rho), \hat{X}_i; i = 1, \dots, N\}$, the regulator is able to identify σ_i^2 for each bank i .

Proposition 6. Given a load factor ρ with $a_i(\rho) > 0, i = 1, \dots, N$, the variances $\{\sigma_1^2, \dots, \sigma_N^2\}$ can be derived uniquely by the data set $\{a_i(\rho), \hat{X}_i; i = 1, \dots, N\}$.

By Proposition 6, as the regulator offers the capital insurance contracts with vary load factors, the regulator is able to identify the variances, $\sigma_i^2, i = 1, \dots, N$, of the error terms of the loss exposures. Notice that these banks are not necessarily to be TBTF since the load factor might be not the optimal load factor though. However, knowing σ_i^2 , both the "true" covariance:

$$Cov(X_i, X) = Cov(\hat{X}_i, \hat{X}) - \sigma_i^2$$

and the "true" variance:

$$Var(X) = Var(\hat{X}) - \sum_{i=1}^N \sigma_i^2$$

are known accordingly. Therefore, the optimal load factor problem of the regulator is reduced to be:

$$\max_{\{\rho > 0\}} \rho \sum_{i=1}^N \max \left\{ \frac{Cov(\hat{X}_i, \hat{X}) - \sigma_i^2 - \rho \gamma E[\hat{X}], 0}{Var(\hat{X}) - \sum_{i=1}^N \sigma_i^2} \right\}, 0 \quad (8)$$

Problem (8) can be solved exactly as in solving problem (4). Thus, the regulator is able to identify all TBTF banks correctly in this collective moral hazard situation.

2.5.2. Mutual Moral Hazard

In a mutual moral hazard situation, bank i is only aware of its own loss exposure X_i and "reported" loss exposures $\hat{X}_j, j \neq i$, of all other banks. Then, from bank i 's perspective, the aggregate loss exposure is $X_i + \sum_{j \neq i} \hat{X}_j$, which is $\hat{X} - \varepsilon_i$. Consequently, bank i 's terminal wealth in equation (1), after purchasing capital insurance, is replaced by:

$$W_0^i - X_i + a_i(\hat{X} - \varepsilon_i) - (1 + \rho)E[a_i(\hat{X} - \varepsilon_i)].$$

As a result, the first order condition yields the optimal coinsurance coefficient for bank i ,

$$a_i(\rho) = \max \left\{ \frac{Cov(X_i, \hat{X} - \varepsilon_i) - \rho \gamma E[\hat{X} - \varepsilon_i], 0}{Var(\hat{X} - \varepsilon_i)} \right\} \quad (9)$$

Proposition 7. In a positive correlated risk environment in the sense that:

$$Cov(X_i, X_j) \geq 0, \forall i, j = 1, \dots, N$$

the regulator is able to identify TBTF banks correctly in a mutual moral hazard situation. Precisely, given a load factor ρ with $a_i(\rho) > 0, i = 1, \dots, N$, the variances $\{\sigma_1^2, \dots, \sigma_N^2\}$ can be derived uniquely by the data set $\{a_i(\rho), \hat{X}_i; i = 1, \dots, N\}$.

Since the noises' variances $\{\sigma_i^2; i = 1, \dots, N\}$ can be solved by the regulator, the regulator knows

$$Cov(X_i, \hat{X} - \varepsilon_i) = Cov(\hat{X}_i, \hat{X}) - \sigma_i^2$$

and

$$Var(\hat{X} - \varepsilon_i) = Var(\hat{X}) - \sigma_i^2.$$

Then, the optimal load factor for the regulator is reduced to be:

$$\max_{(\rho>0)} \rho \sum_{i=1}^N a_i(\rho) \equiv \rho \sum_{i=1}^N \max \left(\frac{\text{Cov}(\hat{X}_i, \hat{X}) - \sigma_i^2 - \rho \gamma E[\hat{X}]}{\text{Var}(\hat{X}) - \sigma_i^2}, 0 \right) \quad (10)$$

Again, Problem (10) can be solved similarly by a method explained in Appendix A. Therefore, the regulator can identify all TBTF banks in this mutual moral hazard situation.

We have developed an equilibrium model of the capital insurance market and shown the advantages of the proposed capital insurance market in several aspects (Proposition 1 to Proposition 7). Our theoretical results justify that the loss betas capture significant component of the systemic risk. We next illustrate how our theoretical results can be implemented empirically.

3. EMPIRICAL ANALYSIS AND IMPLEMENTATION

To illustrate the presented methodology in Section 2 we implement the model in this section. We apply several capital insurance contracts to identify TBTF banks. Then we discuss some implementation issues and make some comments to extend the model.

3.1. Data

In our empirical analysis, we identify TBTF banks over the period from 2004 to 2012 on the year by year basis. There are 14 big financial institutions during the pre-financial crisis period from 2004 to 2008 in our sample. The institutions have been selected according to their role in the U.S. financial sector. It has been widely recognized in the literature that most of the systemic risk is concentrated in just a few places. For example, Acharya et al (2010) show that just 5 firms provide over 50% of all the systemic risk in the US financial markets, and 15 firms 92% of the risk. The institutions in our sample can be categorized in groups of banks, insurance companies, investment firms and government sponsored enterprises. They are: Freddie Mac, Fannie Mac, American International Group, Merrill Lynch, Bank of America, Bear Sterns, Citigroup, Goldman Sachs, JP Morgan, Lehman Brother, Metlife, Morgan Stanley, Wachovia and Wells Fargo. For simplicity, we use the corresponding symbols "3FMCC*1000", "3FNMA", "AIG", "BAC2", "BAC", "BSC.1", "C", "GS", "JPM", "LEHMQ", "MET", "MS", "WB" and "WFC" to represent these 14 big financial institutions, respectively. Only 10 financial institutions out of 14 left in the market after financial crisis, so we report TBTF banks from these ten banks over the pro-crisis period 2009-2012. We obtain information on the bank characteristics such as total assets, total equity and number of shares outstanding from Compustat and stock returns data from CRSP.

Similar to Adrian and Brunnermeier (2016), we compute the asset loss exposure for each financial institution $i, i = 1, \dots, N$. For this purpose, we define the following variables:

- L_t^i : the leverage ratio of institution i at time t , the ratio of total asset value over the total equity value;
- M_t^i : the market capitalization of institution i at time t ;
- Y_t^i : the profit and loss of institution i at time t , that is, $Y_t^i \equiv L_t^i \cdot M_t^i - L_{t-1}^i \cdot M_{t-1}^i$;
- X_t^i : the loss exposure of institution i at time t ,

that is, $X_t^i = \max\{-Y_t^i, 0\}$.

· X_t : the aggregate loss exposure at time $t, X_t = \sum_i X_t^i$.

Therefore, in this empirical analysis the loss exposure, X_t , is measured by the negative changes in the market valued assets of bank i . Since the number of banks in our sample changes before and after financial crisis, we conduct our analysis for two sub-periods pre-crisis (2004-2008) and pro-crisis (2009-2012) separably.

3.2. Identify TBTF Banks Empirically

We make use of two types of capital insurance contracts, deductible insurance and cap insurance contracts, respectively. A *deductible* capital insurance has a payoff structure $Z = \max\{X - L, 0\}$, where L is an exogenously given deductible level. The deductible capital insurance is inspired by the classical deductible insurance contract, which is optimal for the insured with a linear premium principle (Arrow, 1965). On the other hand, a *cap* contract with a payoff structure $Z = \min\{X, L\}$ is shown to be optimal for insurer under some assumptions in Raviv (1979), where L represents a capped level for the loss. Aggregate capital insurance is a special deductible contract with zero deductible level or a special cap contract with infinitely large cap level. For a robust purpose, we examine three different levels of L including $L = 0.1E[X]$, $L = 0.2E[X]$ and $L = 0.5E[X]$ in both deductible and cap insurance contract, where $E[X]$ is the expected aggregate loss exposure across all the banks in our sample. In total, six capital insurance contracts are used in implementing the model.

Our identification of TBTF banks are presented in Table 1 - Table 9 on the year by year basis.

Table 1 displays the procedure of identifying TBTF banks in 2004 with these six different capital insurance contracts, in which TBTF banks are reported for both deductible insurance and cap insurance contracts in red and blue colors, respectively. We highlight m^* and $\bar{\tau}_m^*$ for each contract. By using three deductible insurance contracts, only "BAC" is identified as TBTF. However, there are additional three TBTF banks, 3FNMA, AIG and MS, if cap insurance contracts are employed. To a certain degree, it is not a surprise that there are more TBTF banks from a cap insurance market than a deductible insurance market because a cap contract itself is optimal from issuer's perspective (Raviv, 1979), and we observe similar patterns in Table 2- Table 9 as well. Moreover, these four banks, BAC, 3FNMA, AIG and MS, are TBTF banks in each cap insurance market, and they have the highest loss betas in each deductible market. It demonstrates that these four banks indeed have significant systemic risk exposures. Identifying TBTF banks becomes more interesting and serious in 2005 than in 2004, as reported in Table 2. In Table 2, there are five TBTF banks, 3FNMA, AIG, MS, BAC2 and JPM, in each deductible market. Notice that these five banks are also TBTF banks in each cap insurance market, but the cap insurance market reveals more TBTF banks in 2005. When the cap level is given by $L = 0.1E[X]$, there are ten TBTF banks in total; and there are seven TBTF banks when the cap level is higher ($L = 0.2E[X]$ or $L = 0.5E[X]$). In other words,

five new banks are TBTF banks with the first cap contract and two new banks are TBTF by using other cap insurance contracts. As a summary, at least seven banks are deemed to be too big to fail from the regulator's perspective, by implementing the capital insurance market. In these seven banks, 3FMNA, AIG, MS, BAC, BAC2, JPM and 3FMCCC*1000, two banks, BAC and 3FMCC*1000, are not identified as TBTF banks in deductible insurance market but both of them have large loss betas right next to those other five TBTF banks in each deductible insurance market.

Table 3 displays TBTF banks in 2006. This table also demonstrates some important differences between the deductible contract and the cap insurance contract. As illustrated in Table 3, only one "WFC" is identified as TBTF in each deductible insurance market. On the right side of Table 3, however, there are many more TBTF banks; there are *ten*, *nine*, and *eight* TBTF banks in each cap insurance market with different cap level, respectively. In each cap insurance market, WFC has the highest loss beta so it is TBTF naturally (Proposition 1), but there are at least seven other banks which are deemed to be TBTF banks in each cap insurance market. It is interesting to check positions of LEHMQ in Table 3. LEHMQ is TBTF in each cap insurance market. More importantly, LEHMQ has very high loss beta so as large systemic risk exposure: it has the third largest loss beta persistently in each cap insurance market and the second highest loss beta persistently in each deductible market. The latter point is worth mentioning because LEHMQ is not identified as TBTF just because another bank's loss beta dominates all other banks' loss betas (as explained in Example 3). The year 2007 is important in many aspects to understand the financial crisis because some critical issues regarding the mortgage-backed securities and CDO market have been emerged in the market. The identification of TBTF banks, reporting in Table 4, is fairly consistent with the substantial systemic risk issue occurred in this year. First of all, comparing with only one TBTF bank in 2006 in each deductible insurance market, there are *ten* TBTF banks in 2007 when we make use of the same deductible contracts. Second, these ten TBTF banks are fairly the same as TBTF banks from the cap insurances perspective. Over the entire pre-crisis period, 2007 is the only one year in which deductible markets and cap insurance markets identify TBTF most consistently.

Owing to several dramatic market events in 2008, we have to be deliberate with regard to the data analysis. Because of well known events happened on Bear Sterns (BSC1), Lehman Brother (LEHMQ), Merrill Lynch (BAC2) and Wachovia (WB), the loss exposures of these four banks are under scrutiny. Moreover, because of significant losses across the financial sector in 2008, some cap insurance contracts might not work well in 2008 anymore. For instance, the variance of Z is almost zero when the cap level is set too low in 2008 such $L = 0.1E[X]$ as. Therefore, the top cap insurance market on the right side in Table 5 should be read with diligence because of some negative loss betas. Still, we find that those TBTF banks in 2007 are either TBTF banks or have high level loss betas in each capital insurance market in 2008. By combining Table 4 and Table 5 together, the TBTF issue is so

significant that should be alarmed seriously for the regulator.

Over the post-crisis period (2009-2012), only ten banks left in the original financial sector. The TBTF banks in 2009 are identified and reported in Table 6. As observed, the TBTF issue is still very serious because there are four banks, "AIG", "WFC", "JPM" and "BAC", are deemed to be TBTF banks in each capital insurance market. This is the second year (the first time is on 2007) when both deductible and cap insurance market identify identical TBTF banks. This list of TBTF banks is intuitively appealing because "AIG" plays a crucial role in its CDS issuance and other three are the largest three commercial banks in U.S.

