

ASSET CORRELATION, PORTFOLIO DIVERSIFICATION AND REGULATORY CAPITAL IN THE BASEL CAPITAL ACCORD

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Abstract

In this paper, we analyze the properties of the KMV model of credit portfolio loss. This theoretical model constitutes the cornerstone of Basel II's Internal Ratings Based (IRB) approach to regulatory capital. Our results show that this model tends to overestimate the probability of portfolio loss when the probability of default of a single firm and the firms' asset correlations are low. On the contrary, probabilities of portfolio loss are underestimated when the probability of default of a single firm and asset correlations are high. Moreover, the relationship between asset correlation and probability of loan portfolio loss is only consistent at very high quantiles of the portfolio loss distribution. These are precisely those adopted by the Basel II Capital Accord for the calculations of capital adequacy provisions. So, although the counterintuitive properties of the KMV model do not extend to Basel II, they do restrict its generality as a model of credit portfolio loss.

Keywords: Assets, Basel Accord, Regulatory Capital

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1 Introduction

The idea that regulatory capital requirements should be risk sensitive is at the core of the second Basel Capital Accord (Basel II), BIS (2005a). Even the Basel Committee's current proposals for reform of the Capital Accord (Basel III), which were prompted by the 2008 Credit Crunch, are firmly built on the risk-sensitive framework of Basel II (See BIS, 2009a,b). At the conceptual level there clearly is widespread support for the idea of risk-based capital provisions. However, in order to move this support from the conceptual to the practical level, it is essential that capital provisions accurately reflect credit risk.

The Basel II's Internal Ratings Based (IRB) framework of capital adequacy was built on a credit risk model developed by the KMV Corporation, which was acquired by Moody's in 2002. The KMV model is an extension of Merton (1974) to credit risk (Vasicek, 1987), and more importantly, to loan portfolio risk (Vasicek, 2002). A significant issue in credit risk analysis is how default and asset correlations are taken into account. It is generally accepted that the overall risk of a portfolio can be reduced by diversifying its assets either sectorally or geographically. There is growing evidence that the same principle applies to credit portfolios. Griffith-

Jones et al. (2002a) and Griffith-Jones et al. (2002b), for instance, suggest that the overall risk of a geographically diversified portfolio is lower than that of a geographically concentrated portfolio. García (2002), García et al. (2006), show that the credit risk of a portfolio based on a two-factor model is lower than that of a single-factor portfolio. Tasche (2005) generalised this result to a multi-factor setting. So far, the theoretical and empirical research has taken two main directions. The first approach consists of the empirical estimation of asset correlations and default correlations (Dietsch and Petey, 2004; D'ullmann and Scheule, 2003; Erlenmaier and Gerbach, 2001; Servigny and Renault, 2002). Other papers focus on the theoretical result of the main credit risk model showing that lower asset correlations imply lower default correlations (García, 2002; García et al., 2006; Tasche, 2005, amongst others).

In this paper, we analyse the properties of the KMV model of credit portfolio loss which constitutes the cornerstone of the IRB approach to regulatory capital. We find that the KMV model represents the probability of portfolio losses very poorly at low p , and low p . More specifically, it tends to overestimate the probability of portfolio loss when the probability of default of a single firm and the firms' asset correlations are low. On the contrary, probabilities of

portfolio loss are underestimated when the probability of default of a single firm and asset correlations are high.

The paper is organised as follows. After this Introduction, Section 2 the KMV model is presented and its properties analysed. In Section 3 special emphasis is placed on the relationship between the KMV model and Basel II's IRB approach to regulatory capital provisions. Section 4 concludes.

2 Latent variable models and Basel II's IRB Approach

The theoretical foundations of the IRB approach of the 2005 Basel Capital Accord was first developed by KMV Corporation² as an extension of Merton (1974)'s model of corporate debt pricing, and was later published in Vasicek (1987, 1991, 2002). This model belongs to the class of latent variable models and one of its results is a factor representation of the determinants of individual default. The main property of this factor representation is the independence of individual defaults relative to each other, given the occurrence of the determinants of individual defaults. The KMV model is thus considered as an example of conditionally independent credit risk models. However, as we will show below, it does not adequately capture dependencies between individual default probabilities, and that this failure extends to Basel II's IRB approach.

