APPLICATION OF MARKOWITZ MODEL IN ANALYSING RISK AND RETURN A CASE STUDY OF BSE STOCK

Manas Pandey*

Abstract

In this paper the optimal portfolio formation using real life data subject to two different constraint sets is attempted. It is a theoretical framework for the analysis of risk return choices. Decisions are based on the concept of efficient portfolios. Markowitz portfolio analysis gives as output an efficient frontier on which each portfolio is the highest return earning portfolio for a specified level of risk. The investors can reduce their risks and can maximize their return from the investment, The Markowitz portfolio selections were obtained by solving the portfolio optimization problems to get maximum total returns, constrained by minimum allowable risk level. Investors can get lot of information knowledge about how to invest when to invest and why to invest in the particular portfolio. It basically calculates the standard deviation and returns for each of the feasible portfolios and identifies the efficient frontier, the boundary of the feasible portfolios of increasing returns.

Keywords: Efficient Portfolios, Portfolio Optimization, Efficient Frontier, Variance, Covariance, Risk and Return

* *Department of Business Economics, Faculty of Management Studies, VBS Purvanchal University, Jaunpur (UP), India*
E-mail: manas_pandey@rediffmail.com

Introduction

The portfolio selection is really the process of delineating the efficient portfolio and then selecting the best portfolio from the set. The rational investors will obviously prefer to invest in the efficient portfolios i.e. the selection of the optimal portfolio thus depend on the investor's risk aversion or conversely on risk tolerance.

Portfolio optimization is a key idea in investing and Markowitz model (1952) developed by Dr. Harry Markowitz is also known as the

modern portfolio theory or portfolio selection through which the selection of optimal portfolio takes place since the basic elements of modern portfolio theory emanate from a series of propositions concerning rational investor behavior set forth by him and used mathematical programming and statistical analysis in order to arrange for the optimum allocation of assets within portfolio on the basis of a reward risk context. In other words, the optimal portfolio for an investor would be the one at the point of tangency between the efficient frontier and his risk return utility and he considered the variance in the expected return from investments and their relationship to each other in constructing portfolios.

Markowitz model assumed that investors move toward low level of return from high level of return for a given level of the risk , individual estimates the risk on the basis of return, in the analysis of risk and

return to select the efficient portfolio the role of the Markowitz model is very significant and the investors decision based on the concept of efficient frontier. For optimal portfolio analysis, first step is that an investor needs to specify the list of securities eligible for selection or inclusion in the portfolio then to generate the risk return expectations for these securities expressed as the expected rate of return (mean) and the variance or standard deviation of the return. The expected return of assets is the weighted average of the return of the individual securities held in the portfolio. The variance and standard deviation of return are alternative statistical measures that are used for measuring risk in investment. This depends on their interactive risk i.e. how the returns of a security move with the returns of other securities in the portfolio and contribute to the overall risk of the portfolio. That is expressed in the following formula- Minimize portfolio risk

$$\sigma_p = \sqrt{\left[\sum_{x=1}^{N} w^2 \sigma^2 x + \sum_{x=1}^{N}\sum_{x=1}^{N} w_x w_y c_{xy} \right]}$$

Where

Wx = amount invested in asset x
Wy = amount invested in asset y
Cxy = covariance between security x and y
N = number of security
σ^2x = variance or risk

From the above formula selection of optimum portfolio require the return for the period of holding for each of the securities included in the portfolio , the standard deviation of the return for each security and the covariance (or correlation coefficient) between each pair of securities among all securities from which the portfolio have to form.

Review of Literature

Michael J. Hartley and Gurdip S. Bakshi October 25, 1998 reported that they have developed an econometric methodology associated with the inverse of the portfolio selection problem. In particular, given a time series of actual observed portfolio of risky assets for a sample of investors, a set of socio-economic characteristics for each investor in the sample, and a time series of preceding rates of returns for the set of risky assets, the algorithm determines the parameter values in each investor's utility function and the associated parameters in the returns generating process. It also determines the optimal current portfolios at the same time for all sample members. The proposed econometric framework can, therefore, accommodate both homogeneous or heterogeneous expected returns and covariance matrix however; predictions about individual or panels of portfolio holdings can be provide powerful tests of asset pricing theories. Consequently, a natural question, in the present framework, is how to go from a micro-theory of the individual or institutional investor's asset- portfolio mix to the aggregate behavior of the market for risky and risk-free assets. If the set of 36 panel data represents a strained sample of individual/institutional investors, then by applying suitable sampling weights to the numerical solution values in the sample, one may develop an internally-consistent micro-macro model with no further restrictions on the functional forms of the underlying functional relationships-the investor's returns-generating process and the utility function-than the customary stipulations.