The TBTF issue has been reduced considerably after 2009 according up to our empirical analysis. As shown in Table 7-9, only GS is identified as TBTF between 2009-2012. This fact might result from our construction of asset loss exposure, because the leverage ratio is of essential in this construction and GS has relatively large leverage ratio. Given its substantially large loss beta comparing with all other banks, only the bank, GS, with the highest loss beta is TBTF (as illustrated in Example 3). From the regulatory perspective, it shows some positive signs on the TBTF issues but they should pay a closer attention to GS to reduce its leverage ratio.

Our empirical results can be summarized as follows.

- Deductible capital insurance markets with different deductible levels identify TBTF banks *consistently* in each year.
- Cap insurance markets with vary cap levels also identify TBTF banks fairly *consistently*.
- In general, TBTF banks in deductible market are very likely TBTF banks in cap insurance market, but not vice versa. When a bank is deemed to be TBTF bank in both deductible and cap insurance market, it should have large systemic risk.
- The regulator should be alerted when both the deductible and the cap market identify a large number of TBTF banks consistently (say in 2007 and 2009).
- When one bank has significantly large loss beta comparing with all other banks, only this bank is TBTF according to our presented methodology. In this case, other banks with large loss betas should be analyzed in diligent as well.
- The regulator should conduct the TBTF analysis by using several different capital contracts. The regulator should also be careful to construct loss exposures to analyze the systemic risk.
- The TBTF issues has been considerably reduced in the pro-crisis time period.

3.3. Implementation and Comments

In this section, we show how the previous discussions can be modified or extended in a more general setting. We first discuss different specifications of the bank loss exposure. Next, we explore how to address the background risk. We also incorporate a richer indemnity structure of the capital insurance as well as the general specification of the load factor into the setting.

3.3.1. Loss Exposure

Essential to our methodology is the loss exposure of each bank as input to identify TBTF banks. The presented approach takes each bank's loss exposure as given exogenously without imposing specific assumptions on its distribution, so we are able to apply other systemic risk measures or the regulatory proposals to construct the loss exposure with potential systemic risk component. For instance, incorporating the assessment methodology of BCBS to identify TBTF banks, bank i 's loss exposure can be estimated by the negative changes of the total exposure measure used as a proxy of bank size in the Basel III leverage ratio. This will allow us to capture two key measures - correlations and sizes - of systemic importance.¹³ Specifically, define E_t^i as the total exposure measure of bank i at time t . Then the loss exposure of institution i at time t is defined as $X_t^i \equiv \max\{-E_t^i, 0\}$. As another example, we look at the liquidity because of its importance to the proper functioning of financial market. One of the Basel Committee's key reforms to develop a more resilient banking sector is to promote the short-term resilience of the liquidity risk profile of banks measured by the Liquidity Coverage Ratio (LCR).¹⁴ To emphasizing the importance of the liquidity losses, bank i 's loss exposure can be chosen to be the negative changes in the unencumbered high-quality liquid assets (HQLA) as defined by BCBS. In doing so, we are able to adequately incorporate recent changes on banking supervision into the equilibrium framework of capital insurance to address systemic risk and identify TBTF banks.

3.3.2. Background Risk

Since the loss exposure construction is related to its systemic risk exposure, the background risk can not be ignored. For instance, when the mortgage-based securities risk is a big concern as in 2007-2008, we can choose X to be the loss exposure concentrated on mortgage-based risk only. In this way, the initial wealth with other possible risk exposures is not deterministic anymore. Assume the time period starts from time t and all loss exposures of banks are realized at the next time period $t+1$. Let F_t denote the information set at time t which is observed by all banks and regulator. The wealth of bank i at time t is $W_{i,t}$. Due to the background risk, $W_{i,t}$ could be correlated with the loss exposure $X_{i,t+1}$. Let $X_{t+1} = \sum_{i=1}^N X_{i,t+1}$ denote the aggregate loss exposure in the time period $[t, t+1]$, and the capital insurance contract proposed in this time period is a multiple of $Z_{t+1} = I(X_{t+1})$.

First of all, the bank i 's terminal wealth at time $t+1$ is:

$$W_{i,t+1} = W_{i,t} - X_{i,t+1} + a_{i,t} Z_{t+1} - (1 + \rho_t) E_t[a_{i,t} Z_{t+1}],$$

where $E_t[\cdot]$ denotes the conditional expectation operator with respect to the information set F_t and $a_{i,t}$ is the optimal coinsurance coefficient for bank i . Secondly, let $Cov_t(\cdot)$ denote the conditional covariance with respect to the information set F_t . By using of the method in Section 2, the optimal coinsurance parameter at time t for bank i is:

$$a_{i,t}(\rho_t) = \max\left\{\frac{Cov_t(X_{i,t} - W_{i,t}, Z_{t+1}) - \rho_t E_t[Z_{t+1}] \gamma}{Var_t(Z_{t+1})}, 0\right\} \quad (11)$$

By comparing equation (2) with equation (11), it suffices to replace the loss exposure in equation (2) by the difference between the loss exposure and the initial wealth at time t . Lastly, the regulator determines the best load factor, ρ_t , at time t , by solving the conditional-based optimization problem:

$$\max_{\{\rho_t > 0\}} \sum_{i=1}^N \rho_t \max\{Cov_t(X_{i,t} - W_{i,t}, Z_{t+1}) - \rho_t E_t[Z_{t+1}] \gamma, 0\} \quad (12)$$

The problem (12) can be solved similarly at time t , given the information set F_t .

3.3.3. Payoff Structure

While we develop the theory for a class of capital insurance contract, $I_i(X, X_i) = a_i I(X)$, for some function forms of $I(\cdot)$, the payoff structure can be quite general. $I_i(X, X_i)$ can be designed in a way that both the aggregate loss X and the individual loss exposure X_i are involved for bank i , or $I_i(X, X_i)$ even depends on the entire set of loss exposures, $\{X_1, \dots, X_N\}$. For instance, $I_i(X, X_i) = a_i (X - X_i)$, is a contract proposed in Kashyap et al (2008). In general, the indemnity can be chosen as:

$$I_i(X, X_i) = a_i I(b_1 X_1 + \dots + b_N X_N), \quad (13)$$

where the parameters b_1, \dots, b_N capture some firm-specific features of the banks. It is worth mentioning that the methodology developed in Section 2 is different from the classical insurance literature even for a classical coinsurance contract, $I_i(X, X_i) = a_i X_i$. In classical insurance literature, the insureds' loss exposures are assumed to be independent from each other, so the law of large number is applied. Panttser and Tian (2013) develops an equilibrium analysis following the same methodology in Section 2 for classical coinsurance contracts at the presence of dependent structure among loss exposures.

3.3.4. Load Factor

Finally, we consider the load factor in the form of $\rho_t = \rho(\theta, X)$ to incorporate the firm-specific information such as size, credit risk, liquidity, and its complexity, where θ is a set of parameters and $\rho(\theta, X)$ is used to compute the insurance premium for bank i . The equilibrium analysis can be developed similarly. For instance, for the capital insurance contract, $I_i(X, X_i) = a_i Z$, bank's i optimization problem is still the same as in equation (1) and the optimal coinsurance coefficient is given by:

¹³ The consultative version of Basel III's leverage ratio framework and disclosure requirements published in June 2013 defines exposure measure as the sum of the book values of the following items and provides a specific treatment of each categories : (a) on-balance sheet exposures, (b) derivative exposures, (c) securities financing transaction exposures, and (d) other off-balance exposures.

¹⁴ The liquidity ratio is the ratio of value of the unencumbered high-quality liquid assets (HQLA) that can be converted easily and immediately in private markets into cash to meet their liquidity needs for a 30 calendar day liquidity stress scenario and total net cash outflows over the next 30 calendar days.

$$a_i(\theta, \rho(\theta, X_i)) = \max \left\{ \frac{\text{Cov}(X_i, Z) - \rho(\theta, X_i)E[Z] \gamma}{\text{Var}(Z)}, 0 \right\}. \quad (14)$$

Therefore, the regulator's optimization problem is

$$\max_{\{\theta, \rho(\theta, X_i) > 0\}} \sum_{i=1}^N \rho(\theta, X_i) \max \{ \text{Cov}(X_i, Z) - \rho(\theta, X_i)E[Z] \gamma, 0 \}. \quad (15)$$

The equilibrium is solved similarly to the optimization problem described in equation (4).

4. CONCLUSIONS

This paper proposes a new methodology of studying systemic risk from an insurance perspective. By developing an equilibrium model of the capital insurance, we show that this capital insurance proposal is promising to examine some systemic risk issues because of the following results. (1) The insurer (say, a regulator) is better off to issue the capital insurance and the systemic risk on the market level is reduced. (2) Banks are better off to increase their expected utilities and their systemic risk components are reduced ex post. (3) This capital insurance program enables the regulator to identify which banks are deemed to be TBTF irrespective of absence of moral hazard or not. (4) The TBTF issues can be mainly captured by a high level of loss beta, a new systemic risk measure introduced in this equilibrium approach. These theoretical results have some important policy implications and practical appeals. The regulator can design several optimal capital insurance contracts and identifies TBTF banks. The insurance premium received by the regulator can be viewed as a new type of capital - insurance capital, to protect the insured financial institutions in the face of crisis. Finally, the insurance capital can be also used to assess the implied guarantee subsidy for TBTF banks.

REFERENCES

1. Acemoglu, D., Ozdaglar, A., & Tahbaz-Salehi, A. (2015). Systemic Risk and Stability in Financial Networks. *American Economic Review*, 105, 564-608. <https://doi.org/10.1257/aer.20130456>
2. Acharya, V. (2009). A theory of systemic risk and design of prudential bank regulation. *Journal of Financial Stability*, 5, 224 - 255. <https://doi.org/10.1016/j.jfs.2009.02.001>
3. Acharya, V., Pedersen, L., Philippon, Th., & Richardson, M. (2010). A tax on systemic risk. *Post-crisis regulatory reforms to secure financial stability: 2010 KDI international conference*, 3-39.
4. Acharya, V.V., Engle, R. & Richardson, M. (2012). Capital shortfall: A new approach to ranking and regulating systemic risks. *American Economic Review*, 102, 59-64. <https://doi.org/10.1257/aer.102.3.59>
5. Adrian, T., & Brunnermeier, M. (2016). CoVaR. *American Economic Review*, 106, 1705-1741. <https://doi.org/10.1257/aer.20120555>
6. Aiyar, C., Calomiris, C., & Wieladek, T. (2014). A primer on bank capital regulation: theory, empirics, and policy. *IMF Economic Review*, forthcoming.
7. Allen, F., & Gale, D. (1994). *Financial Innovation and Risk Sharing*. MIT Press, Cambridge, MA.
8. Allen, F., & Gale, D. (2000). Financial contagion. *Journal of Political Economy*, 108, 10-33. <https://doi.org/10.1086/262109>
9. Arrow, K. (1971). *Essays in the Theory of Risk Bearing*. Chicago: Markham Publishing Co.
10. Bawa, V., & Lindenberg, E. B. (1977). Capital market equilibrium in a mean-lower partial moment framework. *Journal of Financial Economics*, 5, 189-200. [https://doi.org/10.1016/0304-405X\(77\)90017-4](https://doi.org/10.1016/0304-405X(77)90017-4)
11. Battiston, S., Gatti, D., Gallegati, M., Greenwald, B., & Stiglitz, J. E. (2012). Liaisons dangereuses: increasing connectivity, risk sharing, and systemic risk. *Journal of Economic Dynamics and Control*, 36, 1121-1141. <https://doi.org/10.1016/j.jedc.2012.04.001>
12. BCBS (2013). Global systemically important banks: updated assessment methodology and the higher loss absorbency requirement. Basel Committee on Banking Supervision Consultative Document.
13. Benoit, S., Colletaz, G., Hurlin, C., & Perignon, C. (2012). A theoretical and empirical comparison of systemic risk measures. Working paper, HEC.
14. Billio, M. Getmansky, S., Lo, A. & Pelizzon, L. (2012). Econometric measures of connectedness and systemic risk in the finance and insurance sectors. *Journal of Financial Economics*, 104, 535-559. <https://doi.org/10.1016/j.jfineco.2011.12.10>
15. Biais, D., Flood, M., Lo, A.W., & Valavanis, S. (2012). A survey of systemic risk analytics. Working paper, Office of Financial Research.
16. Borch, K. (1962). Equilibrium in a Reinsurance Market. *Econometrica*, 30, 424-444. <https://doi.org/10.2307/1909887>
17. Brownless, C., & Engle, R. (2011). Volatility, correlation and tails for systemic risk measurement. Tech. rep., SRRN.
18. Dewatripoint, M., & Tirole, J. (1994). *The Prudential Regulation of Banks*. MIT Press.
19. Diamond, D., & Dybvig, P. (1983). Bank runs, deposit insurance and liquidity. *Journal of Political Economy*, 91, 401-419. <https://doi.org/10.1086/261155>
20. Elliott, M., Golub, B., & Jackson, M. O. (2014). Financial networks and contagion. *American Economic Review*, 104, 3112-3153. <https://doi.org/10.1257/aer.104.10.3115>
21. Farhi, E., & Tirole, J. (2012). Collective Moral Hazard, Maturity Mismatch, and Systemic Bailouts. *American Economic Review*, 102, 60-93. <https://doi.org/10.1257/aer.102.1.60>
22. Greens/EFA Report (2014). Implicit Subsidy in the EU Banking Sector. January.
23. Hansen, L. P. (2012). Challenges in identifying and measuring systemic risk. M.K. Brunnermeier and A. Krishnamurthy, Eds. *Risk Topography: Systemic Risk and Macro Modeling*, Chapter 1, University of Chicago Press. <https://doi.org/10.3386/w18505>
24. Hanson, S. G., Kashyap, A. K., & Stein, J. C. (2011). A macroprudential approach to financial regulation. *Journal of Economic Perspectives*, 25, 3-28. <https://doi.org/10.1257/jep.25.1.3>
25. Harris, M., & Raviv, A. (1995). The Role of Games in Security Design. *Review of Financial Studies*, 8, 327-367. <https://doi.org/10.1093/rfs/8.2.327>
26. Hellwig, M. (2009). Systemic risk in the financial sector: An analysis of the subprime-mortgage financial crisis. *De Economist*, 157, 129-207. <https://doi.org/10.1007/s10645-009-9110-0>
27. Hogan, W., & Warren, J. (1974). Toward the Development of an Equilibrium Capital-Market Model Based on Semivariance. *Journal of Financial and Quantitative Analysis*, 9, 1-11. <https://doi.org/10.2307/2329964>
28. Huberman, G., Mayers, D., & Smith, C. (1983).

