CLIMATE AND ENVIRONMENTAL RISK FACTORS IN THE MARKET RISK FIELD: AN EXTENDED MODEL

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Abstract

The extension of the risk management models to the broad sustainability concept is an open issue in both the academic and financial communities. The current state of the art for the risk measurement models is not satisfactory. There are many weaknesses in the data feasibility and the debate about what the new models should measure is still open. We propose a model that aims to improve the existing market risk models by capturing the sustainability risk sources. The starting point is the incremental risk charge (IRC) model, namely a 1 year 99.9 percent value at risk that covers default and migration risk. We extend the traditional model by defining the environmental incremental risk charge (E-IRC), with two enhancements: 1) by some data analysis and statistical techniques we introduce some new environmental, social, and governance (ESG) risk factors to better explain the portfolio behavior; 2) we adjust the default probabilities provided by the rating agencies by combining the green premium (lower spread) observed in the markets with the available ESG score for each obligor. The new model was tested on a real portfolio by a Montecarlo engine. The model does not affect too much the existing IRC results, so allowing continuity in the reporting process. The main advantage of E-IRC is the availability of a more effective risk decomposition process, where the ESG contributions can be properly highlighted.

Keywords: Risk Management, Financial Regulation, Climate Change, Statistical Regression, Risk Measures, Financial Reporting

Authors' individual contribution: Conceptualization — M.B., A.M., and L.T.; Methodology — M.B. and L.T.; Software — A.M.; Investigation — M.B. and A.M.; Data Curation — A.M.; Validation — M.B. and L.T.; Writing — Original Draft — M.B. and L.T.; Writing — Review & Editing — M.B.

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1. INTRODUCTION

In the last 10 years, awareness about sustainability issues has rapid growth, along with some seminal high-level initiatives such as the "2030 Agenda" by the United Nations and the European Green Deal.

In the financial sector, a huge effort was put down by the European regulators and by the financial authorities to give a framework for standardize the way to detect, measure, and manage risks coming from the (un)sustainability items.

From a regulatory perspective, the Sustainable Finance Disclosure Regulation (SFDR) Regulation (European Parliament, 2019) tries to give a common playfield where the financial players (the product manufacturers) and financial advisors must inform the clients about how the environmental, social, and governance (ESG) risks could affect the risk and returns of the product and how the sustainability issues are taken in to account in the investment strategies of product such as the funds. Products that have a well-defined ESG purpose must be declared in the documentation for the clients (Prospectus, KID, etc.).

The Taxonomy Regulation (European Parliament, 2020) specifies all the data and attributes to standardize the huge universe of ESG data, pushing for an open free platform where one can get all the required data about any product.

Nevertheless, the data puzzle is still unsolved. We remember that the well-known short name adopted in the sustainability area is ESG. These three components (environment, social, governance) are very heterogenous among themselves. Furthermore, each of them is related to many different concepts that are difficult to collect in a unified and well-weighted workflow. Just as an example, the "E" should take into account global warming due to carbon emissions, along with earthquakes, diseases from pollution, flood, and storms. At the same time, one should consider acute risks driven by an event and also long-term changes in climate patterns.

All these factors should be inserted in a risk management model to amend the risk measurement process for the portfolio (mainly from the bank perspective) or for the specific financial products that are being issued in the market, typically in the asset management sector.

To achieve such a new framework, the main challenge comes from the data issues: missing data, poor data quality, lack of standardization, short time series, the transformation of the single data points into properly standardized indices, etc.

From a more specific bank portfolio perspective, the European Central Bank (ECB) set out in 2020 its 11 expectations about the banks' capability to adopt good risk management processes in the climate and environment (C&E) area. C&E is a large subset of the "E" component.

At the beginning of 2022, a climate stress test was launched among significant institutions. The stress scenarios are defined according to the physical vs. transition risks, with an increase in the temperature, respectively, of 1.5 (orderly transition), 2°(disorderly transition) 3 (hot house world scenario) (ECB, 2022a). Briefly, the stress test exercise shows that the banks are still in an early stage as concerns the integration of C&E in the credit and market risk measurement processes. Most of the banks just try to assess the exposure to these risks, by some sector categorization, e.g., by using the Statistical Classification of Economic Activities in the European Community (*Nomenclature statistique des Activités économiques dans la Communauté Européenne* — NACE) economic sector.

Despite all the above summarized regulatory requests and the formal compliance by the banks, the current situation regarding the inclusion of ESG logic in the risk management models and processes is still unsatisfactory.

In fact, ECB. in 2022, performed a thematic review on a large sample of 186 banks, and several weaknesses in the state of the art of the banks were identified, respectively for the credit, market, and operational risk. More specifically, it was observed that even if most of the banks are running the materiality assessment, to evaluate if they are significantly exposed to C&E risks, almost all the banks use basic or judgmental methodologies to quantify the risks, and only 25% have some quantitative methodologies in place (ECB, 2022b). To improve such drawbacks, ECB set the end of 2023 as the deadline to finally achieve good risk management practice for the C&M area, partially stated in ECB (2022c).

There are also some open points at a more conceptual high level. It is worth recalling here the lack of agreement about the ESG ratings. What are the ESG ratings? What do they aim to capture? How are they built? Are they orthogonal with the credit rating or overlapped? Which degree of overlapping? Alternatively, should the credit rating integrate the ESG risk sources to exploit all the sources of the default risk?

As proof of this debate, the ESG ratings across the leading rating providers exhibit high discrepancy due to the difference in the risk factors, weights, and algorithms for the ESG rating attribution.

A large data set of listed companies is analyzed by Billio et al. (2021) with some clear evidence about this discrepancy.