2.1 KMV model

The main objective of the KMV model is the derivation of the probability distribution function of the loss of a portfolio of loans. The model first derives the probability distribution function of a single default. Single loans are then aggregated into a portfolio of loans, and assumptions concerning default correlations are made at this stage. Finally, Vasicek (2002) presents Monte Carlo simulations of the limiting distribution function of portfolio loss distribution function.

It is worth emphasizing at this stage that we are modeling corporate rather than retail (consumer credit) default, and that we take the perspective of the borrowing firms rather than that of the lender when using the term "assets". In the literature on financial regulation or in the documents published by the Basel Committee on Banking Supervision, assets usually refer to loans, which are the firms' liabilities. Throughout this paper, banking loans will always be referred to as debts or liabilities.

Assumptions

A portfolio consists of M loans of equal amounts, one loan per firm. For each firm $i = 1, \dots, M$ the following assumptions hold

1. Default occurs when the value of a firm's assets fall below the value of its debt, at maturity of its

debt T_i . Formally, $A_i T_i < D_i$, where $A_i T_i$ represents the value of firm i 's assets at the loan maturity time T_i and D_i the value of firm i 's liabilities. $A_i T_i$ is the latent variable of the model.

2. The value of a firm's assets is described by the stochastic differential equation $dA_i = A_i(\mu_i dt + \sigma_i dx_{it})$ where x_{it} is a standard Brownian Motion. Moreover, $E[dx_{it}]^{18} = dt$, and $E[dx_{it}][dx_{jt}] = \rho dt$ for $i \neq j$. μ_i and σ_i are constants. They may be interpreted as the drift and the volatility of the asset value of firm i , respectively. Finally, a Brownian motion has a Normal distribution with mean 0 and variance dt .
3. (Portfolio homogeneity) For the sake of simplicity, Vasicek (1987, 1991, 2002) assumes that all borrowers are identical. This implies that (i) all loans have the same maturity, $T_i = T_j = T$, $i, j = 1, \dots, M$, (ii) asset correlations are identical, $\rho_i = \rho_j = \rho$ (iii) debt values D_i are identical, $D_i = D_j = D$.

In the Vasicek setting, the point in time where the occurrence of default is considered is the maturity of the debt, T_i in Assumption (1). In Assumption (2), ρ represents the two-by-two correlation of borrowing firms' assets, but not necessarily default correlation. Default correlation and asset correlation can differ significantly, as shown in Schönbucher (2000), Zhou (2001), and mainly, Frey et al. (2001). Finally, it should be emphasized that asset correlation is exogenous in the KMV model. It is assumed to exist but its value is not obtained from the model. As a result, it may take any arbitrary value. In a consultative paper published by the Basel Committee in 1999, the asset correlation was set to 20%. Theoretically, ρ is the usual correlation formula

¹⁸ Kealhofer, McQuown, Vasicek Development, L. P.

$$p = \text{Corr}[Y_{1T}, Y_{2T}] = \frac{\text{Cov}[Y_{1T}, Y_{2T}]}{\sqrt{\text{VAR}[Y_{1T}]}\sqrt{\text{VAR}[Y_{2T}]}} \quad (1)$$

where $\text{Cov}[Y_{1T}, Y_{2T}]$ is the covariance of Y_{1T} and Y_{2T} and the square root of $\text{VAR}[Y_{1T}]$ is its standard deviation. The covariance is given by

$$\text{Cov}[Y_{1T}, Y_{2T}] = E[Y_{1T} * Y_{2T}] - E[Y_{1T}] * E[Y_{2T}] \quad (2)$$

where $E[]$ is the expectation operator.