- Optimal Insurance Policy Indemnity Schedules. *Bell Journal of Economics*, 14, 415-426. <https://doi.org/10.2307/3003643>
29. IMF Report (2014). How Big is the Implicit Subsidy for Banks Considered Too Important To Fail?. April.
30. Kashyap, A., Rajan, R., & Stein, J. (2008). Rethinking capital regulation. *Maintaining Stability in a Changing Financial System*. Federal Reserve Bank of Kansas City, 431-471.
31. Lehar, A. (2005). Measuring systemic risk: A risk management approach. *Journal of Banking and Finance*, 29, 2577 - 2603. <https://doi.org/10.1016/j.jbankfin.2004.09.007>
32. Mace, R. (1999). Full insurance in the presence of aggregate uncertainty. *Journal of Political Economy*, 99, 928-956. <https://doi.org/10.1086/261784>
33. O'Hara, M., & Shaw, W. (1990). Deposit insurance and wealth effects: the value of being "Too Big To Fail". *Journal of Finance*, 45, 1587-1660. <https://doi.org/10.2307/2328751>
34. Panttser, E., & Tian, W. (2013). A welfare analysis of capital insurance. *Risks*, 1, 57-80.
35. Raviv, A. (1979). The design of an optimal insurance policy. *American Economic Review*, 69, 84-96.
36. Rochet, J.C. (2009). Regulating systemic institutions. *Finnish Economic Papers*, 22, 35-46.
37. Simsek, A. (2013). Speculation and risk sharing with new financial assets. *Quarterly Journal of Economics*, 128, 1365-1396. <https://doi.org/10.1093/qje/qjt007>
38. Shin, H. S. (2008). Risk and liquidity in a system context. *Journal of Financial Intermediation*, 17, 315-329. <https://doi.org/10.1016/j.jfi.2008.02.003>
39. Sundaresan, S., & Wang, Z. (2015). On the design of contingent capital with a market trigger. *Journal of Finance*, 70, 881-920. <https://doi.org/10.1111/jofi.12134>
40. Van Oordt, M., & Zhou, C. (2014). Systemic tail risk. *Journal of Financial and Quantitative Analysis*, forthcoming.
41. Zhou, H., Huang, X., & Zhu, H. (2009). A framework for assessing the systemic risk of major financial institutions. *Journal of Banking and Finance*, 33, 2036-2049. <https://doi.org/10.1016/j.jbankfin.2009.05.017>

APPENDIX A

Solution of the Optimization Problem (4)

We present a solution of the optimization problem (4) and the equilibrium in a general situation with different risk aversion parameters γ_i . We re-order the bank sector such that:

$$\frac{Cov(X_1, Z)}{\gamma_1 Var(Z)} \geq \frac{Cov(X_2, Z)}{\gamma_2 Var(Z)} \geq \dots \geq \frac{Cov(X_N, Z)}{\gamma_N Var(Z)}.$$

Moreover, we assume that $Cov(X_i, Z) > 0$ for each bank $i = 1, \dots, N$, because those banks with negative covariance $Cov(X_i, Z)$ have no contribution to (4); thus, those banks with negative or zero covariance $Cov(X_i, Z)$ should be removed from this setting.

Write:

$$f(\rho) = \sum_{i=1}^N \max\{Cov(X_i, Z)\rho - \rho^2 \gamma_i E[Z], 0\}, \text{ and}$$

$$g_m(\rho) = \sum_{i=1}^m \{Cov(X_i, Z)\rho - \rho^2 \gamma_i E[Z]\} \text{ for each } m=1, \dots, N.$$

Let $A_m = \max_{\rho \in I_m} g_m(\rho)$, where

$$I_m = \begin{cases} \left[\frac{Cov(X_{m+1}, Z)}{\gamma_{m+1} E[Z]}, \frac{Cov(X_m, Z)}{\gamma_m E[Z]} \right], m = 1, \dots, N-1, \\ \left[0, \frac{Cov(X_N, Z)}{\gamma_N E[Z]} \right], m = N. \end{cases}$$

We first demonstrate that, noting that $f(0) = 0$,

$$\max_{\rho \geq 0} f(\rho) = \max_{1 \leq m \leq N} A_m. \tag{1}$$

Therefore, the optimization problem (4) is reduced to a sequence of solving A_m , which in turn are solved by a set of standard optimization problem of $g_m(\rho)$.

On one hand, let ρ^* , be the one such that $\max_{\rho \geq 0} f(\rho) = f(\rho^*)$.

If $Cov(X_i, Z)\rho^* \geq (\rho^*)^2 \gamma_i E[Z]$ for all $i = 1, \dots, N$, we set $m=N$ and then $\rho^* \in I_N$. Otherwise, there exists a unique number $m=1, \dots, N-1$ such that:

$$f(\rho^*) = \sum_{i=1}^m (Cov(X_i, Z)\rho^* - (\rho^*)^2 \gamma_i E[Z]),$$

and m is characterized by the following system of inequalities:

$$\left\{ \begin{array}{l} Cov(X_i, Z)\rho^* - (\rho^*)^2 \gamma_i E[Z] > 0, \text{ for } i = 1, \dots, m \\ Cov(X_i, Z)\rho^* - (\rho^*)^2 \gamma_i E[Z] \leq 0, \text{ for } i = m+1, \dots, N. \end{array} \right. \tag{2}$$

That is, $\rho^* \in I_m$. Hence, $f(\rho^*) = g_m(\rho^*) \leq A_m \leq \max_{1 \leq m \leq N} A_m$.

On the other hand, for any $m=1, \dots, N$ it is evidently that:

$$g_m(\rho) \leq \sum_{i=1}^m \max(Cov(X_i, Z)\rho - \rho^2 \gamma_i E[Z], 0) \leq f(\rho)$$

for any $\rho \geq 0$. Hence, $\max_{1 \leq m \leq N} A_m \leq \max_{\rho \geq 0} f(\rho)$. We have thus proved equation (1).

By virtue of (1), the equilibrium of the capital insurance market can be solved by three steps as follows.

First. Compute A_m and $\bar{\rho}_m \equiv \operatorname{argmax}_{\rho \in I_m} g_m(\rho)$ for each $m=1, \dots, N-1$.

Let $\rho_m = \frac{1}{2E[Z]} \frac{\sum_{i=1}^m Cov(X_i, Z)}{\sum_{i=1}^m \gamma_i}$. Then, we can verify that, for $m=1, \dots, N-1$,

$$\bar{\rho}_m = \min \left(\frac{Cov(X_m, Z)}{\gamma_m E[Z]}, \max \left(\frac{Cov(X_{m+1}, Z)}{\gamma_{m+1} E[Z]}, \rho_m \right) \right) \quad (3)$$

$$\text{and} \\ \bar{\rho}_N = \min \left(\frac{Cov(X_N, Z)}{\gamma_N E[Z]}, \rho_N \right). \quad (4)$$

Second. Compute $\max_{1 \leq m \leq N} A_m$ and $m^* = \operatorname{argmax}_{1 \leq m \leq N} A_m$.

It is possible to have multiple m^* and thus multiple equilibrium, because of the non-concavity feature of the objective function $f(\rho)$ for the regulator. As explained in Section 2, it is natural to choose the smallest one among $\{m^*\}$ if there are more than one optimal solutions.

Third. The optimal load factor $\rho^* = \bar{\rho}_{m^*}$.

The bank i is TBTF if and only if $\rho^* < \frac{Cov(X_i, Z)}{\gamma_i E[Z]}$.

For these too big to fail banks, the premium or the insurance capital is $(1 + \rho^*) a_i (\rho^*) E[Z]$.

Algorithm to identifying TBTF banks in terms of loss beta only:

Assume that $\gamma_i = \gamma$ for each $i = 1, \dots, N$.

Then, $A_m = E[Z] \gamma c^2 \max_{\tau \in J_m} h_m(\tau)$, where $c = \frac{Var(Z)}{\gamma E[Z]}$, $J_m = [\beta_{m+1}, \beta_m]$ for $m=1, \dots, N-1$ and $J_N = [0, \beta_N]$. The

algorithm to identify TBTF banks follows easily from the above characterization of the equilibrium in a general situation.

Proof of Proposition 1:

$$\text{Since } g_1(\rho) = Cov(X_1, Z)\rho - \rho^2 \gamma E[Z], \quad g_1 \left(\frac{Cov(X_1, Z)}{\gamma E[Z]} \right) = 0.$$

Therefore, the optimal load factor ρ^* must be strictly smaller than

$$\frac{Cov(X_1, Z)}{\gamma E[Z]} = \max \left\{ \frac{Cov(X_i, Z)}{\gamma E[Z]}, i = 1, \dots, N \right\}.$$

By definition 1, those banks with the highest loss beta are too big to fail.

Proof of Proposition 2:

By exploring equation (1), it suffices to show that $\max_m A_m > 0$.

Actually, when $\frac{Cov(X_1, Z)}{E[Z]} > \frac{Cov(X_2, Z)}{E[Z]}$, we must have $A_1 > 0$ since $g_1 \left(\frac{Cov(X_1, Z)}{\gamma E[Z]} \right) = 0$.

Assuming $\frac{Cov(X_1, Z)}{E[Z]} = \frac{Cov(X_2, Z)}{E[Z]}$, then $A_2 > 0$ unless $\frac{Cov(X_3, Z)}{E[Z]} = \frac{Cov(X_2, Z)}{E[Z]}$.

Continuing the process we know that one of $A_m, m \in \{1, \dots, N-1\}$, must be positive unless each $\frac{Cov(X_i, Z)}{E[Z]}$ is the same positive number. In the last situation, it is easy to verify that $A_N > 0$. Therefore,

$$\max_{\rho > 0} f(\rho) = \max_{\rho \geq 0} f(\rho) > 0.$$

Proof of Proposition 3:

Note that $E[U(\tilde{W}^i)] - E[U(W^i)]$ is $-a_i \rho E[Z] - \frac{1}{2\gamma_i} \{a_i^2 Var(Z) - 2a_i Cov(X_i, Z)\}$.

For TBTF bank i , $a_i(\rho) = \frac{Cov(X_i, Z) - \rho^* \gamma E[Z]}{Var(Z)} > 0$. By straightforward computation, we have

$$E[U(\tilde{W}^i)] - E[U(W^i)] = \frac{1}{2\gamma \text{Var}(Z)} \left(\text{Cov}(X_i, Z) - \rho^* \gamma E[Z] \right)^2 = \frac{\text{Var}(Z)}{2\gamma} \left(\frac{\text{Cov}(X_i, Z)}{\text{Var}(Z)} - \rho^* \gamma \frac{E(Z)}{\text{Var}(Z)} \right)^2 > 0.$$

Moreover, assuming $a_i(\rho) > 0$, the higher the loss beta, the higher the expected utility enhance, $E[U(\tilde{W}^i)] - E[U(W^i)]$.