Michael J. Hartley and Gurdip S. Bakshi April 2004 reported that there paper has been devoted to a class of dynamic Markowitz's mean-variance portfolio selection problems. Taking into consideration of market trend and other factors, a discrete-time model that is modulated by a Markowitz chain was introduced. Aiming at complexity reduction, we use nearly completely decomposable transition matrices and weak convergence methods to derive the limit mean-variance portfolio selection problem based probability vector and the transition probability matrices under the weak topology. The associated weak convergence and the limit systems can still be obtained. As far as the limit mean-variance problem is concerned, for the cases discussed on the limit, we can design optimal (efficient) portfolios and derive efficient frontier see also the framework of LQ control with indefinite control weights. Then using the efficient portfolios of the limit problem, we constructed portfolios for the original discrete-time model and show that such portfolios are nearly efficient.

Heinz H Moller September 2005 University of Ziirich, reported that Markowitz model i.e. modern portfolio theory has developed to a highly sophisticated field of research. in addition it become more and more obvious that for a large class of insurance problems a separate analysis of actuarial and financial risks is inappropriate. Of course modern portfolio theory is typically applied to common stocks. However it can also be applied to these facts the increasing importance of new financial instruments and the availability of computer capacity explain the growing interest of actuaries in modern portfolio theory.

Beth Bower, June 2008 College of William and Mary, Williamsburg, VA Pamela Wentz, Millersville University, Millersville, reported that they looked at investment portfolio optimization. They created portfolios consisting of five stocks and a six-month bond by randomly selecting the stocks from th S&P 500. We take the data from July 1, 2004 to December 31, 2004 and use the Markowitz minimum variance model as well as the Mean Absolute Deviation model to determine the allocation of funds to each asset in each of the portfolios. We then compare the returns of the portfolios from January 3, 2005 to June 30, 2005 using a series of parametric and non-parametric tests.

Yansen Ali June 2008 Industrial Engineering Honors Program McCormick School of Engineering Northwestern University reported that after completing the study on the computational method in portfolio analysis, I was able to conclude that the single index model worked well in estimating the inputs to the basic Markowitz optimization model. This is shown by the comparable returns produced by the Markowitz model. Although the comparison is not completely accurate as their portfolio stocks are not wholly identical. There are certainly areas of improvement in this project, especially in the development of more advanced extension to the Markowitz optimization model. The current simplistic model can be the reason why the portfolio selection is small for all cases; the optimal portfolio only selects a maximum of 2 industries in all periods. This paper has also not looked in depth for potential biases in the single index model, which might affect the end results. From this project, I found that computational procedure can get as complicated as formulating a solution method, especially when large-scale data is involved. It is imperative that one plans a robust and systematic method in solving a theoretical model. When the right method is applied, it can lead to a better utility of the model and more efficient implementations.

Objectives of the study

- To know how to select optimal portfolio,
- To examine the relation between risk and return,
- To study the utility of Markowitz model,
- To analyze risk and return of the portfolio.

Source of data

The data used in the research is secondary in nature and has been collected from the relevant websites, journals, magazines and some periodicals.

Research design

For studies purpose alphabetically arranged top 30 companies listed in Bombay Stock Exchange (BSE) has been selected and one year's financial information from April 2009 to March 2010 of these companies has been collected. Randomly made six portfolios of the companies by including five companies in each group and selected one company from each portfolio by applying the Markowitz model.

Statistical tools

In the process of selecting the optimal portfolio, many statistical tools have been used for the calculation of the value of the risk and return of the stock. The input data of expected returns and covariance matrix were thus made ready for the next step in the analysis by calculating expected return as the difference between target price and current market price of each security, expressed as a percentage of current market prices. The covariance is calculated with monthly return for each company from the monthly closing prices. The monthly covariance between each pair of securities was converted into annual covariance by multiplying it with 12.

Variance

The variance or risk of a portfolio is not simply weighted average of the variances of the individual securities in the portfolio. The relationship between each security in the portfolio with every other security as measured by the covariance of return has also to be considered. The variance of a portfolio with only two securities in it may be calculated with the following formula.

$$\sigma^2_p = x_1^2\, \sigma^2_1 + x_2^2\, \sigma^2_2 + 2x_1 x_2 (r_{12}\, \sigma_1\, \sigma_2)$$

Where
σ^2_p = Portfolio variance.
x_1^2 = Variance of first security.
σ^2_1 = Variance of second security.

x_1 = Proportion of funds invested in the first security.
x_2 = Proportion of funds invested in the second security.
r_{12} = Correlation coefficient between the returns of first and second security.
σ_1 = Standard deviation of first security.
σ_2 = Standerd deviation of second security.