In Brettenstein et al. (2022), some interesting insights are given about the climate change risk relevance in the general credit ratings calculated by the agencies, some needed improvements are identified, mainly the granularity of climate risk data and the disclosure about how much the climate risk score contributes to the final credit ratings.

To summarize, the sustainability field poses some difficult questions that are embedded in the risk definition itself. Following Novak (2012), the risk is a possibility of an undesirable event. Though such an event is rare, its magnitude can be devastating. But how to consider the two components that arise in any risk quantification, i.e., the probability and the magnitude, in a such disordered, not standardized, chaotic ESG "data lake"?

Coming to the aim of our paper, up to now, most banks are trying to monitor ESG risk through materiality assessment using ESG ratings, or sensitivity analysis via stress test.

According to the ECB guide on climate-related and environmental risks, "Institutions are expected to incorporate climate-related and environmental risks as drivers of established risk categories into their existing risk management framework, with a view to managing and monitoring these over a sufficiently long-term horizon, and to review their arrangements on a regular basis. Institutions are expected to identify and quantify these risks within their overall process of ensuring capital adequacy" (ECB, 2020, p. 4).

Besides, "Institutions are expected to consider how climate-related events could have an adverse impact on business continuity and the extent to which the nature of institutions' activities could increase reputational and/or liability risks" (ECB, 2020, p. 4).

The C&E risk is typically the risk that an issuer could have an adverse impact on its liabilities (equity or bonds) to simply adapt its business model to new regulations or in case of extreme events that could cause financial distress. Hence, we think that the direct consequence of such a phenomenon could be credit downgrading or the default of the issuer.

The risk measure that more than others represents the downgrade and the default risk is the incremental risk charge (IRC) model, calculated by the banks upon the Basel 2.5 reform (Basel Committee of Banking Supervision [BCBS], 2009). The IRC measure works at the boundary between market and credit risk, as it aims to cover the default and migration risk in the trading financial portfolio.

The aim of our paper is then to improve the market risk measurement toolbox by including in the IRC the available ESG (mainly "E") information.

To achieve that, we do not propose new theoretical methodologies but try to exploit in an effective way the statistical techniques well accepted in the market.

An adjusted IRC that considers also environmental rating and/or "E" factors could fit the need to incorporate climate-related risk in market risk measures.

A key point is to define what we expect to get from a market ESG-extended model.

There are two different views in the market risk community:

1. The ESG components could significantly shift the current market risk measures, as the ESG risks are not well covered by the current risk factors.
2. The ESG components should enable to get a more effective risk decomposition for the typical reporting processes, without significant change in the market risk measure. In this view, the current risk factors already span the "risk space" to detect also the ESG risks.

The above points of view depend on the managerial vision more than on quantitative requirements. We prefer to face from a more objective angle both perspectives.

We generally agree with the second perspective. The risk measurement models adopted by the banks already detect accurately the risk level, if properly validated by backtest techniques.

The main advantage of extending these models with ESG pieces should be better knowledge about where the risks come from and the capability to understand the effects of the risk due to the greenness of the portfolio composition. This is the goal of our contribution to the field.

The remainder of this paper is structured as follows. Section 2 reviews the relevant literature on the extension of the risk management models with the ESG factors. Section 3, after a short background about the market risk in the current Basel regulation, analyses the methodological framework of our proposal. Section 4 shows the application and the results. Section 5 contains a discussion of the key points of the model and some potential improvements. Section 6 summarizes the conclusions and further research paths.

2. LITERATURE REVIEW

The ESG scores calculated by several providers (MSCI, Refinitiv, Moodys, and many others) are quite popular in the market for several years, then most literature was devoted to investigating the relationships, if any, between the greenness of a financial instrument such as a bond (or the greenness of its issuer) and the financial features of the instrument in the market, mainly the yield and the liquidity.

The existence of a *green premium* in the bond market has received a lot of attention from both academic research and the financial community.

This premium consists of a lower spread of the bonds (vs. the *risk-free* comparable bonds), given a rating class and sector, with respect to the non-green or brown similar assets/issuers.

Despite many empirical studies, the investigations do not show convergent results.

If we focus on the equity markets, the green premium is generally investigated as an improvement in liquidity or in the long-term return.

These premiums are very hard to measure in a robust statistical way, because of the very difficult process of collecting, cleaning, and mapping data from the markets.

Most of the existing empirical studies seem to show that the green premium exists.

In Dorfeitner et al. (2021), a deep analysis is performed. Briefly, in the period 2011–2020, green bonds (i.e., labelled green bonds) are coupled with similar conventional synthetic bonds issued by the same issuer.

As very few obligors issue pairs of green/conventional bonds with the same features, mainly maturity, the problem is solved by a triplet approach. Two conventional bonds with the same issuer, seniority, and rating are selected, hence the spread of the conventional bonds is interpolated across the different maturities.

Some other data adjustments are applied to the original data to take into account the liquidity of the compared bonds and/or exclude the bonds from the dataset because of poor data quality or mismatching.

The study is finally based on 250 triplets (250 green bonds and 500 conventional bonds) for an amount of about 92,000 observations of each variable over the time window of the study.

The authors performed some different analyses with different subset ad data filtering. Generally, they found that the green premium is positive and stable after 2015, statistically significant even if with a magnitude of few basis points, less than 5 basis points in all the situations.

In Nanayakkara and Colombage (2019), a similar study is made for the 2016–2017 period, selecting green bonds vs. conventional bonds, without a matching systematic procedure.

The authors measure the spread with respect to the risk-free rate by the OAS (option-adjusted spread) provided by Bloomberg. They try to explain the dynamics of the returns of the bond by the binary variable (green, conventional), along with some other features of the bonds, and finally by a short slit of macroeconomic variables, namely the gross domestic product (GDP) and Consumer Price Index (CPI).

