MODELING TAIL-DEPENDENCE OF CRYPTO ASSETS WITH EXTREME VALUE THEORY: PERSPECTIVES OF RISK MANAGEMENT IN BANKS

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Abstract

Cryptocurrencies show some properties that differ from typical financial instruments. For example, dynamic volatility, larger price jumps, and other market participants and their associated characteristics can be observed (Pardalos, Kotsireas, Guo, & Knottenbelt, 2020). Especially high tail risk (Sun, Dedahanov, Shin, & Li, 2021; Corbet, Meegan, Larkin, Lucey, & Yarovaya, 2018; Borri, 2019) leads to the question of whether the methods and procedures established in risk management are suitable for measuring the resulting market risks of cryptos appropriately. Therefore, we examine the risk measurement of Bitcoin, Ethereum, and Litecoin. In addition to the classic methods of market risk measurement, historical simulation, and the variance-covariance approach, we also use the extreme value theory to measure risk. Only the extreme value theory with the peaks-over-threshold method delivers satisfactory backtesting results at a confidence level of 99.9%. In the context of our analysis, the highly volatile market phase from January 2021 was crucial. In this, extreme deflections that have never been observed before in the time series have significantly influenced backtesting. Our paper underlines that critical market phases could not be sufficiently observed from the short time series, leading to adequate backtesting results under the standard market risk measurement. At the same time, the strength of the extreme value theory comes into play here and generates a preferable risk measurement.

Keywords: Crypto Assets, Extreme Value Theory, Backtesting, Basel Traffic Light Approach, Historical Simulation, Variance-Covariance Approach

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1. INTRODUCTION

The evolution of crypto assets began with distributed ledger technology. It led to several virtual currencies like Bitcoin, Ethereum, Tether, Binance Coin, Cardano Ripple, Dogecoin, Litecoin, Chainlink, Tron, etc. A continuously rising market capitalization reflects the rapid growth of these crypto assets. New all-time highs in price and trading volume or market capitalization have been reached (Basel Committee on Banking Supervision [BCBS], 2021).

Although the relative size of the crypto asset market regarding the global financial system is relatively small for financial markets' stability, its absolute size is meaningful from a supervisory point of view. Focused on the significant absolute size of crypto-asset markets, it is necessary to develop an appropriate risk measurement to deal with emerging risks of investing in crypto assets for professional market players like banks and investment firms. The BCBS stated that certain crypto assets have a high degree of volatility and could cause high financial risks for such market participants (BCBS, 2021).

But also, from a risk management perspective and a theoretical point of view, this new asset class, with its high tail dependence, some historical drawdowns, and a high degree of volatility, aims to develop and use suitable risk measurement methods that anticipate these observations. Last but not least, it is essential to create efficient and diversified portfolios and to back them up with appropriate risk capital. In uncertain future volatility due to macroeconomic uncertainty, it is also necessary to consider severe adverse developments.

Concerning most well-known market risk measurements, crypto assets have several statistical characteristics that may not fit such value-at-risk (VaR) measures. Significantly, the high degree of volatility, tail-dependence, and missing stationarity (Pardalos, Kotsireas, Guo, & Knottenbelt, 2020) challenge common risk measurements like historical simulation, variance-covariance model, or Monte Carlo simulation (Mehmke, Cremers, & Packham, 2012). Furthermore, the market conduct of crypto asset investors leads to whether the typically observed correlations represent a new asset class (Allen, 2022; Borri, 2019; Guo, Härdle, & Tao, 2021).

Hence, the following research questions should be answered:

RQ1: Are the common VaR approaches an adequate measurement approach?

First, we perform a statistical analysis of the behavior of crypto assets. Next to the results, we perform a deviation analysis to justify the possible missing fit of common VaR methods.

RQ2: Does an extreme-value-theory-based VaR provide a better fit for the statistical characteristics of crypto assets?

We compare the standard VaR methods to show which approach best fits the emerging risk of investing in crypto assets. Hence, the extreme value theory provides a highly tail-dependence fit in extreme market situations.

With our paper, we contribute to further developing the risk management of cryptocurrencies concerning tail dependence and high volatility and show, in particular, how a risk method can also predict and measure historically unknown observations concerning the risks associated with them. We are also closing a gap between the extreme value theory, known for market and liquidity risks and has not previously been applied to cryptocurrencies. Nevertheless, this method shows good backtesting results, even in times of COVID-19 and the beginning of the war in Ukraine, with high, previously unknown volatility and historically unobserved price changes.

The structure of this paper is as follows. We started with Section 2, in which we and examine the relevance of classical market risk methods like historical simulation, the variance-covariance approach, and some notes in the Monte Carlo simulation in subsection 2.1. Due to the assumption of high tail risk, we also introduce the extreme value theory. In subsection 2.2, we examine the evolution and some main characteristics of cryptocurrencies. Furthermore, we highlight the main research findings relating to our research questions and their relevance for our backtesting. Based on the latest publication of the BCBS regarding cryptocurrencies, we will show the supervisory point of view and its expectations of risk measurements. In Section 3, we explain the chosen data, its length, and some notable aspects relating to our study design, the extreme value theory methodology, and the methodology to analyze our backtesting results. In Section 4, we describe and discuss our effects regarding our two research questions. At least, in Section 5, we highlight the main aspects, new considerations, also for future research, and limitations of our study.

2. LITERATURE REVIEW

2.1. Market risk measurements

Starting with several market risk measurements, mostly common for classic financial/market risks, we focus on the variance-covariance approach, the historical simulation, and the extreme value theory. In addition to the Monte Carlo simulation, historical simulation and the variance-covariance approach, in particular, are standard methods of measuring market price risk (Huschens, 2017; Wiedemann, 2013).

The historical simulation can be calculated using absolute, relative, and logarithmic risk factor changes. The present article focuses on absolute differences. Absolute differences can be described as dependent on the level and thus tends to be unsuitable for trend-related changes in value. In contrast, high changes in times of high volatility are suitable in times of lower volatility and could lead to better forecast quality. Finally, transferring historical changes to the future requires stationarity, i.e., stochastic freedom from trends (Miller, 2018; Huschens, 2017).