The proof of Proposition 4 relies on a simple combinational-type result as follows.

Lemma 1 Given N positive numbers such that $b_1 \geq b_2 \geq \dots \geq b_N$ and $\sum_{i=1}^N b_i = 1$. If there exists an integer i such that:

$$\frac{b_i}{\sum_{k=1}^i b_k} > \frac{1}{2i}, \tag{5}$$

then $b_i > \frac{1}{2N}$. Moreover, if ">" is replaced by \geq in (5), then $b_i \geq \frac{1}{2N}$.

Proof: We prove the first part of this lemma while the proof for the second part is the same. We first consider the case when N is divided by i , that is, $N=mi$ for a positive integer m . Notice that $\sum_{k=1}^N b_k = 1$. Since b_k is decreasing for $k=1, \dots, N$, we have

$$1 = \sum_{k=1}^N b_k \leq m \sum_{k=1}^i b_k. \tag{6}$$

Then

$$\sum_{k=1}^i b_k \geq \frac{1}{m}. \tag{7}$$

Hence, by virtue of (5),

$$b_i > \frac{1}{2i} \sum_{k=1}^i b_k \geq \frac{1}{2i} \frac{1}{m} \geq \frac{1}{2N}. \tag{8}$$

The lemma is proved if N can be divided by such an i .

If N can't be divided by i , write $N=mi+t$ for some $0 < t < i$ and $m \geq 1$. We use the decreasing property of b_k again, we obtain:

$$1 = \sum_{k=1}^N b_k = (b_1 + \dots + b_i) + \dots + (b_{(m-1)i+1} + \dots + b_{mi}) + (b_{mi+1} + \dots + b_{mi+t}) \leq m(b_1 + \dots + b_i) + tb_i.$$

Therefore,

$$\sum_{k=1}^i b_k \geq \frac{1 - tb_i}{m}, \tag{9}$$

then by using (5), we obtain

$$b_i > \frac{1}{2i} \frac{1 - tb_i}{m}, \tag{10}$$

which yields (since $N = mi + t$)

$$b_i > \frac{1}{2mi + t} > \frac{1}{2N}. \tag{11}$$

This lemma is proved.

Proof of Proposition 4:

By using the solution of Problem (4), there are two possibilities for the optimal load factor ρ^* .

Case 1. $\rho^* = \rho_m$ for some m and $\rho_m \leq \frac{Cov(X_m, X)}{\gamma E[X]}$.

In this case, $\rho_m = \frac{Var(X)}{\gamma E[X]} \frac{\sum_{i=1}^m \beta_i}{2m}$ and $\frac{Cov(X_m, X)}{\gamma E[X]} = \frac{Var(X)}{\gamma E[X]} \beta_m$. Therefore, $\beta_m \geq \frac{\sum_{i=1}^m \beta_i}{2m}$. By using Lemma 1, we have $\beta_m \geq \frac{1}{2N}$.

Case 2. $\rho^* = \frac{Cov(X_m, X)}{\gamma E[X]}$ for some $m \geq 2$.

In this case, by using the solution of the equilibrium, we have $\frac{Cov(X_m, X)}{\gamma E[X]} \geq \rho_{m-1}$. Then we have

$$\beta_m \geq \frac{\beta_1 + \dots + \beta_{m-1}}{2(m-1)}$$

which implies that

$$\beta_m > \frac{\beta_1 + \dots + \beta_{m-1}}{2m-1}$$

The last inequality in turn is equivalent to

$$\beta_m > \frac{\beta_1 + \dots + \beta_m}{2m}$$

By using Lemma 1 again, $\beta_m > \frac{1}{2N}$.

Proof of Proposition 5:

Notice that after implementing the capital insurance, the loss exposure is $\tilde{X}_i = -X_i + a_i X - (1 + \rho^*) a_i E[X]$ where $a_i = a_i(\rho^*)$ is the optimal coinsurance coefficient. Thus, the aggregate loss exposure becomes $\tilde{X} = -X + \sum_{i=1}^N a_i X - (1 + \rho^*) \sum_{i=1}^N a_i E[X]$, and the systemic risk $Var(\tilde{X}) = (1-a)^2 Var(X)$, where $a = \sum_{i=1}^N a_i$. To prove that the total systemic risk is reduced, that is, $Var(\tilde{X}) < Var(X)$, it suffices to show that $0 < a < 1$. First, $a > 0$ because of existence of too big to fail by Proposition 1. Second, by using the definition of a_i and the fact that $\rho^* > 0$ in (1), we have

$$a = \sum_{i=1}^m \left(\frac{Cov(X_i, X) - \rho^* \gamma E[X]}{Var(X)} \right) = \sum_{i=1}^m \beta_i - \rho^* \gamma m \frac{E[X]}{Var(X)} < \sum_{i=1}^m \beta_i$$

where those banks $i=1, \dots, m$ are too big to fail banks. The positive correlated assumption yields that $\sum_{i=1}^m \beta_i \leq \sum_{i=1}^N \beta_i = 1$.

The proof of Proposition 6 depends on the following Sherman-Morrison formula in linear algebra.

Lemma 2 Suppose A is an invertible $S \times S$ matrix and u, v are $s \times 1$ vectors. Suppose further that $1 + v^T A^{-1} u \neq 0$. Then the matrix $A + uv^T$ is invertible and

$$(A + uv^T)^{-1} = A^{-1} - \frac{A^{-1} uv^T A^{-1}}{1 + v^T A^{-1} u}. \tag{12}$$

Proof of Proposition 6:

For each $i=1, \dots, N$, we have

$$Cov(X_i, X) - \rho E[X] = a_i(\rho) Var(X). \tag{13}$$

Let

$$\hat{a}_i(\rho) = \frac{Cov(\hat{X}_i, \hat{X}) - \rho E[\hat{X}]}{Var(\hat{X})}. \tag{14}$$

By assumption, it is easy to see $Cov(\hat{X}_i, \hat{X}) = Cov(X_i, X) + \sigma_i^2$ and $E[X] = E[\hat{X}]$.

Replacing $Cov(X_i, X)$ by $Cov(\hat{X}_i, \hat{X}) - \sigma_i^2$ in equation (13) and using equation (14), we obtain $a_i(\rho)Var(X) = Cov(X_i, X) - \rho E[X] = Cov(\hat{X}_i, \hat{X}) - \rho E[\hat{X}] - \sigma_i^2 = \hat{a}_i(\rho)Var(\hat{X}) - \sigma_i^2$. Again, by assumption, $Var(\hat{X}) = Var(X) + \sum_{i=1}^N \sigma_i^2$. Then, for $i = 1, \dots, N$ and let $\sigma^2 = \sum_{i=1}^N \sigma_i^2$, we have

$$a_i(\rho)(Var(\hat{X}) - \sigma^2) = \hat{a}_i(\rho)Var(\hat{X}) - \sigma_i^2. \tag{15}$$

Equivalently,

$$\sigma_i^2 - a_i(\rho)\sigma^2 = (\hat{a}_i(\rho) - a_i(\rho))Var(\hat{X}). \tag{16}$$

The coefficient matrix of the variance vector, $(\sigma_1^2, \dots, \sigma_N^2)^T$, in the last equation is

$$\begin{bmatrix} 1 - a_1(\rho) & -a_1(\rho) & \dots & -a_1(\rho) \\ -a_2(\rho) & 1 - a_2(\rho) & \dots & -a_2(\rho) \\ \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \dots & \cdot \\ -a_N(\rho) & -a_N(\rho) & \dots & 1 - a_N(\rho) \end{bmatrix}$$

which is written as $I + uv^T$, where I is an identity matrix, $u = (-a_1(\rho), \dots, -a_N(\rho))^T$ and $v = (1, 1, \dots, 1)^T$. Furthermore,

$$\sum_{i=1}^N a_i(\rho) = 1 - \rho N \frac{E[X]}{Var(X)} < 1,$$

we have $1 + v^T I^{-1} u = 1 - \sum_{i=1}^N a_i(\rho) > 0$. Then the Sherman-Morrison formula (Lemma 2) ensures that the coefficient matrix $I + uv^T$ is invertible. Therefore, the noises' variance vector, $(\sigma_1^2, \dots, \sigma_N^2)^T$, is *uniquely* determined by the set $\{a_i(\rho), \hat{X}_i; i = 1, \dots, N\}$. The proof is completed.

Proof of Proposition 7:

By assumption,

$$Cov(\hat{X}_i, \hat{X}) = Cov(X_i + \varepsilon_i, X + \sum_{i=1}^N \varepsilon_i) = Cov(X_i, X) + \sigma_i^2, \tag{*}$$

$$Cov(X_i, \hat{X} - \varepsilon_i) = Cov(X_i, X + \sum_{j \neq i} \varepsilon_j) = Cov(X_i, X) \text{ Then:}$$

$$Cov(X_i, \hat{X} - \varepsilon_i) = Cov(\hat{X}_i, \hat{X}) - \sigma_i^2. \tag{17}$$

Moreover, $Var(\hat{X} - \varepsilon_i) = Var(X) + \sum_{j \neq i} \sigma_j^2 = Var(\hat{X}) - \sigma_i^2$. Then, by the definition of $\bar{a}_i(\rho)$, we obtain

$$Cov(\hat{X}_i, \hat{X}) - \sigma_i^2 - \rho E[\hat{X} - \varepsilon_i] = \bar{a}_i(\rho)Var(\hat{X} - \varepsilon_i). \tag{18}$$

Therefore,

$$\hat{a}_i(\rho)Var(\hat{X}) - \sigma_i^2 = \bar{a}_i(\rho)\{Var(\hat{X}) - \sigma_i^2\}, \tag{19}$$

in which we make use of equation (14). Hence, we have

$$\sigma_i^2 - \bar{a}_i(\rho)\sigma_i^2 = \{\hat{a}_i(\rho) - \bar{a}_i(\rho)\}Var(\hat{X}). \tag{20}$$

To determine σ_i^2 uniquely, it thus suffices to show that $\bar{a}_i(\rho) < 1$ under assumption on correlated risk environment. In fact, by definition of $\bar{a}_i(\rho)$ and $E[X] > 0$, we have $\bar{a}_i(\rho)Var(\hat{X} - \varepsilon_i) < Cov(X_i, \hat{X} - \varepsilon_i)$. Notice that $Cov(X_i, X_j) \geq 0$ in a correlated risk environment, then $Cov(X_i, X) \leq Var(X)$ for each $i = 1, \dots, N$. Therefore, $Cov(X_i, \hat{X} - \varepsilon_i) = Cov(X_i, X) - \sigma_i^2 \leq Var(X) - \sigma_i^2 = Var(\hat{X} - \varepsilon_i)$. Therefore, we have proved that

$$0 < \bar{a}_i(\rho) < 1$$

Details of Example 3:

We claim that when τ is small enough such that

$$\tau^{m+1} \leq \frac{1}{1 + 2(1 - \tau)(m + 1)}, m = 0, 1, \dots, N - 1 \tag{21}$$

and

$$\tau^m \leq \frac{\sqrt{m+1} - \sqrt{m}}{\sqrt{m+1} - \tau\sqrt{m}}, m = 1, \dots, N - 1, \tag{22}$$

then only the first bank is too big to fail. In fact, by formula (21), $\tau^{m+1} \leq \frac{1 + \tau + \dots + \tau^m}{2(m+1)}$.

Hence, $\rho_m = \operatorname{argmax}_{\rho \in \mathbb{R}_m} g_m(\rho)$. Moreover, $g_m(\rho_m) = \frac{(1 + \tau + \dots + \tau^{m-1})^2}{4mc}$ for a constant c which independent of m . The condition (22) ensures that $g_m(\rho_m)$ is increasing with respect to m . Therefore, by (1), $\max_{\rho \geq 0} f(\rho) = g_1(\rho_1)$, and the optimal load factor is $\rho^* = \rho_1 = \frac{a}{2E[Z]}$.

Table 1. TBTF Banks in 2004

This table displays a bank sector with 14 financial institutions and identifies “too big to fail” banks in year 2004 following the capital insurance approach explained in Appendix A. The analysis is performed for two types of capital insurance contracts and for three different deductible levels. Two types of contracts are as follows.

DEDUCTIBLE INSURANCE is the deductible capital insurance contract, in which the indemnity of the contract is given by $Z=\max(X-L,0)$.