Their dataset consists of (82, green) + (43, conventional) = 125 bonds, for a total number of about 28,000 observations in the time period of the study.

They claim that the green premium is around 63 basis points, i.e., a very relevant amount.

Despite the many comments in the paper about the modelling approach and the variables inserted in the regression, we doubt that such a huge premium estimate is reliable. No detailed information is given about the closeness between the green and the conventional bonds subset. The instrument attributes such as the sector, the rating, that maturity, the seniority can dramatically affect the spread value, independently for the different shades of green.

Generally, we believe that the approaches based on splitting the bond in green or conventional may have some weaknesses.

Green bonds are issued to finance green projects, but we have many different regulations across countries and geographical areas regarding the regulation, control, and external review of the effectiveness of the green project. Then the green label is not so safe to cluster the bonds.

Furthermore, the greenwashing phenomenon has not yet been solved, again because of the different regulations or the lack of robust controls.

Moving to the liquidity impact in the equity market, Bonagura et al. (2021) analyze a panel of large-cap European stocks, excluding the financial and the real estate sector.

The liquidity is measured by an adjusted bid-ask spread and by the popular Amihud's (2002) *illiquidity ratio*.

The listed companies are categorized into 3 classes (green, non-green, and brown) according to ESG scores provided by Refinitiv and to the sector for the brown case.

The results clearly show for both indicators that the green stocks have more liquidity for the whole period of the study, 2006–2020.

A further perspective is given by Zhu et al. (2022), who analyze the carbon market risk. The carbon market is strictly related to climate change, then to the general environmental risks in the ESG taxonomy.

The authors implement an innovative meta-analysis of the literature by selecting 329 papers related to the carbon market topics and trying to identify the drivers of the carbon market risk.

By several statistical steps, they select 17 variables that pass the tests and are the most significant contributors to the carbon market risk.

With respect to the market returns, work in this direction is done by Chia et al. (2009), which checks if the "green factor" could explain the market dynamics more than the common market risk factors.

The authors found that a green factor seems to exist, by labeling the green stock with a dummy variable and calculating the significance test by some bootstrapping tests, to avid the normality assumptions.

As a summary, we point out that most of the literature about quantitative methods in the ESG field is devoted to the statistical validation of some indicators such as the spread green premium, the liquidity impact, or selecting the risk drivers that affect the market price movements for companies with different ESG scores.

The missing piece is related to the inclusion of the ESG components in quantitative risk measurement models. Along with the regulatory needing, this lack of risk measurement models motivates the role of our research.

3. METHODOLOGY

As outlined in the introduction, our approach consists in extending the IRC model to take into account the "E". This is the first step for extending the model, we focus on "E" because of the urgent regulatory requests that we outlined in Section 1. The methodology could be applied more generally for the "S" and "G" elements, with more effort in the data selection and data management steps.

The incremental risk charge belongs to the broad class of market risk models. It aims to capture losses due to downgrades of the portfolio positions, i.e., the spread jumps, and the losses due to the defaults. Technically, IRC is a quantile-based measure, with a horizon of 1 year and a confidence level of 99.9%.

The focus of this first research is the C&E risk, which typically leads to extreme events, for both the physical and transition risk. Then, the usual value-at-risk models, based on the price volatility in the short term (1 day, 10 days) do not fit the scope of the work.

3.1. Market risk and regulation

In the wide area of regulatory risk management, the so-called "Basel framework", the risks (and the related capital requirements) are categorized according to a *silo* or *building block* logic.

At the highest level of the hierarchy, the risks are given by type: credit risk, market risk, and operational risk.

The next bottom level differs for the different risk types. Credit risk building blocks are defined mainly by counterparty typology: retail exposures, corporate, large corporate, etc.

Market risk is split by asset class: equity risk, interest risk, spread risk (specific risk), and so on.

If we focus on the regulatory market risk measure and refer to the banks that have validated the internal model, the *Pillar I* market risk measure for the trading book, let be internal model capital charge (IMCC) is defined by this expression:

$$\begin{aligned} IMCC = MAX(VaR_T, \beta_1 * VaR_{AVG}) + \\ MAX(SVaR_T, \beta_2 * SVaR_{AVG}) + \\ MAX(IRC_T, IRC_{AVG}) + RNIME \end{aligned} \tag{1}$$

where, respectively:

- *VaR* is the *VaR* (99%, 10d) calculate to cover the risks for the typical asset class of the regulation taxonomy, i.e., interest rate, spread risk, equity, forex, and commodity.

- *SVaR* is the *stressed VaR*, very close to *VaR* from a methodological point of view but calculated by stressed parameters based on a stressed period.
- *IRC* is the *incremental risk charge*, technically speaking a *VaR* (99.9%, 1Y), i.e., the new (incremental) risk measure introduced with the *Basel 2.5* reform, to take into account the risks in the trading book that were missing until the 2008–2009 financial crisis, i.e., potential losses coming from the *default* risk (e.g., for the bonds in the portfolio and optionally equity) and the *migration* risk (losses due to the spread jumps implied by the credit rating downgrade) (BCBS, 2009).
- $\beta_{1,2}$ are multipliers assigned by the authorities that depend on the *backtesting* performance and some other features of the risk management process (model, information and communication technologies (ICT) systems, governance, etc.).
- The indices *(T, AVG)* are referred to point in time calculation (*T*) at the end of each quarter or the average (*AVG*) over the whole quarter, the average of daily observations for *VaR* and *SVaR*, and the weekly calculation for *IRC*.
- *RNIME* is the capital add-on for *risk not in the model engine*, i.e., the risks that are not adequately managed in the model.