The historical simulation is based on historical risk factors changes. Based on these historical risk factors changes, the current portfolio is revalued. Furthermore, considering the historical factor changes means that no distribution or correlation assumption is necessary since this information is taken from the historical change and transferred to the current price level. It is characteristic of the historical simulation that the risk value is counted, i.e., based on the amount of relevant data for the x-worst data set matching the confidence interval. An environment detached from the data record can thus already be recorded, which rank is to be used as the risk value (Wiedemann, 2013).

As a parametric approach, the variance-covariance method assumes a normal distribution of the risk factor changes. In the case of several risk factors, the correlation between them must be considered. The variance-covariance method is widespread due to the low data requirements and good feasibility. However, the variance-covariance approach, particularly the normal distribution assumption, has some disadvantages. The financial crisis has already shown that outliers are usually underestimated and cannot be sustained. It should be added that the normal distribution lacks so-called "fat tails", which can cover larger characteristics at the edge of the distribution. Concerning the correlation assumption, the variance-covariance method uses the Pearson correlation coefficients, which on the one hand, continues the assumption of linearity of the normal distribution and, on the other hand, also has to be checked in terms of stability (Rüder, 2018; Pesaran, 2016; Gleißner & Wolfrum, 2019; Daníelsson, 2006; Romeike & Hager, 2020).

The extreme values in the flanks of the distribution are particularly relevant for modeling risks that rarely occur and which have an enormous risk effect. Due to a high level of extreme risks, these cannot be adequately represented using the flanks of the normal distribution and, sometimes, the empirical distribution. Therefore, a so-called heavy-tailed distribution with thicker and broader sides is required. Due to the rarity of extreme risks and their systemic character, it is possible that these cannot be inferred from history. The extreme value theory deals with extremely high deviations from the mean of a probability distribution. It can be divided into two modeling approaches: the block maxima method and the peaks-over-threshold model (PoT). For both methods, data from the tails of the empirical distribution are used and fitted to an extreme value distribution (Zhao, 2021; Pardalos et al., 2020; Ahelegbey, Giudici, & Mojtahedi, 2021). With the block maxima method, the extreme value distribution is modeled from a series of maxima so that a relatively large data set is required. The PoT method is considered more efficient when modeling with a limited data set, so we use the PoT method for our analysis (McNeil, Frey, & Embrechts, 2015). We explain the details of the PoT methodology in Section 4.

Alternatively, the VaR can also be determined using a generalized autoregressive conditionally heteroscedastic (GARCH) model. In this way, the dynamics of volatility are considered in the model (Angelidis, Benos, & Degiannakis, 2004). However, since the extreme value theory can also be used to estimate risks that were not yet apparent in history, it is preferred for the analysis.

2.2. Market risk management and cryptocurrencies

Virtual currencies, cryptocurrencies, are a digital representation of assets that a central bank or agency does not create. There is, therefore, no connection to legal tender guaranteed by a central bank or authority. Unlike traditional currencies, they are based on the idea of a surrogate currency with a finite supply of money. Cryptocurrencies can be created using a predetermined mathematical process to create new value units. This is called mining (BCBS, 2021). The users face each other peer-to-peer on an equal footing. The respective owner manages the virtual currency (VC) with his private and public key pairs for authentic transactions. All users can transfer their VC to each other within the network and must regularly communicate the respective target addresses outside the network. However, it is impossible to identify which person owns the VC in the network based on the positions. Transactions, once made, are fundamentally irreversible. In addition to the transmission of VC within the web, it is also possible to physically transfer digits and keys between people, bypassing them onto data carriers (Arslanian, 2022).

Most of these cryptocurrencies show significant co-movement because of their similar technology and comparable types of trades. Hence, several studies show patterns of co-movements and correlations. Especially the COVID-19 crisis shows important findings and challenges standard risk measures with changes in crypto asset networks. Furthermore, dApps and protocols are more attractive to investors (Katsiampa, Yarovaya, & Zięba, 2022). Extracting these uncertainty effects of cryptocurrencies offers a wide range of risk management methods to deal with cryptocurrency bets like Bitcoin (Koutmos, King, & Zopounidis, 2018).

In terms of the current state of research, three research areas can be identified:

1) diversification effects with crypto assets,
2) risk/return spillover,
3) volatility forecast.

Most of these papers focus on specific effects but do not scope standard risk measurement methods' ability to assess possible risks of arising crypto assets. Starting with Corbet et al. (2018), the authors analyze the relationship between popular cryptocurrencies and other financial assets. They stated crypto assets could lead to diversification effects and risk-return advantages. Although diversification benefits could be derived, these are only in short investment horizons observable. Borri (2019) focuses on the conditional tail-risk for cryptocurrencies and finds that these are highly exposed to tail-risk within crypto markets (Sun et al., 2021; Corbet et al., 2018; Borri, 2019).

Relating liquidity effects, cryptocurrencies do have a small impact on optimal portfolios. In addition, there is reasonable evidence to imply the existence of downside risk spillover between Bitcoin and four assets (equities, bonds, currencies, and commodities), which seems to be time-dependent. These main findings have implications for participants in the Bitcoin and traditional financial markets for asset allocation and risk management. For policymakers, the results suggest that Bitcoin should be monitored carefully for financial stability (Zhang, Bouri, Gupta, & Ma, 2021). Sun et al.'s (2021) study focuses on cryptocurrencies in private equity (PE) company portfolios and investment factors of PE managers. They find that price volatility does not lower institutional investors' confidence as long as the market can offer timely and accurate price change information to meet investors' price consciousness. Furthermore, especially cryptocurrencies with a high familiarity provide diversification benefits.