CAP INSURANCE is the cap capital insurance contract, in which $Z=\min(X,L)$. We utilize the following three deductible levels, L , defined as the percentage of the expected aggregate loss in the banking sector:

$$L = 0.1E[X], L = 0.2E[X] \text{ and } L = 0.5E[X]. \beta_m = \frac{Cov(X_i,Z)}{Var(Z)}.$$

$$\tau_m = \frac{1}{2m} \sum_{i=1}^m \beta_i \text{ for } i=1,\dots,N. \bar{\tau}_m = \min\{\beta_m, \max(\beta_{m+1}, \tau_m)\} \text{ for } m=1,\dots,N-1 \text{ and } \bar{\tau}_N = \tau_N.$$

The bank i is too big to fail if and only if $\beta_i > \bar{\tau}_m^*$, for each $i = 1, \dots, N$. Thus, the banks marked in red (blue) are too big to fail if the indemnity of the contract follows deductible insurance (cap insurance).

i	Bank Name	β_m	τ_m	$\bar{\tau}_m$	Bank Name	β_m	τ_m	$\bar{\tau}_m$	
DEDUCTIBLE				CAP					
$L = 0.1E[X]$				$m^* = 1$	$L = 0.1E[X]$				$m^* = 4$
1	BAC	0.7447	0.3723	0.3723	3FNMA	2.4512	1.2256	1.9021	
2	3FNMA	0.0746	0.2048	0.0746	BAC	1.9021	1.0883	1.0883	
3	AIG	0.0524	0.1453	0.0524	AIG	1.0112	0.8941	0.9090	
4	MS	0.0431	0.1144	0.0431	MS	0.9090	0.7842	0.7842	
5	JPM	0.0223	0.0937	0.0223	JPM	0.5644	0.6838	0.5644	
6	BAC2	0.0156	0.0794	0.0156	BAC2	0.4495	0.6073	0.4495	
7	3FMCC*1000	0.0106	0.0688	0.0106	3FMCC*1000	0.2942	0.5415	0.2942	
8	WFC	0.0074	0.0607	0.0074	WFC	0.2523	0.4896	0.2523	
9	WB	0.0058	0.0543	0.0058	GS	0.2313	0.4481	0.2313	
10	MET	0.0048	0.0491	0.0048	LEHMQ	0.1893	0.4127	0.1893	
11	LEHMQ	0.0039	0.0448	0.0039	WB	0.1854	0.3836	0.1854	
12	C	0.0034	0.0412	0.0034	MET	0.1481	0.3578	0.1481	
13	BSC.1	0.0032	0.0382	0.0032	BSC.1	0.1055	0.3344	0.1055	
14	GS	-0.0024	0.0353	0.0353	C	0.0883	0.3136	0.0883	
$L = 0.2E[X]$				$m^* = 1$	$L = 0.2E[X]$				$m^* = 4$
1	BAC	0.7665	0.3832	0.3832	3FNMA	1.3133	0.6567	1.0647	
2	3FNMA	0.0718	0.2096	0.0718	BAC	1.0647	0.5945	0.5945	
3	AIG	0.0518	0.1483	0.0518	AIG	0.5642	0.4904	0.5094	
4	MS	0.0425	0.1166	0.0425	MS	0.5094	0.4315	0.4315	
5	JPM	0.0218	0.0954	0.0218	JPM	0.3131	0.3765	0.3131	
6	BAC2	0.0151	0.0808	0.0151	BAC2	0.2478	0.3344	0.2478	
7	3FMCC*1000	0.0104	0.0700	0.0104	3FMCC*1000	0.1577	0.2979	0.1577	
8	WFC	0.0071	0.0617	0.0071	WFC	0.1407	0.2694	0.1407	
9	WB	0.0056	0.0551	0.0056	WB	0.1025	0.2452	0.1025	
10	MET	0.0046	0.0499	0.0046	MET	0.0829	0.2248	0.0829	
11	LEHMQ	0.0038	0.0455	0.0038	LEHMQ	0.0769	0.2079	0.0769	
12	C	0.0033	0.0418	0.0033	GS	0.0746	0.1937	0.0746	
13	BSC.1	0.0032	0.0387	0.0032	BSC.1	0.0502	0.1807	0.0502	
14	GS	-0.0027	0.0359	0.0359	C	0.0471	0.1695	0.0471	
$L = 0.5E[X]$				$m^* = 1$	$L = 0.5E[X]$				$m^* = 4$
1	BAC	0.8662	0.4331	0.4331	3FNMA	0.6057	0.3028	0.6025	
2	3FNMA	0.0569	0.2308	0.0569	BAC	0.6025	0.3020	0.3020	
3	AIG	0.0469	0.1617	0.0469	AIG	0.2946	0.2505	0.2703	
4	MS	0.0370	0.1259	0.0370	MS	0.2703	0.2216	0.2216	
5	JPM	0.0186	0.1026	0.0186	JPM	0.1517	0.1925	0.1517	
6	BAC2	0.0114	0.0864	0.0114	BAC2	0.1326	0.1714	0.1326	
7	3FMCC*1000	0.0083	0.0747	0.0083	3FMCC*1000	0.0823	0.1528	0.0823	
8	WFC	0.0049	0.0656	0.0049	WFC	0.0726	0.1383	0.0726	
9	WB	0.0047	0.0586	0.0047	WB	0.0434	0.1253	0.0434	
10	MET	0.0039	0.0529	0.0039	MET	0.0356	0.1146	0.0356	
11	BSC.1	0.0031	0.0483	0.0031	LEHMQ	0.0355	0.1058	0.0355	
12	C	0.0030	0.0444	0.0030	C	0.0206	0.0978	0.0206	
13	LEHMQ	0.0028	0.0411	0.0028	BSC.1	0.0167	0.0909	0.0167	
14	GS	-0.0030	0.0380	0.0380	GS	0.0071	0.0847	0.0071	

Table 2. TBTF Banks in 2005

This table displays a bank sector with 14 financial institutions and identifies “too big to fail” banks in year 2005 following the capital insurance approach explained in Appendix A. The analysis is performed for two types of capital insurance contracts and for three different deductible levels. Two types of contracts are as follows.

DEDUCTIBLE INSURANCE is the deductible capital insurance contract, in which the indemnity of the contract is given by $Z=\max(X-L,0)$.

CAP INSURANCE is the cap capital insurance contract, in which $Z=\min(X,L)$. We utilize the following three deductible levels, L , defined as the percentage of the expected aggregate loss in the banking sector:

$$L = 0.1E[X], L = 0.2E[X] \text{ and } L = 0.5E[X]. \beta_m = \frac{Cov(X_i,Z)}{Var(Z)}$$

$$\tau_m = \frac{1}{2m} \sum_{i=1}^m \beta_i \text{ for } i=1,\dots,N. \bar{\tau}_m = \min\{\beta_m, \max(\beta_{m+1}, \tau_m)\} \text{ for } m=1,\dots,N-1 \text{ and } \bar{\tau}_N = \tau_N.$$

The bank i is too big to fail if and only if $\beta_i > \bar{\tau}_m^*$, for each $i = 1, \dots, N$. Thus, the banks marked in red (blue) are too big to fail if the indemnity of the contract follows deductible insurance (cap insurance).

i	Bank Name	β_m	τ_m	$\bar{\tau}_m$	Bank Name	β_m	τ_m	$\bar{\tau}_m$	
DEDUCTIBLE				CAP					
$L = 0.1E[X]$				$m^* = 5$		$L = 0.1E[X]$			
$m^* = 9$									
1	3FNMA	0.2205	0.1103	0.1745	3FNMA	0.2617	0.1308	0.1513	
2	AIG	0.1745	0.0987	0.1680	AIG	0.1513	0.1032	0.1277	
3	MS	0.1680	0.0938	0.0943	MS	0.1277	0.0901	0.1179	
4	JPM	0.0943	0.0822	0.0941	BAC	0.1179	0.0823	0.0999	
5	BAC2	0.0941	0.0751	0.0751	3FMCC*1000	0.0999	0.0758	0.0926	
6	3FMCC*1000	0.0655	0.0681	0.0655	JPM	0.0926	0.0709	0.0830	
7	BAC	0.0650	0.0630	0.0630	BAC2	0.0830	0.0667	0.0796	
8	WFC	0.0483	0.0581	0.0483	GS	0.0796	0.0633	0.0636	
9	WB	0.0396	0.0539	0.0396	WB	0.0636	0.0598	0.0598	
10	BSC.1	0.0224	0.0496	0.0224	WFC	0.0562	0.0567	0.0562	
11	C	0.0199	0.0460	0.0199	LEHMQ	0.0337	0.0530	0.0337	
12	MET	0.0163	0.0429	0.0163	MET	0.0285	0.0498	0.0285	
13	GS	0.0160	0.0402	0.0160	BSC.1	0.0278	0.0470	0.0278	
14	LEHMQ	0.0139	0.0378	0.0378	C	0.0223	0.0445	0.0223	
$L = 0.2E[X]$				$m^* = 5$		$L = 0.2E[X]$			
$m^* = 7$									
1	3FNMA	0.2181	0.1091	0.1753	3FNMA	0.7121	0.3560	0.4214	
2	AIG	0.1753	0.0983	0.1705	AIG	0.4214	0.2834	0.3294	
3	MS	0.1705	0.0940	0.0946	MS	0.3294	0.2438	0.3076	
4	BAC2	0.0946	0.0823	0.0940	BAC	0.3076	0.2213	0.2753	
5	JPM	0.0940	0.0752	0.0752	3FMCC*1000	0.2753	0.2046	0.2622	
6	3FMCC*1000	0.0635	0.0680	0.0635	JPM	0.2622	0.1923	0.2289	
7	BAC	0.0625	0.0627	0.0625	BAC2	0.2289	0.1812	0.1812	
8	WFC	0.0479	0.0579	0.0479	GS	0.1747	0.1695	0.1695	
9	WB	0.0383	0.0536	0.0383	WB	0.1631	0.1597	0.1597	
10	BSC.1	0.0224	0.0493	0.0224	WFC	0.1529	0.1514	0.1514	
11	C	0.0200	0.0458	0.0200	LEHMQ	0.0878	0.1416	0.0878	
12	MET	0.0161	0.0426	0.0161	BSC.1	0.0652	0.1325	0.0652	
13	GS	0.0141	0.0399	0.0141	MET	0.0632	0.1248	0.0632	
14	LEHMQ	0.0129	0.0375	0.0375	C	0.0553	0.1178	0.0553	
$L = 0.5E[X]$				$m^* = 5$		$L = 0.5E[X]$			
$m^* = 7$									
1	3FNMA	0.2370	0.1185	0.2237	3FNMA	0.3957	0.1978	0.2006	
2	AIG	0.2237	0.1152	0.2154	AIG	0.2006	0.1491	0.1908	
3	MS	0.2154	0.1127	0.1164	MS	0.1908	0.1312	0.1822	
4	BAC2	0.1164	0.0991	0.1099	BAC	0.1822	0.1212	0.1416	
5	JPM	0.1099	0.0902	0.0902	JPM	0.1416	0.1111	0.1229	
6	3FMCC*1000	0.0695	0.0810	0.0695	3FMCC*1000	0.1229	0.1028	0.1211	
7	WFC	0.0539	0.0733	0.0539	BAC2	0.1211	0.0968	0.0968	
8	BAC	0.0491	0.0672	0.0491	WFC	0.0809	0.0897	0.0809	
9	WB	0.0408	0.0620	0.0408	WB	0.0786	0.0841	0.0786	
10	BSC.1	0.0287	0.0572	0.0287	GS	0.0453	0.0780	0.0453	
11	C	0.0244	0.0531	0.0244	LEHMQ	0.0276	0.0721	0.0276	
12	MET	0.0189	0.0495	0.0189	C	0.0270	0.0673	0.0270	
13	LEHMQ	0.0148	0.0462	0.0148	BSC.1	0.0269	0.0631	0.0269	
14	GS	0.0140	0.0434	0.0434	MET	0.0266	0.0596	0.0266	

Table 3. TBTF Banks in 2006

This table displays a bank sector with 14 financial institutions and identifies “too big to fail” banks in year 2006 following the capital insurance approach explained in Appendix A. The analysis is performed for two types of capital insurance contracts and for three different deductible levels. Two types of contracts are as follows.

DEDUCTIBLE INSURANCE is the deductible capital insurance contract, in which the indemnity of the contract is given by $Z=\max(X-L,0)$.