As detailed in subsection 3.3, our proposal consists in updating the IRC model, by considering both an extended set of risk factors that drive the portfolio dynamics and by amending the default probabilities by the available environmental scores.

3.2. IRC models: Review

The IRC models belong to the category of the internal model, which is developed by each bank and then validated according to the ECB validation process. These models are used both from a Pillar I perspective and can be used for management purposes (Pillar II) as well.

The model must match the quantitative and qualitative constraints stated by the regulation. The main reference is given by ECB (2019).

In the financial industry, a specific approach became quite popular and represents a standard for the IRC measures. The model that we describe below belongs to the *Merton-type* default models because it tried to refer the default events to a structural source.

Some notation is needed. We simplify it with respect to the very many details of the IRC regulatory setup:

- $j = 1 \ldots N$ is the number of *positions*, i.e., the positions in the trading book or banking book for which the IRC has to be calculated. For the sake of simplicity, the very granular position level can be replaced by the *obligor* level, i.e., the different positions (bonds, equities, derivatives) that have the same issuer can be aggregated to a unique value. In most practical cases, N is in the range [100, 1000].
- $r = 1 \ldots R$ is the number of rating classes. Again, for the sake of simplicity, the rating classes defined by the main rating agencies are clustered to get a smaller cardinality. Then assume that $R < 10$ (including the state "D" = default), while the operation rating classes provided by the agencies are about 20.
- E_j = the exposures to each obligor. We use the general term "exposure". In most cases, it is simply the mark-to-market of the position (e.g., the fair value of the bond in the portfolio), while for the derivative positions, it is a proper exposure figure (e.g., the *delta equivalent* for an option).
- L_j = the loss has given default for each obligor, i.e., the fraction in the range [0, 100%] that is not recovered once the default happens. From a general point of view, the loss given default is not a deterministic quantity, but in practice, this random component is replaced by its estimation performed by historical data, and some sensitivity exercises are run to check the model risk. Moreover, this parameter is usually indexed by the product type, along with the seniority of the asset, but to avoid further notation we maintain the j index. In practice, its value is very often in a narrow range of around 60%.
- PD_j = the *default probability* of each obligor. Most of the obligors are large companies, financial institutions, or sovereigns. Hence, the default probabilities are usually assigned given the rating of the obligor, not using the internal rating model of the bank. Then, if we indicate with $R(j)$ the rating of the j-th obligor, a more accurate notation would be $PD_{R(j)}$, but again we omit this non-mandatory detail to keep simple the notation.
- $M = (M_{r,s})$, r, $s = 1 \ldots R$ is the matrix transition probability, i.e., the probability that (say in 1Y for simplicity) an obligor in the current state r moves to the state s. The matrix could be unique in the model, or we could have different matrices according to the high-level sector distinction: government, financial, and corporate.
- $PL = (PL_{r,s}) = r$, $s = 1 \ldots R$ is the *profits and losses* matrix, which specifies the (percent) gain/loss in the mark-to-market values if we observe a migration in the 1Y transition. The values of this matrix are typically estimated from the time series of a large panel of bonds, where the spread jumps due to migration are rescaled to get a straight-to-use PL matrix.
- $k = 1 \ldots K$ is the number of market risk factors to which the creditworthiness of each obligor is related, typically by a statistical regression approach. In most of the banks, K is quite small ($K < 30$) to avoid overfitting effects in the *regression* procedure.
- $X_{t,k}$ is the time series of the returns (or log-returns) of the risk factors that are selected by the banks. Typically, some equity indices in the most popular families (MSCI, BoA, S&P, etc.) are used for modeling corporate and financial behavior, and some credit default swap (CDS) or asset swap index for the sovereign sector.
- $Q = (q_{k,m})$ $m = 1 \ldots K$ is the correlation matrix between the risk factors ($X_{t,k}$).

Equipped with the above notation, let us describe the *IRC* calculation as a steps workflow.

With some offline process (say yearly or quarterly frequency), a statistical regression is performed to build a matrix, let be $B = (b_{j,k})$ that represents the behavior of each obligor with respect to the dynamics of the risk factors. In other words, the model aims to capture the creditworthiness Y_j

of each obligor given the market movements summarized by the risk factors. Formally, we have the below structure:

$$Y_j = \sum_{k=1}^{K} \beta_{j,k} * X_k + \sigma_j * \epsilon_j \quad (2)$$

The last term in Eq. (2) represents the idiosyncratic component of the regression. All the Y_j are usually standardized to an *N(0,1)*, the same for X_k. The regression is usually based (on the left side of Eq. (2)) on the equity returns for the corporate and financial obligors, and on the spread of daily differences for the public sector.

In this model, the default for the *j*-th obligor happens when:

$$Y_j \leq \varphi^{-1}(PD_j) \quad (3)$$

Here, $\varphi^{-1}(x)$ is the inverse of the *N(0,1)* distribution. At the portfolio level, if we indicate with $Loss_{D,PTF}$ the loss originated by the default events, we have the below expression:

$$Loss_{D,PTF} = \sum_{j=1}^{J} 1_{\{D(j)\}} * E_j * L_j \quad (4)$$

By sophisticating the above expression, we can get the loss for the portfolio implied by both the default and the migration events (the matrix PL has to be used).

The first term on the right side of the Eq. (4) is just the *indicator function* of the event in Eq. (3), which takes value 1 for default and 0 for the no default case.

Due to the nonlinearity of Eq. (3) and the quite involved general model, it is not possible to obtain an exact closed formula for the quantile of Eq. (4).