Given the assumptions above, let Y_{iT} be an indicator variable such that $Y_{iT} = 1$ if firm i defaults

at time T and $Y_{iT} = 0$ if firm i does not default at time $t = T$. Then, the probability of default of a single firm is given by

$$P[Y_{iT} = 1] \Leftrightarrow P[A_{iT} < D_i] \quad (3)$$

A_{iT} is obtained by solving the stochastic differential equation in Assumption (2) above,

$$\text{Log}(A_{iT}) = \text{Log}(A_0) + \mu_i T - \frac{1}{2} \sigma_i^2 T + \sigma_i \sqrt{T} X_{it} \quad (4)$$

where A_0 is the value of the firm's assets at time 0, T is maturity time and X_{it} is a variable with Normal (0,T), i.e., with mean 0 and variance T .

Using (4), we can write (3) as

$$p_i = P[\text{log}(A_{iT}) < \text{log}(D_i)] \quad (5)$$

where p_i is the probability of individual default.

$$p_i = P\left[\text{log}(A_0) + \mu_i T - \frac{1}{2} \sigma_i^2 T + \sigma_i \sqrt{T} X_{it} < \text{log}(D_i)\right] \quad (6)$$

The only random element in the left-hand side of this inequality is X_{it} . So we can re-write this expression as

$$p_i = P\left[X_{it} \frac{\text{log}(D_i) - \text{log}(A_0) - \mu_i T + \frac{1}{2} \sigma_i^2 T}{\sigma_i \sqrt{T}}\right] \quad (7)$$

Since X_{it} is a Normal distribution with mean 0 and variance 1, the expression above becomes

$$p_i = \Phi(DD_i) \quad (8)$$

where $DD_i = \frac{\text{log}(D_i) - \text{log}(A_0) - \mu_i T + \frac{1}{2} \sigma_i^2 T}{\sigma_i \sqrt{T}}$ and Φ is the cumulative normal distribution function. DD_i is the "distance-to-default" of firm i (Merton, 1974; Vasicek, 1987, 1991, 2002).

Expression (8) clearly shows that the probability of default of a single firm i does not depend in any way on the probability of default of a single firm j , since the two-by-two correlation of firms' assets, p , does not appear in it. Asset correlations can be endogenized by assuming that the asset value of a single firm follows a multidimensional Brownian motion. In Assumption (2) above the stochastic differential equation describing the asset value becomes $dA_i = A_i(\mu_i dt + \sum_{j=1}^M \sigma_j dx_{jt})$. Recent papers have pursued this direction, for instance, Kafetzaki-Boulamatsis and Tasche (2001), and Nyfeler (2000). Although this approach is conceptually more adequate for modelling joint defaults or default dependencies, it suffers from a major shortcoming. Estimating the asset correlation matrix is practically impossible (see Gottschalk, 2011, for details).

2.2 Probability of loan portfolio loss

From result (8), Vasicek (2002) proceeds to derive the probability of the loss of a loan portfolio. Let L_i denote the gross loss on the i -th loan. The gross loss represents the loss before recoveries. $L_i = 1$ if the i -th firm defaults and $L_i = 0$ if the i -th firm does not default. Let L be the portfolio percentage gross loss, defined as the weighted sum of each individual portfolio percentage gross loss,

$$L = \sum_{i=1}^M L_i \quad (9)$$

where M is the total number of loans in the portfolio. We wish to calculate the probability of n defaults out of the M loans

$$P_n = P\left[L = \frac{n}{M}\right] \quad (10)$$

for $n = 1, \dots, M$. From Assumption 2, each firm's asset follows a Brownian motion, and the two-by-two correlations are identical, i.e., $\rho_{ij} = \rho_{uv} = \rho$, for any firm $i \neq j, u \neq v, \dots, M$. This implies that the Brownian motion variables X_{it} are jointly equicorrelated standard normal variables. A property of this type of probability distribution is the representation

$$X_{it} = \sqrt{p}F_t + \sqrt{1-p}Z_{it} \quad (11)$$

where, F_t and Z_{1t}, \dots, Z_{Mt} are mutually independent standard normal distributions. (11) is a factor representation, the factor being F_t . Vasicek (1991) explicitly points out that expression (11) derives necessarily from the assumption of normality of asset returns, which in turn is a necessary outcome of the hypothesis that asset returns follow a Brownian motion. In many subsequent papers in the literature, (11) is presented as the starting point of the KMV credit risk model, with the assumption of normality replaced by the more convenient hypothesis that the factor F_t follows a Student's t distribution.¹⁹ Vasicek (2002) suggested that F_t can be interpreted as a common macroeconomic factor affecting the whole portfolio of loans. Each firm's sensitivity to this factor is given by $\sqrt{p}F_t$. $\sqrt{1-p}Z_{it}$ stands for the firm's specific risk.