CAP INSURANCE is the cap capital insurance contract, in which $Z=\min(X,L)$. We utilize the following three deductible levels, L , defined as the percentage of the expected aggregate loss in the banking sector:

$$L = 0.1E[X], L = 0.2E[X] \text{ and } L = 0.5E[X]. \beta_m = \frac{Cov(X_i, Z)}{Var(Z)}.$$

$$\tau_m = \frac{1}{2m} \sum_{i=1}^m \beta_i \text{ for } i=1, \dots, N. \bar{\tau}_m = \min\{\beta_m, \max(\beta_{m+1}, \tau_m)\} \text{ for } m=1, \dots, N-1 \text{ and } \bar{\tau}_N = \tau_N.$$

The bank i is too big to fail if and only if $\beta_i > \bar{\tau}_m^*$, for each $i = 1, \dots, N$. Thus, the banks marked in red (blue) are too big to fail if the indemnity of the contract follows deductible insurance (cap insurance).

i	Bank Name	β_m	τ_m	$\bar{\tau}_m$	Bank Name	β_m	τ_m	$\bar{\tau}_m$	
DEDUCTIBLE				CAP					
$L = 0.1E[X]$				$m^* = 1$	$L = 0.1E[X]$				$m^* = 10$
1	WFC	0.3996	0.1998	0.1998	WFC	1.3649	0.6824	1.0263	
2	LEHMQ	0.1471	0.1367	0.1367	MS	1.0263	0.5978	0.9328	
3	MS	0.1061	0.1088	0.1061	LEHMQ	0.9328	0.5540	0.9138	
4	JPM	0.0746	0.0909	0.0746	GS	0.9138	0.5297	0.8830	
5	BAC2	0.0558	0.0783	0.0558	BAC	0.8830	0.5121	0.7260	
6	AIG	0.0541	0.0698	0.0541	JPM	0.7260	0.4872	0.6530	
7	BAC	0.0475	0.0632	0.0475	BAC2	0.6530	0.4643	0.5625	
8	3FNMA	0.0319	0.0573	0.0319	3FMCC*1000	0.5625	0.4414	0.5510	
9	WB	0.0209	0.0521	0.0209	AIG	0.5510	0.4229	0.4881	
10	GS	0.0188	0.0478	0.0188	3FNMA	0.4881	0.4051	0.4051	
11	3FMCC*1000	0.0134	0.0441	0.0134	WB	0.3837	0.3857	0.3837	
12	BSC.1	0.0119	0.0409	0.0119	BSC.1	0.2240	0.3629	0.2240	
13	C	0.0091	0.0381	0.0091	MET	0.1472	0.3406	0.1472	
14	MET	-0.0017	0.0353	0.0353	C	0.0817	0.3192	0.0817	
$L = 0.2E[X]$				$m^* = 1$	$L = 0.2E[X]$				$m^* = 9$
1	WFC	0.4157	0.2078	0.2078	WFC	0.7695	0.3848	0.5890	
2	LEHMQ	0.1515	0.1418	0.1418	MS	0.5890	0.3396	0.5290	
3	MS	0.1080	0.1125	0.1080	LEHMQ	0.5290	0.3146	0.5032	
4	JPM	0.0760	0.0939	0.0760	BAC	0.5032	0.2988	0.4219	
5	BAC2	0.0564	0.0807	0.0564	GS	0.4219	0.2813	0.4144	
6	AIG	0.0550	0.0719	0.0550	JPM	0.4144	0.2689	0.3714	
7	BAC	0.0468	0.0650	0.0468	BAC2	0.3714	0.2570	0.3094	
8	3FNMA	0.0321	0.0588	0.0321	AIG	0.3094	0.2442	0.2442	
9	WB	0.0207	0.0535	0.0207	3FNMA	0.2420	0.2305	0.2305	
10	GS	0.0174	0.0490	0.0174	WB	0.2190	0.2184	0.2184	
11	3FMCC*1000	0.0131	0.0451	0.0131	3FMCC*1000	0.1948	0.2074	0.1948	
12	BSC.1	0.0120	0.0419	0.0120	BSC.1	0.1045	0.1945	0.1045	
13	C	0.0093	0.0390	0.0093	MET	0.0868	0.1829	0.0868	
14	MET	-0.0024	0.0361	0.0361	C	0.0544	0.1718	0.0544	
$L = 0.5E[X]$				$m^* = 1$	$L = 0.5E[X]$				$m^* = 8$
1	WFC	1.7067	0.8534	0.8534	WFC	0.4484	0.2242	0.3161	
2	LEHMQ	0.5889	0.5739	0.5739	MS	0.3161	0.1911	0.2815	
3	MS	0.3864	0.4470	0.3864	LEHMQ	0.2815	0.1743	0.2273	
4	JPM	0.2702	0.3690	0.2702	JPM	0.2273	0.1592	0.2116	
5	AIG	0.1949	0.3147	0.1949	BAC	0.2116	0.1485	0.2012	
6	BAC2	0.1914	0.2782	0.1914	BAC2	0.2012	0.1405	0.1672	
7	BAC	0.1501	0.2492	0.1501	AIG	0.1672	0.1324	0.1598	
8	3FNMA	0.1126	0.2251	0.1126	GS	0.1598	0.1258	0.1258	
9	WB	0.0586	0.2033	0.0586	WB	0.1148	0.1182	0.1148	
10	3FMCC*1000	0.0387	0.1849	0.0387	3FNMA	0.1078	0.1118	0.1078	
11	BSC.1	0.0373	0.1698	0.0373	3FMCC*1000	0.0744	0.1050	0.0744	
12	GS	0.0355	0.1571	0.0355	BSC.1	0.0547	0.0985	0.0547	
13	C	0.0321	0.1463	0.0321	C	0.0303	0.0921	0.0303	
14	MET	-0.0141	0.1353	0.1353	MET	0.0190	0.0862	0.0190	

Table 4. TBTF Banks in 2007

This table displays a bank sector with 14 financial institutions and identifies “too big to fail” banks in year 2007 following the capital insurance approach explained in Appendix A. The analysis is performed for two types of capital insurance contracts and for three different deductible levels. Two types of contracts are as follows.

DEDUCTIBLE INSURANCE is the deductible capital insurance contract, in which the indemnity of the contract is given by $Z=\max(X-L,0)$.

CAP INSURANCE is the cap capital insurance contract, in which $Z=\min(X,L)$. We utilize the following three deductible levels, L , defined as the percentage of the expected aggregate loss in the banking sector:

$$L = 0.1E[X], L = 0.2E[X] \text{ and } L = 0.5E[X]. \beta_m = \frac{Cov(X_i,Z)}{Var(Z)}.$$

$$\tau_m = \frac{1}{2m} \sum_{i=1}^m \beta_i \text{ for } i=1,\dots,N. \bar{\tau}_m = \min\{\beta_m, \max(\beta_{m+1}, \tau_m)\} \text{ for } m=1,\dots,N-1 \text{ and } \bar{\tau}_N=\tau_N.$$

The bank i is too big to fail if and only if $\beta_i > \bar{\tau}_m^*$, for each $i = 1, \dots, N$. Thus, the banks marked in red (blue) are too big to fail if the indemnity of the contract follows deductible insurance (cap insurance).

i	Bank Name	β_m	τ_m	$\bar{\tau}_m$	Bank Name	β_m	τ_m	$\bar{\tau}_m$
DEDUCTIBLE				CAP				
$L = 0.1E[X]$				$L = 0.1E[X]$				
$m^* = 10$				$m^* = 10$				
1	MS	0.1878	0.0939	0.1323	MS	2.0123	1.0061	1.4468
2	BAC2	0.1323	0.0800	0.1084	GS	1.4468	0.8648	1.1686
3	BAC	0.1084	0.0714	0.0925	3FNMA	1.1686	0.7713	1.1598
4	3FMCC*1000	0.0925	0.0651	0.0834	3FMCC*1000	1.1598	0.7234	1.1376
5	3FNMA	0.0834	0.0604	0.0803	BAC2	1.1376	0.6925	1.1294
6	JPM	0.0803	0.0571	0.0589	BAC	1.1294	0.6712	0.9387
7	AIG	0.0589	0.0531	0.0531	JPM	0.9387	0.6424	0.8657
8	LEHMQ	0.0520	0.0497	0.0505	LEHMQ	0.8657	0.6162	0.7710
9	WB	0.0505	0.0470	0.0504	AIG	0.7710	0.5906	0.5906
10	WFC	0.0504	0.0448	0.0448	WB	0.5895	0.5610	0.5610
11	GS	0.0295	0.0421	0.0295	WFC	0.4807	0.5318	0.4807
12	BSC.1	0.0264	0.0397	0.0264	BSC.1	0.4714	0.5071	0.4714
13	C	0.0227	0.0375	0.0227	MET	0.3233	0.4806	0.3233
14	MET	0.0101	0.0352	0.0352	C	0.2737	0.4560	0.2737
$L = 0.2E[X]$				$L = 0.2E[X]$				
$m^* = 10$				$m^* = 9$				
1	MS	0.1898	0.0949	0.1339	MS	1.2654	0.6327	0.8666
2	BAC2	0.1339	0.0809	0.1096	GS	0.8666	0.5330	0.7153
3	BAC	0.1096	0.0722	0.0934	BAC2	0.7153	0.4745	0.7147
4	3FMCC*1000	0.0934	0.0658	0.0841	3FNMA	0.7147	0.4452	0.7102
5	3FNMA	0.0841	0.0611	0.0811	BAC	0.7102	0.4272	0.6703
6	JPM	0.0811	0.0577	0.0594	3FMCC*1000	0.6703	0.4119	0.5903
7	AIG	0.0594	0.0537	0.0537	JPM	0.5903	0.3952	0.4984
8	LEHMQ	0.0524	0.0502	0.0510	LEHMQ	0.4984	0.3770	0.4832
9	WB	0.0510	0.0475	0.0510	AIG	0.4832	0.3619	0.3619
10	WFC	0.0510	0.0453	0.0453	WB	0.3469	0.3431	0.3431
11	GS	0.0289	0.0425	0.0289	WFC	0.3053	0.3258	0.3053
12	BSC.1	0.0266	0.0400	0.0266	BSC.1	0.2688	0.3098	0.2688
13	C	0.0229	0.0378	0.0229	MET	0.1802	0.2929	0.1802
14	MET	0.0101	0.0355	0.0355	C	0.1604	0.2777	0.1604
$L = 0.5E[X]$				$L = 0.5E[X]$				
$m^* = 10$				$m^* = 10$				
1	MS	0.1995	0.0998	0.1419	MS	0.6470	0.3235	0.3819
2	BAC2	0.1419	0.0854	0.1161	BAC2	0.3819	0.2572	0.3715
3	BAC	0.1161	0.0763	0.0978	GS	0.3715	0.2334	0.3447
4	3FMCC*1000	0.0978	0.0694	0.0878	3FMCC*1000	0.3447	0.2181	0.3343
5	3FNMA	0.0878	0.0643	0.0846	3FNMA	0.3343	0.2079	0.3312
6	JPM	0.0846	0.0606	0.0617	BAC	0.3312	0.2009	0.3108
7	AIG	0.0617	0.0564	0.0564	JPM	0.3108	0.1944	0.2462
8	LEHMQ	0.0544	0.0527	0.0540	AIG	0.2462	0.1855	0.2259
9	WB	0.0540	0.0499	0.0537	LEHMQ	0.2259	0.1774	0.1774
10	WFC	0.0537	0.0476	0.0476	WFC	0.1645	0.1679	0.1645
11	BSC.1	0.0275	0.0445	0.0275	WB	0.1605	0.1599	0.1599
12	GS	0.0262	0.0419	0.0262	BSC.1	0.1198	0.1516	0.1198
13	C	0.0240	0.0396	0.0240	C	0.0816	0.1431	0.0816
14	MET	0.0101	0.0371	0.0371	MET	0.0697	0.1353	0.0697

Table 5. TBTF Banks in 2008

This table displays a bank sector with 14 financial institutions and identifies “too big to fail” banks in year 2008 following the capital insurance approach explained in Appendix A. The analysis is performed for two types of capital insurance contracts and for three different deductible levels. Two types of contracts are as follows.

DEDUCTIBLE INSURANCE is the deductible capital insurance contract, in which the indemnity of the contract is given by $Z=\max(X-L,0)$.