One could face the problem by analytical approximation. The variable $Loss_{D,PTF}$ does not allow to apply of the classical *i.i.d.* version of the central limit theorem, but if the quantities E_jL_j in Eq. (4) are not too much concentrated (*granularity* assumption) and we have an average low correlation level, one could expect that a normal approximation for Eq. (4) work well. To such an extent, see the deep study by Lehdili and Givi (2018).

To perform this analytical approach, the moments of loss must be exploited. For the first moment, one very easily gets:

$$E[Loss_{D,PTF}] = \sum_{j=1}^{J} PD_j * E_j * L_j = \frac{\sum_{j=1}^{J} PD_j * E_j * L_j}{\sum_{j=1}^{J} E_j} * \sum_{j=1}^{J} E_j = \mu * E \quad (5)$$

The last right-side formula is just a most useful representation, where μ is the loss rate, while E is the total amount (exposure of the portfolio).

The second moment of the portfolio default losses is quite involved. We omit the index *D,PTF* for brevity and we have:

$$E[Loss] = E\left[\left(\sum_{j=1}^{J} 1_{\{D(j)\}} * E_j * L_j\right)^2\right] = \sum_{j=1}^{J}\left(PD_j * E_j * L_j\right)^2 + \sum_{j \neq i}^{J} P\{D_i, D_j\} * E_j * L_j * E_i * L_i \quad (6)$$

The term $P\{D_i, D_j\}$ is the probability of the joint default of i, j obligors. It can be calculated according to the following formula:

$$P\{D_i, D_j\} = \int_{-\infty}^{F^{-1}(PD_i)} \int_{-\infty}^{F^{-1}(PD_j)} \varphi(y_i, y_j, \theta_{i,j}) * dy_i dy_j \quad (7)$$

In the above formula, $\varphi(x, y, \rho)$ is the bivariate normal density for the *N(0,1)* Gaussian random variable.

The calculation of Eq. (7) requires some preliminary matrix algebra to get the coefficient of the correlation matrix (*Q*). Furthermore, we need the $NI \approx O(K^2)$ double integrals in Eq. (7) to be numerically solved. In the large banks, we can observe $K = O(10^3)$ then we have to solve $NI \approx O(10^6)$ numerical integral.

The computation complexity, along with the requirement of granularity and low correlation properties in the portfolio makes it very hard to set up and maintain such an approach.

For these reasons, most of the banks perform a Montecarlo simulation. The simulation is driven by a Gaussian copula or by a t-student copula.

To achieve the strict requirements of the authorities and considering that the level of 99.9% is a very extreme quantile where the empirical estimator shows high uncertainty, the number of simulations *NS* is very high, in most banks in the range [100K, 1M].

If the bank collects the portfolio results for each simulation, let be $Loss_{ns}$ and also the granular results, $y_{j,ns}$, then by quite standard algorithmic techniques the IRC (quantile) at the portfolio level can be analyzed and decomposed according to the sector, the portfolio, or any other useful dimension.

3.3. The extended IRC model

In what follows, E-IRC will be the short name for the new model, where "E" points out both the extension of the traditional IRC model and the relevance of the environmental drivers.

The E-IRC model consists of 2 components.

We want to combine a pure market risk approach by market movements explained by the underlying risk factors along with the extreme events that are driven by defaults. To summarize, they represent the market piece and the credit piece of the recipe.

3.3.1. The market risk component: Augmented risk factors set

The idea behind this first extension is that the creditworthiness of each obligor is not entirely captured by the risk factors.

The relevance residual term $s_i e_i$ of the regression could be reduced if we introduce a proper (small) set of new regressors, that belong to the ESG family. In our context, we are mainly focused on a new set of C&E risk factors.

The new risk factors (modeled and rescaled to *N(0,1)*) Z_m, $m = 1 \ldots M$ should have the following properties:

- $COV(X_k, Z_m) = E[X_k * Z_m] \approx 0$;
- $COV(Z_m, Z_u) = E[Z_m * Z_u] \approx 0$.

The reason for the above constraints is to avoid over-parametrization and multicollinearity, i.e., the new risk factors must increase the available information, so reducing the unexplained fraction of the regression and increasing the R^2 or any other fitting index (AIC, BIC, etc.).

Formally, the new expression for obligor behavior is given by:

$$Y_j = \sum_{k=1}^{K} \beta_{j,k} * X_k + \sum_{m=1}^{M} \beta_{j,m} * Z_m + \sigma_j * \epsilon_j \quad (8)$$

Nothing changes about the default rule expressed in Eq. (3).

What do we expect as concerns the portfolio level IRC results with this augmented model?

Depending on the increase in the R^2 index, we expect a small increase in the IRC risk measure, as the diversification effect due to the uncorrelated term e_j will have a smaller weight.

From a more managerial point of view, this approach is in line with subsection 3.1, i.e., the current risk factors do not (fully) detect the dynamics of the obligors implied by the ESG factors, hence the structural model must be extended.

3.3.2. The credit risk component: Green-brown default probability adjustment

Our approach consists in the change of the default probability to manage the different shades of green and the related probabilities.

From an algorithmic point of view, no change in the general IRC flow.

We must just split the transition matrix M defined in Section 2 in a set of transition matrices, let be $C = (c_{g,r,s})$.

The new index $g = 1 \ldots G$ represents the greenness level of each obligor. The C wants to emphasize that now we have a *cube* of transition probabilities, not simply a matrix.

Usually, we have $G = 2$ to distinguish green vs. non-green level, or $G = 3$, for the green, non-green, brown triplet.

Coming back to the underlying idea for this approach, in this case, one believes that the ESG ratings are more objective and/or robust than the ESG time series and tries to consider the greenness of the obligor in its key parameter, the default probability.

From a numerical point of view, we cannot have a priori expectation about the impact on the portfolio IRC.