In order to evaluate the probability of n defaults in the portfolio, it is necessary to determine the number of possible combinations of n individual defaults in a portfolio of M loans. Since individual defaults are independent given the occurrence of the factor F_t ,²⁰ the number of possible combinations of defaults is given by the Binomial factor $\binom{M}{n}$. Moreover, we now assume for the sake of simplicity, and following Vasicek (1991, 2002), that the portfolio is homogeneous. This implies that individual probabilities of default are identical, as are the distances-to-default DD and as before the maturities of the debts.

By the law of iterated expectations, the probability of having exactly n defaults is the average of the conditional probabilities of n defaults, averaged over the possible realizations of F_t and weighted by the probability density function of F_t evaluated at u ,

$$P[X = n] = P[X = n | F = u] = \int_{-\infty}^{\infty} \binom{M}{n} (p(u))^n (1-p(u))^{M-n} \phi(u) dF \quad (12)$$

Once the individual defaults can be assumed to occur independently, the Vasicek model reduces to a Binomial model of default.

From (8), $p(u)$ is given by

$$p(u) = P[X_T < |DD|F = u] \quad (13)$$

Substituting X_T in factor representation gives

$$p(u) = P[\sqrt{p}F_t + \sqrt{1-p}Z_{it} < DD | F_t = u] \quad (14)$$

Re-arranging the terms in the left-hand side of the inequality, we obtain

$$p(u) = P\left[Z_{it} < \frac{DD - \sqrt{p}F_t}{\sqrt{1-p}} \mid F_t = u\right] \quad (15)$$

As was seen above, since $Z_{it}, i = 1, \dots, M$, is Normally distributed

$$p(u) = \Phi\left(\frac{DD - \sqrt{p}F_t}{\sqrt{1-p}}\right) \quad (16)$$

Note that we have replaced F_t by its value u . Substituting (16) in (12) yields the probability of n defaults in the portfolio

$$p[L = n] = \int_{-\infty}^{\infty} \binom{M}{n} \left(\Phi\left(\frac{DD - \sqrt{p}F_t}{\sqrt{1-p}}\right)\right)^n \left(1 - \Phi\left(\frac{DD - \sqrt{p}F_t}{\sqrt{1-p}}\right)\right)^{M-n} \quad (17)$$

A limiting distribution of portfolio loss can be obtained by assuming the number of loans in the portfolio tends to infinity. If we maintain the assumptions of homogeneity, $L = \frac{1}{M} \sum_{i=1}^M L_i$ now becomes the fraction of defaulted loans in the portfolio. By the law of large numbers, the fraction of defaults is (almost surely) equal to the individual default probability, $P\left[\frac{1}{M} \sum_{i=1}^M L_i = p(u) \mid F = u\right] = 1$.

¹⁹ The probability of extreme events is higher in the Student's t distribution than in the Normal distribution. Distributions with higher probabilities of extreme events capture more adequately the empirical distributions of financial variables.

²⁰ See Gottschalk (2011) for proof.

$$P[L < x] = E[P(L \leq x | F = u)] = \int_{-\infty}^{\infty} P[p(u) \leq x] \phi(u) du \quad (17)$$

The cumulative distribution function of the fraction L portfolio loss is thus

$$F_{\infty}(x) = P(L \leq x) = \Phi\left(\frac{\sqrt{p} \Phi^{-1}(x) - \Phi^{-1}(p)}{\sqrt{p}}\right) \quad (19)$$

The notation $F_{\infty}(x)$ is used to emphasize that this distribution is valid only when the number of loans in the portfolio becomes infinitely large. The α -percentile of (19), denoted x_{α} , is given by inverting $F(x_{\alpha}) = \alpha$. The α -percentile of the loss distribution, $x_{\alpha}(L) \equiv q_{\alpha}(L)$ is thus

$$q_{\alpha}(L) = \Phi\left(\frac{\Phi^{-1}(p) + \sqrt{p} \Phi^{-1}(\alpha)}{\sqrt{1-p}}\right) \quad (20)$$