CAP INSURANCE is the cap capital insurance contract, in which $Z=\min(X,L)$. We utilize the following three deductible levels, L , defined as the percentage of the expected aggregate loss in the banking sector:

$$L = 0.1E[X], L = 0.2E[X] \text{ and } L = 0.5E[X]. \beta_m = \frac{Cov(X_i,Z)}{Var(Z)}.$$

$$\tau_m = \frac{1}{2m} \sum_{i=1}^m \beta_i \text{ for } i=1,\dots,N. \bar{\tau}_m = \min\{\beta_m, \max(\beta_{m+1}, \tau_m)\} \text{ for } m=1,\dots,N-1 \text{ and } \bar{\tau}_N = \tau_N.$$

The bank i is too big to fail if and only if $\beta_i > \bar{\tau}_m^*$, for each $i = 1, \dots, N$. Thus, the banks marked in red (blue) are too big to fail if the indemnity of the contract follows deductible insurance (cap insurance).

i	Bank Name	β_m	τ_m	$\bar{\tau}_m$	Bank Name	β_m	τ_m	$\bar{\tau}_m$	
DEDUCTIBLE				CAP					
$L = 0.1E[X]$				$m^* = 6$				$L = 0.1E[X]$	
$m^* = 6$				$L = 0.1E[X]$				$m^* = 4$	
1	3FNMA	0.2606	0.1303	0.1323	WB	1.4994	0.7497	1.3649	
2	BAC	0.1323	0.0982	0.1192	AIG	1.3649	0.7161	0.7161	
3	BAC2	0.1192	0.0853	0.1185	MS	0.6151	0.5799	0.6151	
4	JPM	0.1185	0.0788	0.0998	BAC	0.6151	0.5118	0.5118	
5	MS	0.0998	0.0730	0.0799	BSC.1	0.4421	0.4537	0.4421	
6	WFC	0.0799	0.0675	0.0675	MET	0.1922	0.3941	0.1922	
7	AIG	0.0624	0.0623	0.0623	JPM	-0.1538	0.3268	-0.1538	
8	WB	0.0471	0.0575	0.0471	C	-0.3556	0.2637	-0.3556	
9	BSC.1	0.0178	0.0521	0.0178	WFC	-1.1150	0.1725	-1.1150	
10	MET	0.0134	0.0475	0.0134	GS	-1.3456	0.0879	-1.3456	
11	C	0.0132	0.0438	0.0132	3FNMA	-1.5379	0.0100	-1.5379	
12	LEHMQ	0.0105	0.0406	0.0105	LEHMQ	-1.6532	-0.0597	-1.6532	
13	GS	0.0036	0.0376	0.0036	3FMCC*1000	-2.1146	-0.1364	-2.1146	
14	3FMCC*1000	0.0021	0.0350	0.0350	BAC2	-2.3453	-0.2104	-2.3453	
$L = 0.2E[X]$				$m^* = 6$				$L = 0.2E[X]$	
$m^* = 6$				$L = 0.2E[X]$				$m^* = 8$	
1	3FNMA	0.2607	0.1303	0.1323	3FNMA	11.1738	5.5869	10.5320	
2	BAC	0.1323	0.0982	0.1192	JPM	10.5320	5.4265	10.4936	
3	BAC2	0.1192	0.0854	0.1185	BAC	10.4936	5.3666	9.2761	
4	JPM	0.1185	0.0788	0.0998	MS	9.2761	5.1844	8.2246	
5	MS	0.0998	0.0731	0.0799	BAC2	8.2246	4.9700	6.4195	
6	WFC	0.0799	0.0675	0.0675	WFC	6.4195	4.6766	6.2804	
7	AIG	0.0625	0.0624	0.0624	AIG	6.2804	4.4571	5.1676	
8	WB	0.0471	0.0575	0.0471	3FMCC*1000	5.1676	4.2230	4.2230	
9	BSC.1	0.0178	0.0521	0.0178	GS	3.3960	3.9424	3.3960	
10	MET	0.0134	0.0476	0.0134	MET	2.9175	3.6941	2.9175	
11	C	0.0133	0.0438	0.0133	LEHMQ	2.2898	3.4623	2.2898	
12	LEHMQ	0.0105	0.0406	0.0105	WB	1.9683	3.2558	1.9683	
13	GS	0.0036	0.0376	0.0036	C	1.3849	3.0586	1.3849	
14	3FMCC*1000	0.0021	0.0350	0.0350	BSC.1	1.0872	2.8790	1.0872	
$L = 0.5E[X]$				$m^* = 6$				$L = 0.5E[X]$	
$m^* = 6$				$L = 0.5E[X]$				$m^* = 7$	
1	3FNMA	0.2650	0.1325	0.1338	JPM	1.2355	0.6177	1.2006	
2	BAC	0.1338	0.0997	0.1208	3FNMA	1.2006	0.6090	1.1744	
3	BAC2	0.1208	0.0866	0.1196	BAC	1.1744	0.6018	1.0702	
4	JPM	0.1196	0.0799	0.1007	MS	1.0702	0.5851	0.9036	
5	MS	0.1007	0.0740	0.0808	BAC2	0.9036	0.5584	0.7364	
6	WFC	0.0808	0.0684	0.0684	WFC	0.7364	0.5267	0.7304	
7	AIG	0.0629	0.0631	0.0629	AIG	0.7304	0.5036	0.5036	
8	WB	0.0479	0.0582	0.0479	3FMCC*1000	0.4562	0.4692	0.4562	
9	BSC.1	0.0181	0.0527	0.0181	LEHMQ	0.4057	0.4396	0.4057	
10	C	0.0134	0.0481	0.0134	GS	0.3909	0.4152	0.3909	
11	MET	0.0134	0.0444	0.0134	MET	0.2378	0.3883	0.2378	
12	LEHMQ	0.0102	0.0411	0.0102	WB	0.2309	0.3655	0.2309	
13	GS	0.0032	0.0381	0.0032	C	0.1368	0.3427	0.1368	
14	3FMCC*1000	0.0015	0.0354	0.0354	BSC.1	0.1275	0.3227	0.1275	

Table 6. TBTF Banks in 2009

This table displays a bank sector with 10 financial institutions and identifies “too big to fail” banks in year 2009 following the capital insurance approach explained in Appendix A. The analysis is performed for two types of capital insurance contracts and for three different deductible levels. Two types of contracts are as follows.

DEDUCTIBLE INSURANCE is the deductible capital insurance contract, in which the indemnity of the contract is given by $Z=\max(X-L,0)$.

CAP INSURANCE is the cap capital insurance contract, in which $Z=\min(X,L)$. We utilize the following three deductible levels, L , defined as the percentage of the expected aggregate loss in the banking sector:

$$L = 0.1E[X], L = 0.2E[X] \text{ and } L = 0.5E[X]. \beta_m = \frac{Cov(X_i, Z)}{Var(Z)}.$$

$$\tau_m = \frac{1}{2m} \sum_{i=1}^m \beta_i \text{ for } i=1, \dots, N. \bar{\tau}_m = \min\{\beta_m, \max(\beta_{m+1}, \tau_m)\} \text{ for } m=1, \dots, N-1 \text{ and } \bar{\tau}_N = \tau_N.$$

The bank i is too big to fail if and only if $\beta_i > \bar{\tau}_m^*$, for each $i = 1, \dots, N$. Thus, the banks marked in red (blue) are too big to fail if the indemnity of the contract follows deductible insurance (cap insurance).

i	Bank Name	β_m	τ_m	$\bar{\tau}_m$	Bank Name	β_m	τ_m	$\bar{\tau}_m$	
DEDUCTIBLE				CAP					
$L = 0.1E[X]$				$m^* = 4$				$L = 0.1E[X]$	
$m^* = 4$				$L = 0.1E[X]$				$m^* = 4$	
1	GS	0.3287	0.1643	0.2133	GS	1.2916	0.6458	1.0060	
2	C	0.2133	0.1355	0.1987	C	1.0060	0.5744	0.9089	
3	JPM	0.1987	0.1235	0.1468	JPM	0.9089	0.5344	0.7893	
4	MS	0.1468	0.1109	0.1109	3FMCC*1000	0.7893	0.4995	0.4995	
5	MET	0.0534	0.0941	0.0534	MS	0.3723	0.4368	0.3723	
6	WFC	0.0271	0.0807	0.0271	MET	0.2945	0.3886	0.2945	
7	AIG	0.0083	0.0697	0.0083	WFC	0.2046	0.3477	0.2046	
8	BAC	0.0073	0.0615	0.0073	AIG	0.1619	0.3143	0.1619	
9	3FNMA	0.0060	0.0550	0.0060	BAC	0.0938	0.2846	0.0938	
10	3FMCC*1000	0.0015	0.0495	0.0495	3FNMA	0.0324	0.2578	0.2578	
$L = 0.2E[X]$				$m^* = 4$				$L = 0.2E[X]$	
$m^* = 4$				$L = 0.2E[X]$				$m^* = 4$	
1	GS	0.3455	0.1728	0.2225	GS	1.2916	0.6458	1.0060	
2	C	0.2225	0.1420	0.2076	C	1.0060	0.5744	0.9089	
3	JPM	0.2076	0.1293	0.1522	JPM	0.9089	0.5344	0.7893	
4	MS	0.1522	0.1160	0.1160	3FMCC*1000	0.7893	0.4995	0.4995	
5	MET	0.0546	0.0983	0.0546	MS	0.3723	0.4368	0.3723	
6	WFC	0.0276	0.0842	0.0276	MET	0.2945	0.3886	0.2945	
7	AIG	0.0071	0.0727	0.0071	WFC	0.2046	0.3477	0.2046	
8	BAC	0.0064	0.0640	0.0064	AIG	0.1619	0.3143	0.1619	
9	3FNMA	0.0061	0.0572	0.0061	BAC	0.0938	0.2846	0.0938	
10	3FMCC*1000	0.0004	0.0515	0.0515	3FNMA	0.0324	0.2578	0.2578	
$L = 0.5E[X]$				$m^* = 4$				$L = 0.5E[X]$	
$m^* = 4$				$L = 0.5E[X]$				$m^* = 4$	
1	GS	0.4261	0.2131	0.2756	GS	0.6497	0.3248	0.4289	
2	C	0.2756	0.1754	0.2568	C	0.4289	0.2696	0.3989	
3	JPM	0.2568	0.1598	0.1809	JPM	0.3989	0.2462	0.3400	
4	MS	0.1809	0.1424	0.1424	MS	0.3400	0.2272	0.2272	
5	MET	0.0678	0.1207	0.0678	MET	0.1156	0.1933	0.1156	
6	WFC	0.0284	0.1030	0.0284	WFC	0.0889	0.1685	0.0889	
7	AIG	0.0096	0.0889	0.0096	3FMCC*1000	0.0882	0.1507	0.0882	
8	BAC	0.0063	0.0782	0.0063	AIG	0.0880	0.1374	0.0880	
9	3FNMA	-0.0025	0.0694	-0.0025	BAC	0.0188	0.1232	0.0188	
10	3FMCC*1000	-0.0143	0.0617	0.0617	3FNMA	0.0156	0.1116	0.1116	

Table 7. TBTF Banks in 2010

This table displays a bank sector with 10 financial institutions and identifies “too big to fail” banks in year 2010 following the capital insurance approach explained in Appendix A. The analysis is performed for two types of capital insurance contracts and for three different deductible levels. Two types of contracts are as follows.

DEDUCTIBLE INSURANCE is the deductible capital insurance contract, in which the indemnity of the contract is given by $Z=\max(X-L,0)$.