It depends on the results in the calibration of the new probabilities (how much the green probabilities go down, how much the brown increase) and on the portfolio composition, i.e., what fraction of the exposure is labelled according to the different g-values.

In the following subsection, we will show a practical application for a real-world portfolio, where the two outlined approaches have been implemented separately and then combined.

3.4. Model validation

In the financial sector, the validation of a risk management model is referred to as backtesting.

Each model to measure the risk aims to have forward-looking features, i.e., to predict the extreme losses in an accurate way to the confidence level.

Unfortunately, for the models that work with a long horizon, 1 year as the IRC, the classical backtest, to compare the ex-ante risk measures with the observed losses, is not a feasible exercise, because of 3 facts:

1. The long-year horizon does not allow to collect a large sample of realized losses.
2. Default is a rare event, as the banks typically hold portfolios with small PDs.
3. The confidence level is very high, 99.9%. As the confidence level increases, one needs a larger sample size for accurate quantile empirical estimation.

For this reason, also the Basel Committee, referring to IRC, requires some other checks in order to guarantee the model's adequateness.

In BCBS (2019, para. 33.34), it is then prescribed that the IRC model should be validated (checked) by some alternative tools, such as sensitivity analysis, scenario analysis, and stress test, on an ongoing basis.

With a very detailed list, these controls for the soundness of the model are well specified (ECB, 2019).

For the above reason, we do not propose a formal statistical validation of the model outcome, as we believe that rigorous statistical work in selecting the variables and estimating the parameters of the model could ensure, given that for an IRC-type model other soundness checks must be implemented.

4. RESULTS

4.1. Portfolio and datasets: Some details of E-IRC implementation

The portfolio was extracted as of 30th September 2022. The term "portfolio" usually meant as a vector is not very accurate in practice as it is a *tree*, where the nodes categorize the accounting category (banking book vs. trading book), the branch of the banking group to which the single position belongs, the desks, and so on.

From now on, we refer to the most relevant subportfolio in the whole dataset, i.e., the *banking book* portfolio of the main holding legal entity.

We have the following values:

- market value ≈ €30 billion;
- number of positions ≈ 750;
- number of obligors ≈ 150;
- the market value of the top 5 obligors: 80% (all belong to the sovereign sector).

As concerns the risk factor list, in the existing IRC model we have 27 credit drivers, that are categorized according to 3 families:

- Stock exchange indices, such as Eurostoxx 50, S&P 500, FTSE MIB, 5 indices.
- Equity geographic or sector indices, mainly belonging to the MSCI categories, globally 16 indices.

• Asset swap spread indices, used for the sovereign positions, 6 indices.

To implement the market risk component described in subsection 3.2, we extended the risk factors set with new 6 ESG risk factors, that are built as described:

• Three synthetic *green* risk drivers. $E_{SINT,G}$ defined separating from the indices (MSCI Europe ESG Screened, MSCI USA ESG Screened, MSCI EM ex Fossil Fuels) the systematic risk already captured by the related general MSCI indices, i.e., MSCI Europe, MSCI USA, MSCI EM. Technically they were obtained with the residual of the regression of the ESG indices vs. the general indices, namely $E_{SINT,G} = GEN_G - b_{E,G}$, where the index G indicates the geographic area and $b_{E,G}$ is the regression coefficient.

• Three *brown* risk drivers were selected in a combined statistical/expert way, by taking the stock indices of countries that do not adopt an industrial policy to achieve environmental sustainability, i.e., Bove Spa Brazil, Hang Seng China, FTSE South Africa, and that maximize the fitting properties of the model. These indices were selected after analysing and comparing a longer set of stock indices of other countries.

The bond sample that was used to implement the default probability shift illustrated in subsection 3.2 is a bond in the maturity range [2Y, 7Y] because it is the most liquid segment of the bond market.

The bonds were sampled at different dates in 2022, to capture the different magnitude of their spread values, with about 27,000 total observations, based on:

• more than 600 issuers;
• 14 observation dates;
• more than 2,000 different bonds (each bond does not appear in all the observation dates);
• average maturity of 5.74 years.

We point out that with respect to the works cited in Section 2, we have a larger number of bonds, and a smaller number of observation dates, with a similar panel data approach.

Indeed, we believe that the number of bonds is the most relevant parameter to reduce the noise coming from market data with respect to the frequency of the observation.

The bonds were labelled as "green" vs. "non-green" (hence, $G = 2$), by defining a threshold in an internal "E" score, that combines data coming from some different providers.

To avoid the discrepancy exhibited by the different ESG providers, most of the large banks smooth this effect by averaging the scores. The threshold was chosen to have enough data points for each cluster.

Then, our approach differs from most of the literature, as the green tag is related to the quality of the issuer, not derived from the green label of the bond.

Our choice is motivated partially by the fragmentation of the green bond definition and quality as explained in Section 2. About the fragmentation over the different jurisdictions of the green bond concept, see the comprehensive review by Agliardi and Agliardi (2023).

Moreover, the IRC model is driven by the obligor events (default, rating migration), then we prefer to work at the issuer level, not at the instrument bottom level.

The below table summarized the data.

Table 1. The bonds panel for PD calibration

Rating class	*Green*	*Non-green*	*Total*
1. AAA	3.536	4	3.540
2. AA	3.362	443	3.805
3. A	9.223	1.026	10.249
4. BBB	6.792	1.354	8.146
5. BB	571	335	906
6. B	68	199	267
7. C	58	26	84
Total	**23.610**	**3.387**	**26.997**

The grey cells indicate that due to small sample issues, for the related rating classes, no change was applied for the "AAA" rating class, while for the classes "B" and "C" we decided to update the default probability by applying the apply the same adjustment parameters calculated for "BB".