Finally, the derivative of (19) with respect to x gives the density distribution function

$$f_{\infty}(x) = \sqrt{\frac{1-p}{p}} \exp\left\{\frac{1}{2p} \left(\Phi^{-1}(p) - \sqrt{1-p} \Phi^{-1}(x)\right)^2\right\} - \frac{1}{2p} \left(\Phi^{-1}(x)\right)^2 \quad (21)$$

where $\exp(\cdot) \equiv e^{(\cdot)}$ is the exponential function.

Vasicek (2002) and Gordy (2003) show that (20) is also valid when the weights of single loans in the portfolio are allowed to differ, i.e., when $L = \frac{1}{M} \sum_{i=1}^M \omega_i L_i$, where $\sum_{i=1}^M \omega_i = 1$. However, a necessary and sufficient condition is that no single loan may dominate the portfolio, which implies that $\sum_{i=1}^M \omega_i \rightarrow 0$. This result is particularly important in the light of Basel II's formulae to calculate regulatory capital.

The properties of the cumulative distribution function of portfolio loss (19) are summarized in Vasicek (2002), and a couple illustrative plots are presented in Schönbucher (2000) and García (2002). A more useful reference is Bluhm et al. (2003), where the properties of (19) are more thoroughly described. When $\rho \rightarrow 0$, $F_{\infty}(x; p; \rho)$ converges to a one-point distribution concentrated at $L = p$. When $\rho \rightarrow 1$ the distribution flattens and converges to a zero-one distribution with probabilities ρ and $1 - \rho$.

and 91%. The fixed parameter is the individual probability of default p . It is worth remembering at this stage that (19) hinges on the assumption that all the firms in the portfolio have the same probability of default.

Figure 1 clearly shows that for a probability of default equal to 1% $F_{\infty}(x; p; \rho)$ always collapses to p , when asset correlations are quite low (1% to 41%). Figure 1 also shows that at low p and low ρ , the probability of any fraction of the portfolio defaulting is 100%, irrespective of the level of asset correlation. This is quite counterintuitive since one would expect the probability of portfolio loss to be low when the probability of individual default and the asset correlations are low.

2.3 Is the KMV model adequate for modeling the probability of portfolio losses?

In Figures 1 and 2 we simulate (19) to illustrate some of these properties.²¹ In all figures, the left-hand graph shows $F_{\infty}(x; p; \rho)$ for asset correlations between 1% and 41%. The right-hand side graph plots $F_{\infty}(x; p; \rho)$ for asset correlations between 51%

²¹ Results for other values of p and ρ can be found in Gottschalk (2011)

Figure 1. Probability of portfolio loss - $p=1\%$

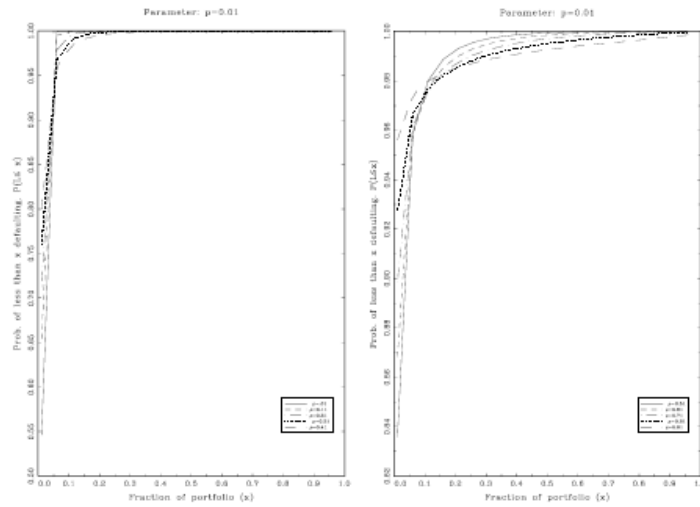
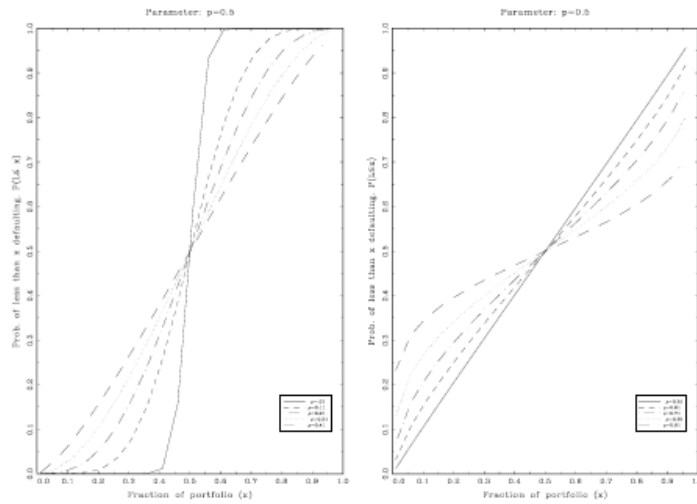