Most of such research investigates one or two specific cryptocurrencies. Most research is done on the Bitcoin time series. This could be motivated by the familiarity and the length of the time series of

Bitcoin. Many studies have fully emphasized and analyzed the VaR in the Bitcoin market, providing many valuable tools for risk measurement (Ardia, Bluteau, & Rüede, 2019; Stavroyiannis, 2018; Troster, Tiwari, Shahbaz, & Macedo, 2019; Ahelegbey et al., 2021; Gao, Ye, & Guo, 2022; Jiménez, Mora-Valencia, & Perote, 2020).

Ahelegbey et al. (2021) examine the specific tail risks of cryptocurrencies and use particular measurements. Therefore, they use the extreme downside hedge (EDH) and the extreme downside correlation (EDC) methods and focus on the cryptocurrencies' relationship. They show a positive and statistically significant relationship between the tail risk of the crypto assets and the weighted average market index. Based on the EDH, they identified two groups of assets with a characteristic attribute. One group has speculative behavior like Bitcoin, EOS, and Litecoin and are "givers" of tail risk. The other group could be characterized by a professional outlook like Ethereum, Tron, and Ripple and are mainly receivers of contagion. Based on the EDC, the two groups are split up into four groups:

1) "speculative" and "diversification", e.g., Bitcoin;
2) "professional" and "complementary", e.g., Ethereum;
3) "speculative" and "complementary", e.g., EOS and Litecoin; and
4) "professional" and "diversification", e.g., Ripple and Tron.

Concerning our research question about specific tail risks of crypto assets, Bitcoin is mainly an agent of tail contagion and leads to vulnerable assets, e.g., Ethereum or Litecoin. Focusing on our empirical research, we take this classification into account. Based on the clusters, there is speculation, diversifying, complementary, and professional tail risk and backtesting results (Ahelegbey et al., 2021).

Gao et al. (2022) focus their research on VaR and expected-shortfall (ES) forecasting and modeling Bitcoin risk with a regime-switching conditional autoregressive value-at-risk (CAViaR) model. The paper is based on the empirical finding of dynamic tail risk and prior evidence that bubbles, e.g., a bubble index, contain essential information on systemic risk. Based on daily Bitcoin data between 2013 and 2021, the authors perform in-sample estimates and out-of-sample forecasts of Bitcoin returns and find out that tail risks are observable and lead to under- or overestimation. The authors construct a Markow regime-switching (MS) model with the time-varying transition probability considering asset price bubble information. The backtesting shows that the modeled impact of a bubble index leads to good VaR and ES results. Furthermore, there is some evidence of a form of regime change (Gao et al., 2022).

Jiménez et al. (2020) use different semi-nonparametric and parametric distributions, such as volatility models, for modeling Bitcoin risk. They focused on semi-nonparametric risk management, which was never used on cryptocurrency before, and compared the forecast quality with generalized autoregressive score (GAS) models and GARCH processes. The results show that the semi-nonparametric models (SNP) technique takes skewness, kurtosis, and extreme events into account and is superior to GAS and GARCH models. Although GAS and GARCH models also provide good results, the authors stated a time-consuming measurement and parametrization process. Considering model complexity, the simple semi-nonparametric approach outperforms the less flexible parametric methods (Jiménez et al., 2020).

Summing up our literature review, most papers research the downside or tail risk of crypto assets. Only Gkillas and Katsiampa (2018) use the extreme value theory in cryptocurrency. There is a wide range of different risk measurement models observable. While some researchers prefer parametric and complex risk measurement methods, others pointed out that well-parametrized simpler models outperform more time-consuming measurements. At least, none of those mentioned research uses the extreme value theory to measure the tail risk of crypto assets. Mainly, Ahelegbey et al. (2021) characterize four types of crypto assets that could explain different backtesting results of various cryptocurrencies.

2.3. Supervisory treatment of crypto assets

Considering the regulatory handling of crypto assets in financial market regulation, there are apparent difficulties. At least in the context of Pillar 1 of the Basel III framework, there are no special requirements for capital backing. Capital Requirements Regulation (CRR) III primarily defines counterparty and market risks. The latter can be subdivided into interest rate risks in the trading book, share price risks, and foreign currency risks in various financial instruments. In this context, cryptocurrency price and volatility risks cannot be assigned to market risks or specific financial instruments. It should also be added that the existing requirements for cash, commodities, or foreign exchange (FX) positions cannot reflect the volatility of crypto assets. The accounting requirements must therefore be taken into account. In line with IAS 38, cryptocurrencies are classified as intangible assets (European Central Bank [ECB], 2019). According to Article 38 of Regulation (EU) No. 575/2013, intangible assets are included as deductions from Common Equity Tier 1 capital. This equates to an risk weighted assets (RWA) weighting of 1.250% multiplied by 8% equity backing, corresponding to an estimated 100% for crypto assets as intangible assets (European Parliament and of the Council, 2013).

This also corresponds to the view of the BCBS, which uses BCBS 519 to divide crypto assets into two main categories (BCBS, 2021). Table 1 provides an overview of this.

Table 1. Overview of the prudential treatment of crypto assets

Prudential requirements	***Group 1 crypto assets***		***Group 2 crypto assets***
	Group 1a: Tokenised traditional assets	**Group 1b:** Crypto assets with stabilization mechanisms (i.e., Stablecoin)	Crypto assets that do not qualify as Group 1 (e.g., Bitcoin)
Credit and market risk requirements	Capital requirements are at least equivalent to those of traditional assets (with further consideration for capital add-ons).	New guidance on the application of current rules to capture the risks relating to stabilization mechanisms (with further consideration for capital add-ons).	New conservative prudential treatment based on a 1.250% risk weight applied to the maximum of long and short positions.
Other minimum requirements (leverage ratio, significant exposures, liquidity ratios)	Application of the existing Basel framework requirements with additional guidance where applicable.		
Supervisory review	Additional guidance to ensure that risks not captured under minimum (Pillar 1) requirements are assessed, managed, and appropriately mitigated (including through capital add-ons.		

Source: BCBS (2021).