These choices were motivated by the sample size and by the statistical test results about the significance of the observed green premium.

The recalibration consists of the following steps. First of all, the average spread for each rating class is calculated, without the green labeling:

$$\bar{S}^r = \sum\nolimits_{k \in i} \frac{S_k^r}{N},\ r = 1, \dots, 7 \tag{9}$$

Then, we calculated the average according to the cluster that we achieve with the green and non-green split:

$$\bar{S}^{r,e} = \sum\nolimits_{k \in \{i,e\}} \frac{S_k^{r,e}}{M},\ r = 1, \dots, 7; \quad e = G,\ NoG \tag{10}$$

From the spread, we derive the market-implied (risk-neutral) PDs for each cluster, with and without the green label, by the usual relationship $PD_{imp} = 1 - exp(-spread/LGD)$.

Finally, we update the real-world (*RW*) probabilities associated with the rating classes by the ratios calculated as follows:

$$\delta\%PD_{imp}^{r,e} = \frac{PD_{imp}^{r,e}}{PD_{imp}^{r}} \tag{11}$$

$$PD_{RW}^{r,e} = PD_{RW}^{r} * \delta\%PD_{imp}^{r,e} \tag{12}$$

For the sake of simplicity, we omit the index s, which records the macro sectors: corporate, financial, and government.

Let us point out the subtle concept implied in the above expressions. Because of the risk premium, the risk-neutral and real-world probabilities may differ a lot, mainly for the good rating classes. For this reason, we do not calculate in an *additive* way the green premium (very common in the literature). We prefer to define the *multiplicative* ratio coming for the market spreads and then apply it to the rating agencies' probability.

We are aware do not have a theoretical consolidated foundation for this calculation, but we deem it is the most reasonable.

On one hand, especially for good ratings and low default probabilities, the additive approach could affect too much the original PDs, due to the noise effect in the data. On the other hand, the proportional rescaling that we adopt is aligned with the most accepted practices in the data adjusting in the market, for example, the transformation from the raw prices time series to the *total return* time series to consider the dividends over time.

4.2. Main results

4.2.1. Model parameters and intermediate indicators

As concerns the market risk component of the E-IRC model, we observe that on a time window of 4 years of daily data in the time window 2018–2022, the R^2 fitting index of the linear regression (equity returns and spread variations vs the risk drivers) increases from around 50% to 53.5%. We recall that we moved from the existing model with 27 explanatory variables to the E-IRC model with 33 variables.

Considering that we plugged 6 new ESG indices in an existing set of $K = 27$ indices, this is a quite satisfactory result.

It is worth noting that the new indices did not change too much the (Beta) matrix β, which summarizes the sensitivity of each obligor to the set of risk factors. In the below factors, the (few) green cells indicate the cases where the difference in the Beta coefficient, with and without the ESG indices, was statistically significant. We recall that the Beta matrix contains *(Number of obligors)* × *(Number of risk factors)* elements, in our case of (27 or 33) × (150).

By running a statistical test of the extended model vs. the existing one, hence limited to the (27 × 150) beta coefficients, for only about 4% of them the results about statistical significance change.

Generally, we observe that in the augmented set of risk drivers, some good results to avoid overfitting have been achieved.

Indeed, if we measure the statistical significance of the correlation coefficient *between* the blocks, i.e., the old 27 risk drivers vs. the 6 new risk drivers, only in 10% of the cases (27 × 6 = 162 cases), the correlation is significantly different from zero. In other words, the new risk factors are quite orthogonal with the existing ones, they do not tell the same story.

From a descriptive perspective, the average correlation *between* the 2 blocks, 27 risk existing risk factors, and the new 6 risk factors, is very low, around 4.1%.

The average correlation *within* the new (6 × 6) block for the "E" risk factors is again low, 7.5%.

The PD adjusting that we described in the previous subsection gives the following results for the corporate and financial sectors. For the sovereign positions, no adjustment must be applied, due to the smaller size of the sample. In the left column, the original PD coming from the provider.

Table 2. PDs before and after the adjustment process

CORP 1Y	*PD*	*Green*	*Non-green*	*FIN 1Y*	*PD*	*Green*	*Non-green*
1. AAA	0.01%	0.01%	0.01%	1. AAA	0.01%	0.01%	0.01%
2. AA	0.01%	0.01%	0.14%	2. AA	0.07%	0.05%	0.19%
3. A	0.01%	0.01%	0.14%	3. A	0.16%	0.16%	0.22%
4. BBB	0.07%	0.01%	0.42%	4. BBB	0.58%	0.51%	0.93%
5. BB	0.21%	0.21%	0.42%	5.BB	1.04%	1.01%	1.19%
6. B	1.40%	1.36%	1.60%	6. B	2.29%	2.23%	2.63%
7. C	8.53%	8.29%	9.73%	7. C	9.76%	9.49%	11.12%
8. D	100.00%	100.00%	100.00%	8. D	100.00%	100.00%	100.00%

The values of 0.01% that we have for the best rating are due to a regulatory floor. Even if the estimated *PD* is less than 0.01%, this prudential level must be used.

4.2.2. Risk measures results for the E-IRC model

Finally, we describe the results coming from the new E-IRC model. The actual portfolio of the bank is portfolio No. 3 (PTF #3). For non-disclosure reasons, we rescale the results and assign the say *baseline* IRC model a standard value of 100, to highlight the impact of the E-IRC proposal. The results were obtained with $NS = 1$ million simulations in the Montecarlo algorithm, hence very stable with low uncertainty.

The goal was to understand how much the green/brown features of the portfolio can affect the risk measures that are usually adopted in the banks, i.e., the IRC, namely the *VaR* (99.9%, 1Y).