Figure 2. Probability of portfolio loss - $p=50\%$



The KMV model performs a bit better at higher levels of asset correlation (40% to 91%). Portfolio default probabilities are more spread out, and more dependent on the level of asset correlation. Nonetheless, it is evident from the two figures that the KMV model represents the probability of portfolio losses very poorly at low p , and low ρ . This fact was pointed out by Schonbucher (2000), and Bluhm et al. (2003), amongst others.

In Figure 2 the probability of portfolio loss when the probability of default of a single firm is now $p=50\%$. The left-hand side figure shows a more interesting result. When the asset correlation is 1%, $\rho = 0.1$, up to 40% of the firms in the portfolio do not default, even though the individual probabilities of default are quite high. The fraction of the portfolio that does not default obviously decreases inversely with asset correlation. When asset correlation is 41%, all the portfolio defaults.

3 The KMV model and regulatory capital

The IRB approach assumes heterogeneous portfolios of loans. This implies that each borrower may have a distinct probability of default p_i , each loan has a distinct maturity M_i , the weight of each loan in the portfolio is different ω_i , and that the percent loss on each loan can be different, s_i . In the Basel Committee's publications, s_i is referred to as the loss given default, and is equal to 1 minus the recovery rate. For the sake of simplicity, it is assumed in the KMV model that the loss is total, so $s_i = 1$, for all loans in the portfolio.

According to the rules of Basel II, regulatory capital is needed only to cover unexpected losses, given that banks are supposed to cover for expected losses as part of their on-going activities. Regulatory capital is thus given by $K(L) = q_\alpha(L) - E(L)$, where $E(L) = \sum_{i=1}^M s_i p_i$

and $q_\alpha(L)$ is given by expression (20) in the main text, factored by the loss given default s_i .

$$K(L) = \sum_{i=1}^M s_i \left[\Phi \left(\frac{\Phi^{-1}(\rho) + \sqrt{\rho} \Phi^{-1}(\alpha)}{\sqrt{1-\rho}} \right) - \rho_i \right] * m_i \quad (22)$$

To take into account the impacts of differing maturities on credit risk, the IRB rules require that the following expression be multiplied to the regulatory capital formula (22)

$$m_i = \frac{1 + (M - 2.5) * b(\rho_i)}{1 - 1.5 * b(\rho_i)} \quad (23)$$

where

$b(\rho_i) = (0.11852 - 0.05478 * \log(\rho_i))^2$. M is the maturity of the loan for which regulatory capital is being calculated, and ρ_i is the borrower's probability of default. In the KMV context, ρ_i is given by (3)²²

It is clear from (22) that regulatory capital in the IRB approach is fundamentally the α -percentile of the asymptotic cumulative distribution function of the portfolio loss in the KMV model, (20). In the IRB approach, α is set to 99.99 percent. The inclusion of asset correlation in (22) raises the issue of portfolio diversification on the IRB approach. A growing number of papers throw some light on the relationship between the KMV model and Basel II's IRB approach. Amongst those, Kjersti (2005), Hamerle et al. (2003) are the most straightforward without compromising technicality. Gordy (2003) is an important paper showing that Basel's capital adequacy rules can be reconciled with a class of credit risk models which are portfolio-invariant. Gordy (2003) is explicitly mentioned in the Basel Committee's official documents as one of the theoretical cornerstone of their IRB approach, along with Vasicek (2002) BIS (see 2005a).