Group 2 seems particularly relevant for the present article since this includes the typical crypto assets such as Bitcoin, Ethereum, etc. In addition to the regulatory capital deposits of Pillar 1, there are also requirements for Pillar 2. There are various risks to which banks, but also other market players, are exposed:

- market risk,
- liquidity risk,
- credit risk (especially counterparty risk),
- operational risk (including fraud and cyber threats),
- money laundering/terrorist financing risk; other,
- legal and reputation risks.

Summing up the supervisory review, there are high volatility risks, mainly focusing on market price risks.

3. DATA AND METHODOLOGY

Our research focuses on Bitcoin, Ethereum, and Litecoin because of their comparatively long time series within a minimum of 5 years overall cryptos. We use an identical time series length (from March 5, 2017 to March 4, 2022). This short time series is typical for cryptocurrencies since cryptocurrencies as an asset class for professional traders are a pretty young portfolio component in professional portfolios. In addition, only a length of 5 years is available for the most common cryptocurrencies. Of course, some of these cryptocurrencies offer a more extended time series. To compare the backtesting, we must apply our market risk measurements on an identical time series length. This leads to comparable historical observations and reveals matching results. Furthermore, the co-movement of cryptocurrencies motivates a similar length of history. Vice versa, a more extended history could provide historical observations which are not available for all cryptos and lead to advantages in backtesting. Next to the length of times series, we motivated our choice of cryptocurrencies with the research of Ahelegbey et al. (2021) and their systematization of investors. With Bitcoin as "speculative" and "diversification", Ethereum as "professional" and "complementary", and Litecoin as "speculative" and "complementary", we represent three of four investor types. Relating to the fourth characterization of "diversification" and "complementary", several data problems like structural breaks, data errors, and unexplainable price changes lead us to reduce our research on these three types, which are, in our point of view relating to the knowledge of several professional portfolios, most common in cryptocurrency trading.

Starting with the daily and 10-day returns, we perform several normal distribution tests (Kolmogorov–Smirnov test, Shapiro–Wilk test, and Anderson–Darling test) to investigate potential tail risk. The normal distribution assumption is sufficient to estimate risk with the variance-covariance approach. Therefore, we use absolute returns.

Considering risk modeling using historical simulation and the variance-covariance approach, we believe in three different confidence intervals of 95.0%, 99.0%, and 99.9%. We distinguish between a cumulative data history and a rolling data history of 365 days with the underlying data series. While the built-up data history includes more observations and can take different market phases into account, the rolling data history reacts more sensitively to short-term market changes. In stress phases, the rolling history can adapt to market changes, such as increased volatility at short notice.

In the second step, we extend our risk measurement to include extreme value theory. The extreme value theory is based on the two central convergence theorems of Fisher–Tippett and Pickands–Balkema–de Haan. The PoT method, as a method of extreme value theory, uses both of these convergence theorems (Embrechts, Klüppelberg, & Mikosch, 2003):

- The Fisher and Tippett convergence theorem describes standardized samples' convergence against the generalized extreme value distribution. After proper renormalization, the maxima of a sample of independent identically distributed (iid) random variables converge to an extreme value distribution (Gumbel distribution, Fréchet distribution, or Weibull distribution). If for a non-degenerate distribution H, the constants $c_n > 0$ and $d_n \in \mathbb{R}$ are existing, such that:

$$c_n^{-1}(M_n - b_n) \xrightarrow{d} H \tag{1}$$

holds, then H belongs to one of the three distribution types:

$$\text{Fréchet: } \Phi_\alpha(x) = \begin{cases} 0 \\ e^{-x^{-\alpha}} \end{cases} \tag{2}$$

$$\text{Weibull: } \Psi_\alpha(x) = \begin{cases} e^{-(-x)^{\alpha}} \\ 1 \end{cases} \tag{3}$$

$$\text{Gumbel: } \Lambda(x) = e^{-e^{-x}} \tag{4}$$

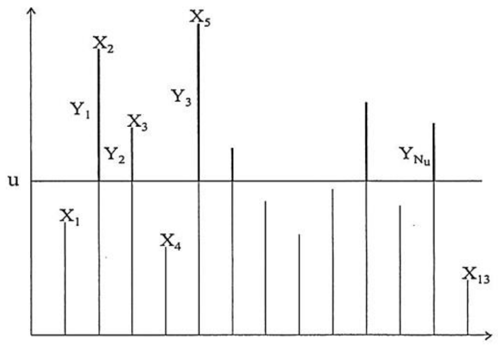
The distribution types can be summarized in the generalized extreme value distribution H_ξ by replacing the distribution parameter α with the parameter ξ:

$$H_\xi(x) = \begin{cases} e^{-(1+\xi x)^{-1/\xi}} \\ e^{-e^{-x}} \end{cases} \tag{5}$$

• The Pickands–Balkema–de Haan theorem states that the distribution of excesses above a sufficiently high threshold u that fall within the region of attraction of the generalized extreme value distribution can be approximated by the generalized Pareto distribution (GPD):

$$G_\xi(y) = \begin{cases} 1-(1+\frac{\xi y}{\beta})^{-1/\xi} \\ 1-e^{-y/\beta} \end{cases} \tag{6}$$

Figure 1. The PoT method $X_1, X_2, \ldots, X_n$ and excesses $Y_1, Y_2, \ldots, Y_{N_u}$



Source: Zeranski (2005).