CAP INSURANCE is the cap capital insurance contract, in which $Z=\min(X,L)$. We utilize the following three deductible levels, L , defined as the percentage of the expected aggregate loss in the banking sector:

$$L = 0.1E[X], L = 0.2E[X] \text{ and } L = 0.5E[X]. \beta_m = \frac{Cov(X_i, Z)}{Var(Z)}.$$

$$\tau_m = \frac{1}{2m} \sum_{i=1}^m \beta_i \text{ for } i=1, \dots, N. \bar{\tau}_m = \min\{\beta_m, \max(\beta_{m+1}, \tau_m)\} \text{ for } m=1, \dots, N-1 \text{ and } \bar{\tau}_N = \tau_N.$$

The bank i is too big to fail if and only if $\beta_i > \bar{\tau}_m^*$, for each $i = 1, \dots, N$. Thus, the banks marked in red (blue) are too big to fail if the indemnity of the contract follows deductible insurance (cap insurance).

i	Bank Name	β_m	τ_m	$\bar{\tau}_m$	Bank Name	β_m	τ_m	$\bar{\tau}_m$
DEDUCTIBLE				CAP				
$L = 0.1E[X]$				$L = 0.1E[X]$				
$m^* = 1$				$m^* = 1$				
1	GS	0.4945	0.2472	0.2472	GS	5.9744	2.9872	2.9872
2	C	0.1343	0.1572	0.1343	C	0.6010	1.6438	0.6010
3	JPM	0.1240	0.1255	0.1240	JPM	0.5413	1.1861	0.5413
4	MS	0.1103	0.1079	0.1079	3FMCC*1000	0.5280	0.9556	0.5280
5	MET	0.1001	0.0963	0.0963	MS	0.4642	0.8109	0.4642
6	WFC	0.0113	0.0812	0.0113	MET	0.1811	0.6908	0.1811
7	AIG	0.0072	0.0701	0.0072	WFC	0.1583	0.6034	0.1583
8	BAC	0.0059	0.0617	0.0059	AIG	0.1106	0.5349	0.1106
9	3FNMA	0.0021	0.0550	0.0021	BAC	0.1072	0.4814	0.1072
10	3FMCC*1000	-0.0004	0.0495	0.0495	3FNMA	0.0794	0.4373	0.4373
$L = 0.2E[X]$				$L = 0.2E[X]$				
$m^* = 1$				$m^* = 1$				
1	GS	0.4981	0.2490	0.2490	GS	5.9744	2.9872	2.9872
2	C	0.1372	0.1588	0.1372	C	0.6010	1.6438	0.6010
3	JPM	0.1266	0.1270	0.1266	JPM	0.5413	1.1861	0.5413
4	MS	0.1124	0.1093	0.1093	3FMCC*1000	0.5280	0.9556	0.5280
5	MET	0.1019	0.0976	0.0976	MS	0.4642	0.8109	0.4642
6	WFC	0.0113	0.0823	0.0113	MET	0.1811	0.6908	0.1811
7	AIG	0.0073	0.0711	0.0073	WFC	0.1583	0.6034	0.1583
8	BAC	0.0059	0.0625	0.0059	AIG	0.1106	0.5349	0.1106
9	3FNMA	0.0019	0.0557	0.0019	BAC	0.1072	0.4814	0.1072
10	3FMCC*1000	-0.0006	0.0501	0.0501	3FNMA	0.0794	0.4373	0.4373
$L = 0.5E[X]$				$L = 0.5E[X]$				
$m^* = 1$				$m^* = 1$				
1	GS	0.5075	0.2538	0.2538	GS	3.0825	1.5412	1.5412
2	C	0.1483	0.1640	0.1483	C	0.2994	0.8455	0.2994
3	JPM	0.1363	0.1320	0.1320	JPM	0.2637	0.6076	0.2637
4	MS	0.1201	0.1140	0.1140	MS	0.2430	0.4861	0.2430
5	MET	0.1100	0.1022	0.1022	MET	0.2217	0.4110	0.2217
6	WFC	0.0115	0.0861	0.0115	WFC	0.0781	0.3490	0.0781
7	AIG	0.0074	0.0744	0.0074	3FMCC*1000	0.0480	0.3026	0.0480
8	BAC	0.0060	0.0654	0.0060	AIG	0.0465	0.2677	0.0465
9	3FNMA	0.0015	0.0583	0.0015	BAC	0.0379	0.2400	0.0379
10	3FMCC*1000	-0.0008	0.0524	0.0524	3FNMA	0.0342	0.2178	0.2178

Table 8. TBTF Banks in 2011

This table displays a bank sector with 10 financial institutions and identifies “too big to fail” banks in year 2011 following the capital insurance approach explained in Appendix A. The analysis is performed for two types of capital insurance contracts and for three different deductible levels. Two types of contracts are as follows.

DEDUCTIBLE INSURANCE is the deductible capital insurance contract, in which the indemnity of the contract is given by $Z=\max(X-L,0)$.

CAP INSURANCE is the cap capital insurance contract, in which $Z=\min(X,L)$. We utilize the following three deductible levels, L , defined as the percentage of the expected aggregate loss in the banking sector:

$$L = 0.1E[X], L = 0.2E[X] \text{ and } L = 0.5E[X]. \beta_m = \frac{Cov(X_i, Z)}{Var(Z)}.$$

$$\tau_m = \frac{1}{2m} \sum_{i=1}^m \beta_i \text{ for } i=1, \dots, N. \bar{\tau}_m = \min\{\beta_m, \max(\beta_{m+1}, \tau_m)\} \text{ for } m=1, \dots, N-1 \text{ and } \bar{\tau}_N = \tau_N.$$

The bank i is too big to fail if and only if $\beta_i > \bar{\tau}_m^*$, for each $i = 1, \dots, N$. Thus, the banks marked in red (blue) are too big to fail if the indemnity of the contract follows deductible insurance (cap insurance).

i	Bank Name	β_m	τ_m	$\bar{\tau}_m$	Bank Name	β_m	τ_m	$\bar{\tau}_m$
DEDUCTIBLE				CAP				
$L = 0.1E[X]$				$L = 0.1E[X]$				
$m^* = 1$				$m^* = 1$				
1	GS	0.8334	0.4167	0.4167	GS	7.0885	3.5443	3.5443
2	C	0.0492	0.2206	0.0492	C	0.6449	1.9334	0.6449
3	JPM	0.0439	0.1544	0.0439	JPM	0.3427	1.3460	0.3427
4	MS	0.0272	0.1192	0.0272	3FMCC*1000	0.3012	1.0472	0.3012
5	MET	0.0218	0.0976	0.0218	MS	0.2196	0.8597	0.2196
6	WFC	0.0147	0.0825	0.0147	MET	0.1466	0.7286	0.1466
7	AIG	0.0101	0.0715	0.0101	WFC	0.1104	0.6324	0.1104
8	BAC	0.0018	0.0626	0.0018	AIG	0.0949	0.5593	0.0949
9	3FNMA	-0.0003	0.0557	-0.0003	BAC	0.0085	0.4976	0.0085
10	3FMCC*1000	-0.0103	0.0496	0.0496	3FNMA	-0.0037	0.4477	0.4477
$L = 0.2E[X]$				$L = 0.2E[X]$				
$m^* = 1$				$m^* = 1$				
1	GS	0.8492	0.4246	0.4246	GS	7.0885	3.5443	3.5443
2	C	0.0497	0.2247	0.0497	C	0.6449	1.9334	0.6449
3	JPM	0.0449	0.1573	0.0449	JPM	0.3427	1.3460	0.3427
4	MS	0.0278	0.1214	0.0278	3FMCC*1000	0.3012	1.0472	0.3012
5	MET	0.0223	0.0994	0.0223	MS	0.2196	0.8597	0.2196
6	WFC	0.0150	0.0841	0.0150	MET	0.1466	0.7286	0.1466
7	AIG	0.0103	0.0728	0.0103	WFC	0.1104	0.6324	0.1104
8	BAC	0.0018	0.0638	0.0018	AIG	0.0949	0.5593	0.0949
9	3FNMA	-0.0003	0.0567	-0.0003	BAC	0.0085	0.4976	0.0085
10	3FMCC*1000	-0.0113	0.0505	0.0505	3FNMA	-0.0037	0.4477	0.4477
$L = 0.5E[X]$				$L = 0.5E[X]$				
$m^* = 1$				$m^* = 1$				
1	GS	0.9053	0.4527	0.4527	GS	3.2685	1.6343	1.6343
2	C	0.0509	0.2391	0.0509	C	0.2890	0.8894	0.2890
3	JPM	0.0481	0.1674	0.0481	JPM	0.1575	0.6192	0.1575
4	MS	0.0298	0.1293	0.0298	MS	0.1003	0.4769	0.1003
5	MET	0.0242	0.1058	0.0242	MET	0.0659	0.3881	0.0659
6	WFC	0.0162	0.0895	0.0162	WFC	0.0492	0.3275	0.0492
7	AIG	0.0110	0.0775	0.0110	3FMCC*1000	0.0422	0.2838	0.0422
8	BAC	0.0020	0.0680	0.0020	AIG	0.0411	0.2509	0.0411
9	3FNMA	-0.0003	0.0604	-0.0003	BAC	0.0042	0.2232	0.0042
10	3FMCC*1000	-0.0131	0.0537	0.0537	3FNMA	-0.0009	0.2008	0.2008

Table 9. TBTF Banks in 2012

This table displays a bank sector with 10 financial institutions and identifies “too big to fail” banks in year 2012 following the capital insurance approach explained in Appendix A. The analysis is performed for two types of capital insurance contracts and for three different deductible levels. Two types of contracts are as follows.

DEDUCTIBLE INSURANCE is the deductible capital insurance contract, in which the indemnity of the contract is given by $Z = \max(X - L, 0)$.

CAP INSURANCE is the cap capital insurance contract, in which $Z = \min(X, L)$. We utilize the following three deductible levels, L , defined as the percentage of the expected aggregate loss in the banking sector:

$$L = 0.1E[X], L = 0.2E[X] \text{ and } L = 0.5E[X]. \beta_m = \frac{Cov(X_i, Z)}{Var(Z)}$$

$$\tau_m = \frac{1}{2m} \sum_{i=1}^m \beta_i \text{ for } i=1, \dots, N. \bar{\tau}_m = \min\{\beta_m, \max(\beta_{m+1}, \tau_m)\} \text{ for } m=1, \dots, N-1 \text{ and } \bar{\tau}_N = \tau_N.$$

The bank i is too big to fail if and only if $\beta_i > \bar{\tau}_m^*$, for each $i = 1, \dots, N$. Thus, the banks marked in red (blue) are too big to fail if the indemnity of the contract follows deductible insurance (cap insurance).

i	Bank Name	β_m	τ_m	$\bar{\tau}_m$	Bank Name	β_m	τ_m	$\bar{\tau}_m$
DEDUCTIBLE				CAP				
$L = 0.1E[X]$				$L = 0.1E[X]$				
$m^* = 1$				$m^* = 1$				
1	GS	0.8872	0.4436	0.4436	GS	4.4281	2.2141	2.2141
2	JPM	0.0423	0.2324	0.0423	JPM	0.3238	1.1880	0.3238
3	MS	0.0206	0.1584	0.0206	MS	0.1848	0.8228	0.1848
4	MET	0.0177	0.1210	0.0177	MET	0.1500	0.6358	0.1500
5	C	0.0145	0.0982	0.0145	3FNMA	0.1366	0.5223	0.1366
6	3FNMA	0.0143	0.0831	0.0143	3FMCC*1000	0.1331	0.4464	0.1331
7	WFC	0.0125	0.0721	0.0125	C	0.1086	0.3904	0.1086
8	AIG	0.0084	0.0636	0.0084	WFC	0.1016	0.3479	0.1016
9	BAC	0.0000	0.0565	0.0000	AIG	0.0593	0.3125	0.0593
10	3FMCC*1000	-0.0027	0.0507	0.0507	BAC	0.0008	0.2813	0.2813
$L = 0.2E[X]$				$L = 0.2E[X]$				
$m^* = 1$				$m^* = 1$				
1	GS	0.9311	0.4655	0.4655	GS	4.4281	2.2141	2.2141
2	JPM	0.0434	0.2436	0.0434	JPM	0.3238	1.1880	0.3238
3	MS	0.0209	0.1659	0.0209	MS	0.1848	0.8228	0.1848
4	MET	0.0180	0.1267	0.0180	MET	0.1500	0.6358	0.1500
5	C	0.0148	0.1028	0.0148	3FNMA	0.1366	0.5223	0.1366
6	3FNMA	0.0147	0.0869	0.0147	3FMCC*1000	0.1331	0.4464	0.1331
7	WFC	0.0128	0.0754	0.0128	C	0.1086	0.3904	0.1086
8	AIG	0.0087	0.0665	0.0087	WFC	0.1016	0.3479	0.1016
9	BAC	0.0000	0.0591	0.0000	AIG	0.0593	0.3125	0.0593
10	3FMCC*1000	-0.0043	0.0530	0.0530	BAC	0.0008	0.2813	0.2813
$L = 0.5E[X]$				$L = 0.5E[X]$				
$m^* = 1$				$m^* = 1$				
1	GS	0.7848	0.3924	0.3924	GS	12.9880	6.4940	6.4940
2	JPM	0.0347	0.2049	0.0347	JPM	0.8337	3.4554	0.8337
3	MS	0.0160	0.1393	0.0160	MS	0.4698	2.3819	0.4698
4	MET	0.0145	0.1063	0.0145	3FNMA	0.3652	1.8321	0.3652
5	C	0.0119	0.0862	0.0119	MET	0.3558	1.5013	0.3558
6	3FNMA	0.0108	0.0727	0.0108	C	0.2821	1.2746	0.2821
7	WFC	0.0103	0.0631	0.0103	WFC	0.2446	1.1099	0.2446
8	AIG	0.0069	0.0556	0.0069	3FMCC*1000	0.2118	0.9844	0.2118
9	BAC	0.0000	0.0494	0.0000	AIG	0.1659	0.8843	0.1659
10	3FMCC*1000	-0.0056	0.0442	0.0442	BAC	0.0011	0.7959	0.7959