3.1 Default correlation and portfolio diversification

Following Markowitz(1952) a portfolio is efficient if there is no other portfolio with lower risk and an at least equal expected return, and no portfolio with a higher expected return and at most equal risk. In this context diversification is a means to change the risk of the portfolio. The portfolio risk is measured as the standard deviation from expected returns, and, by definition, is the sum of the variances of each component of the portfolio from the expected return and the correlations (ρ_{ij}) between components $i \neq j$. Risk diversification can be achieved if

$-1 \leq \rho_{ij} < 1$, but not when the components of the portfolio are perfectly correlated ($\rho_{ij} = 1$). Moreover, increasing the number of components in the portfolio decreases its overall variance, regardless of the sign of cross-correlations,²³ since in a large portfolio, cross-correlations among assets determine the portfolio variance. The variance of each asset then contribute little to portfolio risk (see Ingersoll, 1987).

Extending this setting to credit portfolios is not straightforward. In the case of credit, the concept of risk is not solely associated from that of variance. Risk is an inherent characteristic of a loan, and can be proxied by the probability of default. In the Vasicek model the probability of default is determined by the behaviour of the latent variable, and increasing the number of loans may not necessarily lead to lower probability of default.

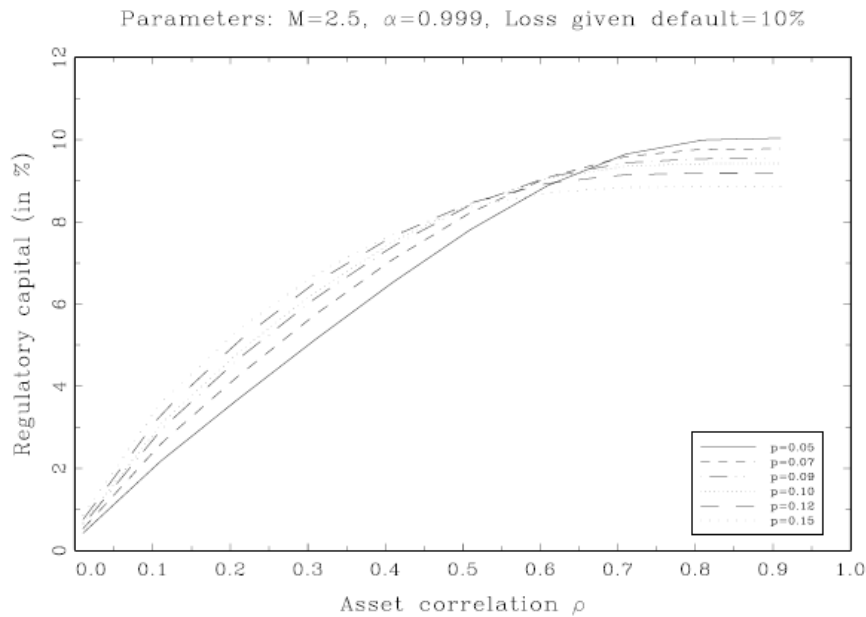
First, default correlation is positively related to asset correlation. Second, asset correlations in the Vasicek model can only assume positive values, unlike conventional portfolios. As a result, the opportunities for credit risk diversification are limited to assets presenting cross- correlations converging to zero.

Moreover, Figures 1 and 2 suggest that the Vasicek model yields a negative relationship between asset correlation and the probability of portfolio loss, for certain levels of probability. For instance, for $P[L \leq 0.96]$, Figure 1 shows that the fraction of the portfolio that is lost is lower when $\rho = 0.91$ (dashed curve) than when $\rho = 0.51$ (solid curve). An analogous result can be seen for $P[L \leq 0.90]$. The dashed curve corresponds to $\rho = 0.41$, whilst the solid curve is associated to $\rho = 0.01$. This counterintuitive result is much more pronounced for higher values of the individual probability of default, as Figure 2 illustrates. For $\alpha = 0.95$ the fraction of portfolio lost is lower when $\rho = 0.91$ (dashed line) than when $\rho = 0.51$ (solid line). This implies that at these levels of probability, increasing asset correlation actually reduces the overall risk of the portfolio.