The data $X_1, X_2, \ldots, X_n$ are assumed to be iid realizations of the random variable *X*. If a realization of *X* exceeds the threshold u, the realization is called exceedance, and the difference $Y_i = X_i - u$ is called excess (see Figure 1). The determination of the threshold value u faces a trade-off problem. The selected threshold should be in the upper-value range of the data. The Pickands–Balkema–de Haan theorem also shows that the GPD can approximate the desired distribution of the excesses if u is chosen large enough. On the other hand, if a high threshold is determined, there are not enough data to estimate the distribution parameters. These circumstances can lead to a very high variance in the estimate. The mean excess function (MEF) is used to determine u. The MEF is defined as the expected value of all excesses:

$$e(u) = E(X-u|X>u), \qquad u \geq 0 \tag{7}$$

For a sample of $X_1, X_2, \ldots, X_n$, the empirical MEF function is defined as:

$$e_n(u) = \frac{1}{N_u}\sum_{i\in\Delta_n(u)}^{n}(X_i - u) \tag{8}$$

where N_u is the number of excesses. The excess means value function *e(u)* of the selected functions for a progressive u can be displayed graphically in the mean excess plot (Berge, Fröhlich, & Locarek-Junge, 2006; Embrechts et al., 2003; Saeed Far & Abd. Wahab, 2016). In addition to the threshold, we need to estimate the GPD parameters ξ and β. The maximum likelihood method (ML method) for determining estimates of the GPD is the most widely used plot (Berge et al., 2006; Embrechts et al., 2003; Saeed Far & Abd. Wahab, 2016).

To determine the maximum value change under the risk probability $p \in (0,1)$, the results from the confidence level to be maintained $\alpha = 1-p$ as the p-quantile Q_p. The estimator of the p-quantile is expressed by $\hat{Q}_p$ and found by inverting:

$$\hat{Q}_p = u + \frac{\hat{\beta}}{\hat{\xi}}\left(\left(\frac{n}{N_u} * p\right)^{-\hat{\xi}} - 1\right) \tag{9}$$

With this theory, we are pursuing the goal of being able to estimate extremely rare and high-loss events. Therefore, we only consider a holding period of one day to avoid interim loss compensation through longer times and potential autocorrelation effects. It is inherent in the method that a rolling data history does not entail any advantages. The adjustment of the extreme value theory arises from determining the threshold, which also requires a sufficiently long data history. The threshold determination identifies values above the threshold so that values below are ignored, regardless of the length of the data history. We take a new threshold estimate in each observation point if statistically necessary. This forms the basis of our risk assessment.

We use the Basel traffic light approach to evaluate the risk models. The Basel traffic light ranks the number of violations of the predicted values based on the probability of the first type of error (probability that a correct model is wrongly rejected) into green, yellow and red zone. While the yellow zone indicates random outliers, a systematic error can be assumed in the red zone, requiring a model adjustment. Depending on the zone in which a model is assigned, the equity to be covered is determined with an increased multiplier. Due to the holding period of 365 days, we adjust the Basel traffic light approach by a binomial distribution and the confidence level.

4. RESULT AND DISCUSSION

4.1. Common VaR approaches

Starting with the normal distribution test of the three crypto-assets Bitcoin, Ethereum, and Litecoin, we use absolute changes for a holding period of 1 and 10 days. All normal distribution tests (Kolmogorov–Smirnov test, Shapiro–Wilk test, and Anderson–Darling test) show that the null hypothesis for the absolute 1- and 10-day returns is significant. The assumption of the normal distribution has, therefore, been rejected.

In addition to the assumption of normal distribution, we examine whether the returns are stationary over time. This is essential for the *ex-ante* representativeness of the *ex-post* data. Otherwise, the historical simulation, in particular, is incorrectly specified since the development of the historical data is subject to trends and *ex-ante* risk forecasts are likely to be distorted. For this purpose, an augmented Dickey-Fuller test was carried out. The null hypothesis of a non-stationary process was confirmed for absolute 1-day and 10-day returns. These lead to trend-affected processes.

Concerning the basic description of the underlying distribution and the stationarity of the time series, it can first be stated that the rejection of the normal distribution hypothesis means that the variance-covariance approach may have a poorer forecast quality.

Starting with the backtesting, a 1-day holding period and absolute returns for all tested confidence intervals of 95.0%, 99.0%, and 99.9% show an overall poor forecast quality for all crypto assets. The model violations of the red area are sometimes over 20.0% or 30.0%. Except for the 99.9% confidence level, the violations are partly higher. In most observations, Ethereum and Litecoin show a better forecast quality in the variance-covariance approach than in the historical simulation. Although the assumption of normal distribution had to be negated, the assumption of normal distribution seems to lead to better forecast results than the historical distribution of the historical simulation. It can be subsumed that the historical observations result in a trend that immediately harms the quality of the forecast and, concerning the impacts, superimposes the strong assumption of normal distribution.

Concerning a generally better forecast quality under the assumption of normal distribution, this cannot be maintained with a 10-day holding period. The best forecast quality is shown with a confidence interval of 95.0%.

In distinguishing between cumulative and rolling history, we show better backtesting results for the rolling history with a 1-day holding period. Only with Litecoin can this statement not be kept. However, it should be pointed out that Litecoin requires the best forecast quality in backtesting. In contrast, the forecast quality for Bitcoin is worst suited. Interestingly, Litecoin has a significantly better forecast quality for a 1-day than a 10-day holding period.

In summary, it can also be stated that none of the VaR show sufficiently conservative forecast results that are appropriate from a regulatory point of view.

Table 2. Forecast quality of historical simulation and variance-covariance approach within a 95.0% confidence level

95.0% confidence level		*1-day holding period*				*10-day holding period*			
		Historical simulation		*Variance-covariance*		*Historical simulation*		*Variance-covariance*	
		Cum.	*Rolling*	*Cum.*	*Rolling*	*Cum.*	*Rolling*	*Cum.*	*Rolling*
Bitcoin	green	65.7%	68.2%	66.1%	71.3%	68.4%	69.9%	70.9%	70.1%
	yellow	2.6%	11.2%	4.7%	18.5%	3.0%	15.5%	2.4%	16.2%
	red	31.8%	20.5%	29.3%	10.2%	28.5%	14.5%	26.7%	13.7%
Ethereum	green	65.8%	61.6%	68.8%	66.5%	73.7%	61.7%	74.0%	80.3%
	yellow	2.6%	6.6%	4.7%	18.9%	1.9%	16.1%	2.5%	19.7%
	red	31.6%	31.8%	26.5%	14.6%	24.4%	22.2%	23.6%	0.0%
Litecoin	green	73.3%	72.7%	100.0%	79.0%	73.2%	73.2%	78.8%	75.4%
	yellow	18.3%	11.8%	0.0%	21.0%	2.1%	12.1%	21.2%	10.9%
	red	8.4%	15.5%	0.0%	0.0%	24.7%	14.6%	0.0%	13.6%

Source: Authors' calculations.