Covariance

Covariance is a measure of how returns of two securities move together. It is the statistical measure that indicates the interactive risk of a security relative to others in a portfolio of securities. The covariance between two securities x and y may be calculated using the following formula:

$$Cov_{xy} = \sum_{i=1}^{n} \left[Rx - \overline{Rx} \right]\left[Ry - \overline{Ry} \right]$$

Where
Cov_{xy} = Covariance between x and y portfolio.
Rx = Return of security x.
Ry = Return of security y.
$\overline{Rx}$ = Mean return of security x.
$\overline{Ry}$ = Mean return of security y.
N = Number of observations.

Return

Return of a portfolio of assets is simply the weighted average of the individual securities held in the portfolio. The weight applied to each return is the fraction of the portfolio invested in that security. The formula for the calculation of expected portfolio return may be expressed as shown below:
Return of the portfolio

$$\left(\overline{R_p}\right) = \sum_{I=1}^{N} Xi \times \overline{Ri}$$

Where $\overline{Rp}$ = expected return of the portfolio,
Xi = proportion of funds invested in security I,
$\overline{Ri}$ = return of security i.

Analysis and Interpretation

Portfolio optimization is a key idea in investing. The selection of the optimal portfolio thus depend on the investor's risk aversion or conversely on risk tolerance. In the process of selecting the company for the portfolios the analysis of the return as well as risk has been taken place which is reflected by the relationship between increasing or decreasing of return and risk. Thus six companies from each month on the basis of their risk and return has been found.

Now these six companies of each month considered as a portfolio. In this way now there are twelve portfolios over a year. After this for the further analysis select one company's stock from each portfolio of whole month on the basis of performance of the securities. Now there were only twelve companies' securities. These twelve companies are now separated into the two semester then two companies from these two semester selected on the basis of the performance of securities in term of maximum or minimum risk and returns. After this process compare to these two securities to each other which are well performing in comparison to other companies or in comparison which company is performing well overall year and select that as a efficient portfolio and take decision for investment in that companies security. In this model, the objective function is to maximize total returns, constrained by maximum allowable risk level. Therefore the entire process of analysis and interpretation has been completed in the two parts which are discussed as follows:

Return of a portfolio

As a first step in portfolio analysis an investor needs to specify the list of securities eligible for selection or inclusion in the portfolio. Next he has to generate the risk return expectations for these securities. These are typically expressed as the expected rate of return (mean) and the variance or standard deviation of the return .The expected return of assets is the weighted average of the return of the individual securities held in the portfolio. Calculation of return, deviation and square of closing price of each security for one year from April 2009 to March 2010 by using the formula

$$\text{Return \%} = \frac{\text{Value of current month-value of previous month}}{\text{Value of previous month}} \times 100$$