²² Note that there is a discrepancy between two Basel Committee's publications regarding the Normal distribution used in expression (22). In BIS (2005a, p.7) $\Phi(\cdot)$ is the Normal distribution function $N(\cdot)$. However, in BIS (2005b, p.60 footnote 71) $\Phi(\cdot)$ is the cumulative Normal distribution. Since BIS (2005a) is the main document of the Basel Capital Accord and thus supercedes BIS (2005a), we use its formula here.

²³ provided the assets are not perfectly correlated.

Figure 3. Regulatory Capital



However, for $\alpha = 0.98$ and $\alpha = 0.95$, respectively, the more intuitive relationship between asset correlation and risk holds. The higher asset correlation, the higher default risk. In the context of Basel II's IRB Approach, these findings have hardly any implications. First, the Basel Committee decided that banks should make capital provisions for losses occurring with a probability of less than 0.01%. Second, the formulae adopted by the Basel Committee for the asset correlation restrict its value to the interval [0.12;0.24], (see BIS, 2005a).

In the IRB formulae, the correlation between individual assets and the macroeconomic factor is given by the expression (24) for bank and sovereign borrowers,

$$p_i = 0.12 * \frac{1 - e^{-50p_i}}{1 - e^{-50}} + 0.24 * \left(-\frac{1 - e^{-50p_i}}{1 - e^{-50}} \right) \quad (24)$$

The correlation between individual asset and the macroeconomic factor for corporate borrowers are derived from (24). $\rho_c = p_i - 0.04$ for $S_i \leq 5$. $\rho_c = p_i - 0.04 * (1 - S_i - 5)/45$ for $5 \leq S_i \leq 50$, and $\rho_c = p_i$ for $S_i \geq 50$. ρ_c is the correlation coefficient for corporate borrowers and p_i is given by (24). S_i are annual sales of firms i . Clearly, ρ_c is reduced for small firms. (24) assumes a downward relationship between the asset/factor correlation and default probability. The higher the individual probability of default, the lower the asset/factor correlation, since in this case, the idiosyncratic factors Z_i are assumed to dominate the macroeconomic factor. In other words, the higher the probability of default the higher the likelihood that default will be determined by factors specific to the borrower rather than macroeconomic conditions. The

IRB's assumption of a negative relationship between asset/factor correlation and default probability stems from Lopez (2004). However, several subsequent studies have produced results that contradict this assumption. Amongst other, Dietsch and Petey (2004), D'ullmann and Scheule (2003), and Hamerle et al. (2003).

Figure 3 plots the regulatory capital given by (22), for various levels of individual probability of default $p \in [0.05; 0.15]$, a loan maturity of 2.5 years (standard value in BIS (2005b)), $\alpha = 99.99$ and a loss given default of 10%. Clearly, the higher the probability of default, the higher the necessary amount regulatory capital, for asset correlations lower than 0.65.

4 Conclusions

In this paper, we present the theoretical model that constitutes the cornerstone of Basel II's Internal Ratings Based (IRB) approach to regulatory capital. This model was developed by KMV Corporation, and later published in Vasicek (1987, 1991, 2002). We then analyse the properties of the KMV model of credit portfolio loss, for distinct value of single firm default probability and asset correlations. Our results show that this model tends to overestimate the probability of portfolio loss when the probability of default of a single firm and the firms' asset correlations are low. On the contrary, probabilities of portfolio loss are underestimated when the probability of default of a single firm and asset correlations are high. Moreover, the relationship between asset correlation and probability of loan portfolio loss is only consistent at very high quantiles of the portfolio loss distribution. These are precisely those adopted by

the Basel II Capital Accord for the calculations of capital adequacy provisions. So, although the counterintuitive properties of the KMV model do not extend to Basel II, they do restrict its generality as a model of credit portfolio loss.

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