Table 3. Forecast quality of historical simulation, variance-covariance approach, and extreme value theory within a 99.0% confidence level

99.0% confidence level		*1-day holding period*				*Extreme value theory*	*10-day holding period*			
		Historical simulation		*Variance-covariance*			*Historical simulation*		*Variance-covariance*	
		Cum.	*Rolling*	*Cum.*	*Rolling*		*Cum.*	*Rolling*	*Cum.*	*Rolling*
Bitcoin	green	62.7%	71.8%	62.2%	61.9%	62.8%	64.4%	43.6%	38.5%	43.3%
	yellow	4.2%	11.1%	2.4%	2.4%	6.2%	3.8%	26.7%	27.4%	18.7%
	red	33.1%	17.1%	35.4%	35.8%	31.0%	31.8%	29.7%	34.1%	38.0%
Ethereum	green	65.7%	51.9%	63.7%	63.7%	73.0%	66.0%	43.8%	66.0%	43.4%
	yellow	4.0%	21.7%	2.4%	2.4%	2.5%	5.2%	39.1%	2.1%	30.3%
	red	30.3%	26.4%	33.9%	33.9%	24.5%	28.9%	17.1%	31.9%	26.3%
Litecoin	green	74.7%	70.6%	67.6%	67.6%	83.6%	74.9%	46.3%	74.8%	65.4%
	yellow	4.1%	15.2%	7.8%	7.8%	16.4%	1.1%	36.6%	1.0%	5.1%
	red	21.2%	14.1%	24.6%	24.6%	0.0%	24.0%	17.1%	24.2%	29.5%

Source: Authors' calculations.

4.2. Extreme-value-theory-based VaR

Starting with the risk measurement of the extreme value theory, the comparison must first be adjusted. Due to the aim of using the extreme value theory to forecast high and very rare claims, the 95.0% confidence level is unnecessary. For better comparability, the 10-day holding period is also omitted. In this way, no interim loss compensation can be considered to reduce risk. Furthermore, the extreme-value-theory-based VaR is only calculated with a growing history since a data history is required to determine the threshold value. The "omission" of data points cannot lead to a higher dynamic than the other VaR methods.

With a confidence level of 99.0%, Bitcoin shows the best risk measurement with the historical simulation and a rolling 1-year history. With 17.1% in the red zone, the rolling historical simulation has the best risk measurement. This finding also continues with Ethereum, although the rolling historical simulation only reveals slight advantages in comparing the alternative measurements. In the case of Ethereum, the extreme value theory shows a comparable forecast result. With the addition of the yellow and the red regions, the extreme value theory offers the best results. Hence, this approach shows a significantly better forecast quality for Litecoin than other methods.

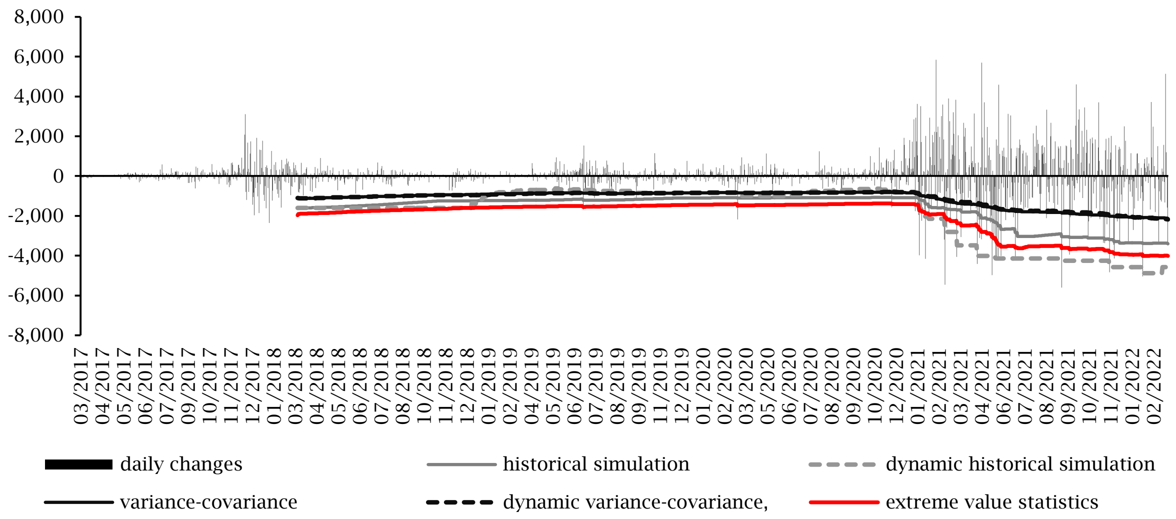
Table 4. Forecast quality of historical simulation, variance-covariance approach, and extreme value theory within a 99.9% confidence level

99.9% confidence level		*1-day holding period*				*Extreme value theory*	*10-day holding period*			
		Historical simulation		*Variance-covariance*			*Historical simulation*		*Variance-covariance*	
		Cum.	*Rolling*	*Cum.*	*Rolling*		*Cum.*	*Rolling*	*Cum.*	*Rolling*
Bitcoin	green	37.5%	11.8%	10.4%	6.7%	75.6%	64.4%	43.6%	38.5%	43.3%
	yellow	33.2%	66.9%	51.4%	31.1%	24.4%	3.8%	26.7%	27.4%	18.7%
	red	29.3%	21.4%	38.2%	62.2%	0.0%	31.8%	29.7%	34.1%	38.0%
Ethereum	green	61.9%	24.1%	61.9%	18.5%	75.1%	66.0%	43.8%	66.0%	43.4%
	yellow	13.4%	54.4%	0.9%	15.1%	24.9%	5.2%	39.1%	2.1%	30.3%
	red	24.7%	21.5%	37.2%	66.3%	0.0%	28.9%	17.1%	31.9%	26.3%
Litecoin	green	72.8%	39.6%	61.9%	8.3%	75.6%	74.9%	46.3%	74.8%	65.4%
	yellow	5.7%	38.3%	8.9%	33.7%	24.4%	1.1%	36.6%	1.0%	5.1%
	red	21.5%	22.1%	29.3%	58.0%	0.0%	24.0%	17.1%	24.2%	29.5%

Source: Authors' calculations.