In other words, to evaluate the sensitivity of the risk measures to different asset allocations that depict different levels of greenness.

To do that, we start from the list of obligors and assets that composed the real portfolio and defined some *model portfolios* according to different combinations. Below is the summary table of the model portfolios.

Table 3. Model portfolios

Model portfolio	ESG weights		Macro sectors			Risk KPI	
	Green	Non-reen	Government	Financial	Corporate	Average PD (Original)	Adjusted PD
PTF #1	80.0%	20.0%	80.0%	10.0%	10.0%	0.19%	0.22%
PTF #2	80.0%	20.0%	80.0%	10.0%	10.0%	0.90%	1.03%
PTF #3	80.0%	20.0%	50.0%	25.0%	25.0%	0.08%	0.09%
PTF #4	80.0%	20.0%	50.0%	25.0%	25.0%	0.90%	1.06%
PTF #5	50.0%	50.0%	80.0%	10.0%	10.0%	0.17%	0.19%
PTF #6	50.0%	50.0%	80.0%	10.0%	10.0%	3.09%	3.62%
PTF #7	50.0%	50.0%	50.0%	25.0%	25.0%	0.07%	0.07%
PTF #8	50.0%	50.0%	50.0%	25.0%	25.0%	3.11%	3.66%

The original *PD* is meant before the adjusting process described in the previous sections.

The E-IRC results are rescaled with respect to any portfolio without the E-extension to have sharp evidence of the impacts of the ESG features. Below are the results (Table 4). In the table, migration risk (MGR) indicates the quantile of the losses due only to the migration effect, and default risk charge (DRC) is the risk measure for the default-only case.

Table 4. E-IRC results for the model portfolios (Rescaled results)

Model portfolio	IRC (Original model)	E-IRC	MGR (Original model)	E-MGR	DRC (Original model)	E-DRC
PTF #1	-100.00	-100.61	-89.75	-90.16	-34.89	-34.89
PTF #2	-100.00	-102.65	-80.11	-79.91	-33.63	-33.63
PTF #3	-100.00	-100.34	-53.88	-53.52	-73.00	-76.69
PTF #4	-100.00	-102.83	-60.55	-60.43	-66.21	-68.09
PTF #5	-100.00	-100.96	-99.44	-99.03	-9.97	-9.97
PTF #6	-100.00	-103.87	-57.27	-59.07	-69.03	-69.85
PTF #7	-100.00	-100.38	-76.09	-76.19	-66.98	-66.98
PTF #8	-100.00	-101.69	-35.68	-34.9	-95.99	-97.89

5. DISCUSSION

The impact of the extended E-IRC model does not always show the same sign and the magnitude is quite small. It depends on the mixed combination of the PD adjusting (lower risk for highly green portfolios) and the larger risk drivers set (higher systematic risk, higher risk at portfolio level). Typically for green portfolios with higher average PD the PD adjusting effect prevails, so getting a bigger risk measure, as in PTF #2 and PTF #4. Further research and numerical experiments are requested to be safe about the calibration of the green/brown boundary and the PD shifts.

The results are in line with the theoretical expectations. The introduction of the new risk factors allows for to reduction of the unexplained (uncorrelated) component of the statistical regression. If we represent the portfolio as a portfolio of risk factors, it becomes more correlated, hence riskier. This is a more prudential and accurate representation. As concerns the PDs adjusting impact, the magnitude of the impact would be higher for medium-high risk portfolios, but sometimes it is nearly offset by the new risk factors' systematic effect, see PTF #7.

From a managerial perspective, the combined E-IRC model does not significantly change the whole IRC figures, but it gives a more accurate estimation at the portfolio level, and by the usual statistical techniques the approach allows to decompose of the risk figures detecting the contribution of the ESG (or "E") sources of risk separating them from the classical risk factors. We refer to the risk decomposition techniques such as the *Component VaR* and *Marginal VaR or Incremental VaR*. The seminal work was by Garman (1997). Many other estimators have been furtherly developed.

With respect to the validation of the model, having implemented in the global algorithm the new parameters and the new steps, we could also perform the scenario, sensitivity analysis illustrated in subsection 3.4. To achieve that, one can easily define a more granular "grid" of different portfolios and check the E-IRC results among themselves and also compare E-IRC with IRC results for each cluster of portfolios, so detecting potential weaknesses of the model.

6. CONCLUSION

The extension of the risk management model to consider ESG risks is still a pioneering area. Despite a wide literature about the green premium in the markets and the wide practical use of the ESG ratings, the quantification techniques do not achieve, so far, a satisfactory level of standardization and reliability, as pointed about by several surveys by the financial authorities.

In this very challenging field, we define the IRC model as a good starting point to extend the current model.

Focusing on the "E" part, we extend it by adding some accurately chosen risk factors and by adjusting the default probabilities of the issuers according to their environmental qualities and the market empirical evidence.

The new E-IRC model does not affect too much the existing IRC results for a real-world portfolio, so giving continuity to the reporting process. Its main advantage is the feasibility of a more effective risk decomposition process, where the ESG contributions can be properly highlighted.

As we pointed out with the analysis about the risk measures sensitivity to the green level of the portfolio some further work is needed to define a more rigorous statistical process to include the new risk factors and to extend the bonds sample used to infer the spread premiums.

Some further work is also needed to have a robust theoretical foundation for the PD adjustment that combines spread with the default probabilities, i.e., data coming from risk-neutral and real-world settings.

To validate the model, just more numerical work has to be done.

If we focus on the E-IRC model structure, we believe it can capture in a practical feasible way the needing for risk measure models that include the ESG information, so contributing to a more effective risk segmentation and attribution at the managerial reporting level.

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