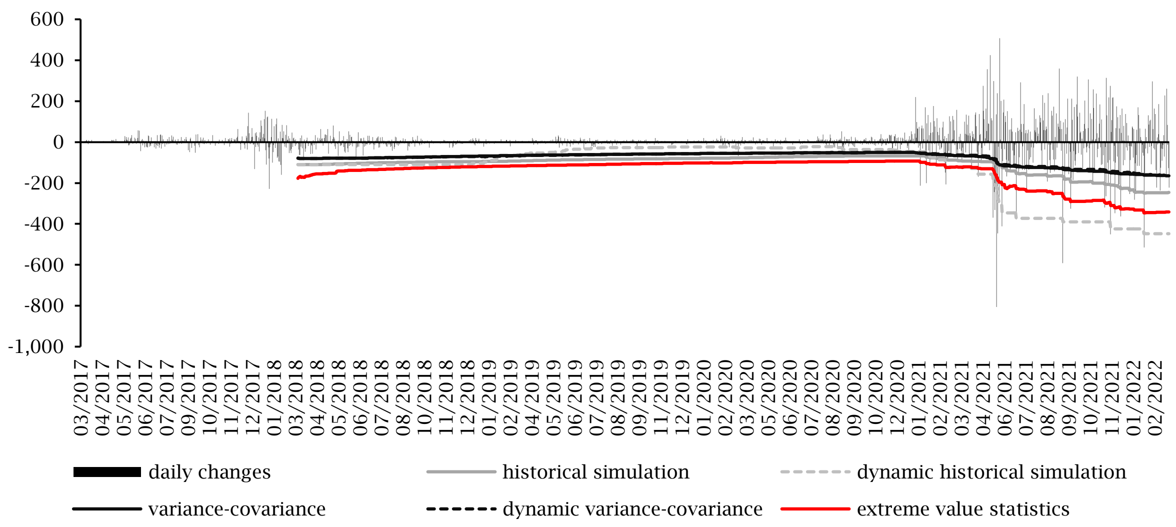
Nevertheless, there are no adequate backtesting results for any crypto assets considered. Regarding the temporal distribution of the backtesting outliers, it is obvious that the forecasts become poor from January to March 2021. In this way, 90% and more of the outliers in the observation period from January 2021 can be located across all risk measurement methods. This also goes hand in hand with the general observation that crypto assets have a highly volatile market phase from this time onwards.

Figure 2. Bitcoin: Comparison of risk measurement methods (99.0% confidence level, 1-day holding period)



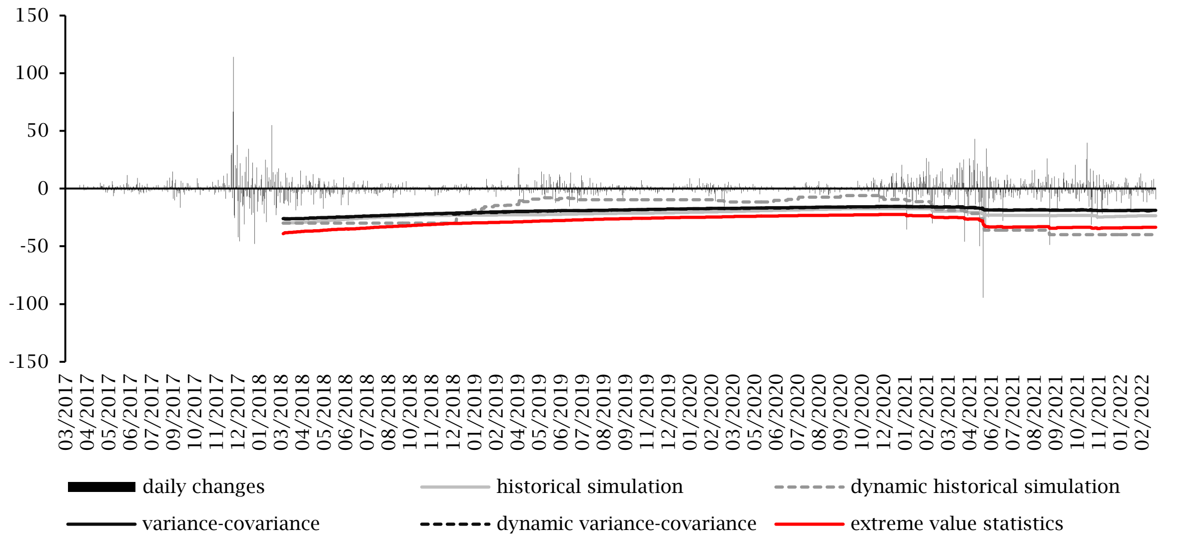
Source: Authors' calculations.

Figure 3. Ethereum: Comparison of risk measurement methods (99.0% confidence level, 1-day holding period)



Source: Authors' calculations.

Figure 4. Litecoin: Comparison of risk measurement methods (99.0% confidence level, 1-day holding period)



Source: Authors' calculations.

Using a confidence level of 99.9% shows an improvement in the forecast quality compared to a confidence level of 99.0%. It is also noticeable that there are no longer any outliers in the red area, which means that an overall satisfactory risk measurement can be attested. The extreme value theory thus also shows clear advantages in backtesting compared to historical simulation and the variance-covariance approach.

Due to the significant improvement in the forecast quality, it can be summarized that the risk measurement of crypto assets, especially in high confidence intervals, appears to be sufficiently conservative and represents a suitable measurement approach. Nevertheless, the historical simulation and the variance-covariance method do not appear to be entirely suitable for adequately measuring cryptocurrency risks.

Table 5. Backtesting violations within a 99.0% confidence level and a 1-day holding period

99.0% confidence level		*Historical simulation*		*Variance-covariance*		*Extreme value theory*
		Cumulative	*Rolling*	*Cumulative*	*Rolling*	
Bitcoin	total	41	22	69	73	30
	since January 2021	38	13	62	65	27
Ethereum	total	39	29	67	70	22
	since January 2021	39	19	63	66	22
Litecoin	total	15	20	22	22	7
	since January 2021	15	12	22	22	7

Source: Authors' calculations.

Table 6. Backtesting violations within a 99.9% confidence level and a 1-day holding period

99.9% confidence level		*Historical simulation*		*Variance-covariance*		*Extreme value theory*
		Cumulative	*Rolling*	*Cumulative*	*Rolling*	
Bitcoin	total	7	7	45	21	1
	since January 2021	6	4	42	14	1
Ethereum	total	7	6	46	29	2
	since January 2021	7	3	46	19	2
Litecoin	total	3	5	13	19	1
	since January 2021	3	4	13	13	1

Source: Authors' calculations.

Bitcoin tends to show the worst backtesting results, while Litecoin shows the best backtesting results in the two classic methods. Referring to Ahelegbey et al. (2021), Bitcoin can be classified as "speculative" and "diversification". It can be concluded from this that it generates good efficiency advantages, especially in the diversification function. This also goes hand in hand with further research that highlights the diversification benefits of Bitcoin. This is not opposed to the fact that Bitcoin has the worst singular risk measurement in backtesting. It is more astonishing that Litecoin as "speculative" and "complementary" shows better backtesting than Ethereum as "professional" and "complementary". A possible explanation is that the "speculative" Litecoin went through a more pronounced volatile market phase at the beginning of 2018 and thus caused more conservative input parameters. In contrast, volatility was less noticeable for Ethereum in early 2018.

Price jumps and volatility of many crypto assets are extreme. Price increases, like in the past with Bitcoin, often have a self-reinforcing effect. Falling prices, such as Bitcoin and Ethereum recently, are not necessarily a good time to invest. Because further course development is not foreseeable, as our analysis showed. For example, in the case of crypto assets, where the number or total value of the coins or tokens on the market is relatively low, there may be very few buyers and no trading opportunities (Wei, 2018; King & Koutmos, 2021). Much information on crypto assets comes from data quality and completeness sources that are difficult to verify.

This dynamic price and volatility development is also reflected in the recent macroeconomic crises such as the war in Ukraine and the COVID-19 pandemic. While the volatility for Litecoin increased at the time of the outbreak and can also be described as relatively stable in the further course of the year, the volatility level for Bitcoin and Ethereum has increased significantly, especially in 2021. In this respect, it can be generally assumed that the market-wide COVID-19 event, in particular, did not have the same effect on cryptos. In contrast, there were significantly increased risks in 2021, which are also reflected in a significant deterioration in the backtesting results. However, it should be noted that these are not attributable to general market developments and influences. The price dynamics for Litecoin are significantly lower. Hence, the extreme value theory can also be motivated by market development since this can best reflect the general market development and dynamically triggered level shifts of volatility and price changes.

5. CONCLUSION

In summary, it can be said that the historical simulation and the variance-covariance method do not appear to be suitable for measuring the risk of crypto assets. On the one hand, this is due to the lack of stationarity in the time series. On the other hand, the normal distribution as the central assumption of the variance-covariance method is not fulfilled either. The unsuitability of the two measurement approaches is particularly evident in a highly volatile market phase from January 2021 since almost all outliers are recorded during this time.

The first research question *"Are the common VaR approaches an adequate measurement approach?"* is to be answered negatively.

Concerning the second research question *"Does an extreme-value-theory-based VaR provide a better fit to the statistical characteristics of crypto assets?"*, it has to be answered differentiated. Thus, the research question cannot be confirmed for a confidence interval of 99.0%. For a confidence interval of 99.9%, the extreme value theory shows significantly better backtesting so that the hypothesis can be confirmed here.

Regarding the research limitations, the available variety of risk measurement and backtesting methods should be pointed out. Other goodness-of-fit measures or adjustment tests are as conceivable as risk measurement with Monte Carlo, Copula, PoT, etc. Concerning our investigation, the 1-day holding period, in particular, should be named as a research limitation. While a more extended holding period has sometimes prevailed in practice, the extreme value theory method shows weaknesses with a more extended holding period depending on the perspective.

It is also noticeable here that the three tested cryptocurrencies do not draw completely uniform backtesting results. On the one hand, this may lie in their specific characteristics. On the other hand, the work of Ahelegbey et al. (2021) can also be taken up, according to which the various assets have different specifications. It can be subsumed that "professional" or less "speculative" crypto assets could tend to include less extreme loss events in the time series. Poorer results in this respect accompany this.

So, it is necessary to adjust the risk measurement depending on the focused currency. In particular, the lack of stationarity and the lack of loss events in the available history should be taken into account. In particular, the extreme value theory can adequately depict the latter property.

Our paper shows that extreme value theory can make a valuable contribution to risk measurement, particularly in the case of phases of high volatility or dynamic price trends that are still unknown or cannot be derived historically. Along with this, we first compare the one-day changes in backtesting for the study.

This also offers numerous starting points for further research. In particular, against the background of different investor types or crypto properties, a different approach to risk measurement can offer efficiency advantages. Furthermore, it will be investigated how herding behavior can be included in risk measurement using the extreme value theory. This also impacts other risk measurements of different financial instruments since this is where the strength of the extreme value theory is generally shown.

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