$$\overline{Rx} = \frac{\sum Rx}{N}$$

$$\text{deviation } (\sigma) = \text{Return \%} - \overline{Rx}$$

Table 1. List of return of 30 stocks for one year time period

	April 2009	May 2009	June 2009	July 2009	August 2009	Sept 2009	Oct 2009	Nov 2009	Dec 2009	Jan 2010	Feb 2010	March 2010
ACC	13.24	19.91	-1.80	14.62	-8.24	1.31	-8.75	6.55	9.40	-0.75	5.99	2.98
B.Airtel	19.73	9.39	-2.14	-48.22	3.45	-1.46	-30.06	2.39	9.71	-6.78	-8.89	4.91
BHEL	9.80	31.62	1.35	1.08	3.89	0.45	-4.65	1.24	7.20	0.01	-2.25	2.44
DLF	38.10	74.66	-22.94	27.46	7.08	3.25	-15.52	-4.97	2.71	-7.84	-10.71	3.95
Grasim Indus.	12.79	18.42	9.70	18.60	-2.11	3.22	-21.24	9.20	4.06	4.70	3.99	4.31
HDFC	22.70	26.24	7.30	7.91	-2.22	12.20	-4.32	3.89	-298	-11.03	4.94	8.51
HDFC Bank	13.73	31.04	3.42	0.53	-2.02	11.77	-1.28	9.33	-4.07	-4.09	4.52	13.37
Hero Honda	10.66	13.23	4.25	14.85	-5.86	10.47	-6.22	9.91	-0.26	-9.19	13.69	9.62
Hindalco Ind.	3.76	57.29	2.07	15.91	5.64	21.73	-5.36	13.20	16.44	-8.40	9.51	12.68
Hindustan Lever	-1.53	-1.47	15.58	9.02	-10.77	1.15	7.65	0.81	-7.19	-7.80	-3.42	1.25
ICICI Bank	43.64	55.04	-2.52	5.13	-1.26	20.72	-12.73	9.46	1.32	-5.17	4.99	9.27
Infosys	13.84	6.28	10.92	16.15	3.31	8.26	-4.46	8.10	9.28	-4.93	5.04	0.52
ITC Ltd.	2.33	-1.30	3.70	31.29	-47.53	0.30	10.03	10.03	1.04	-2.70	-0.24	-7.27
Jaiprakash asso.	64.85	49.50	-1.18	17.46	-5.69	4.45	-11.42	7.38	-34.90	-6.26	-3.96	13.12
L& T	30.76	59.81	11.58	-3.93	4.05	7.37	-6.89	3.00	4.04	-15.15	9.95	3.80
M&M Ltd.	26.89	38.82	2.59	23.73	0.38	2.46	4.62	11.53	5.11	-5.68	-1.18	-45.88
Maruti Suzuki	5.24	25.24	4.30	32.64	1.66	18.25	-17.42	11.31	-0.31	-10.87	5.28	-3.24
NTPC	5.52	13.31	-9.47	10.54	-1.37	0.49	-1.08	-0.78	12.37	-9.10	-5.25	1.97
ONGC	11.00	35.86	-9.25	9.13	1.78	-1.17	-3.30	5.86	-1.79	-6.60	1.57	-1.66
Reliance comm..	23.11	42.27	-5.20	-4.92	-5.50	18.23	-42.87	-2.24	0.55	-1.76	-7.36	8.48
Reliance Inds.	18.35	26.34	-11.16	-3.27	2.40	9.83	-12.26	-44.97	2.50	-3.93	-6.55	9.88
Reliance Infra.	34.97	83.67	-6.21	0.63	-4.9	6.59	-13.36	-0.95	9.41	-9.89	-2.86	-0.56

SBI	19.80	46.29	-6.80	4.13	-3.91	25.97	-0.21	2.15	1.40	-9.32	-3.99	5.22
Sterlite Inds.	15.01	52.23	-2.38	6.10	4.71	14.82	-0.49	11.15	0.47	-12.67	3.79	8.51
Sun Pharma.	14.64	-5.14	-9.82	7.42	1.54	17.59	-1.26	5.45	3.44	-2.50	4.86	16.14
Tata Consult.	15.41	12.28	-44.31	35.08	0.11	17.52	1.11	9.74	9.10	-1.91	3.47	2.60
Tata Moters	34.41	38.93	-13.53	44.79	16.08	20.84	-4.46	16.97	19.93	-12.40	2.41	6.28
Tata Power	16.81	19.73	7.42	13.25	0.52	0.81	1.80	0.43	2.15	-5.24	-7.11	13.16
Tata Steel	15.56	70.68	-3.81	18.40	-8.30	20.28	-7.59	22.01	7.33	-7.87	0.82	10.29
Wipro	34.68	15.45	-1.02	29.92	12.25	9.26	0.98	3.50	8.03	-4.71	4.53	4.45

Risk of a portfolio

The variance of return and standard deviation of return are alternative statistical measures that are used for measuring risk in investment. These statistics measure the extent to which returns are expected to vary around an average over time. The calculation of variance of a portfolio is a little more difficult than determining its expected return. The variance per standard deviation of an individual security measures the riskiness' of a security in absolute sense. For calculating the risk of a portfolio of securities the riskiness of each security within the context of the overall portfolio has to be considered. This depends on their interactive risk i.e. how the returns of a security move with the returns of other securities in the portfolio and contribute to the overall risk of the portfolio.

Calculations of risk through Markowitz formula for minimization of risk

The calculation of risk by using Markowitz model in a following way - Minimize portfolio risk

$$\sigma_p = \sqrt{\left[\sum_{x=1}^{N} w^2 \sigma^2 x + \sum_{x=1}^{N}\sum_{y=1}^{N} w_x w_y c_{xy}\right]}$$

Where
Wx = amount invested in asset x,
Wy = amount invested in asset y,
Cxy = covariance between security x and y,
N = number of security,
$\sigma^2 x$ = variance or risk.

For performing the portfolio analysis using the Markowitz method, we need the return for the period of holding for each of the securities to be considered for inclusion in the portfolio. We also require the standard deviation of the return for each security. In addition we have to know the covariance (or correlation coefficient) between each pair of securities among all securities from which we have to form the portfolio. In this method risk calculated for each portfolio and compared with the return percentage. In this process of comparison we selected that security from the each portfolio, which has the maximum return at minimum level of risk.

Table 2. List of deviation of 30 stocks for one year time period

	April 2009	May 2009	June 2009	July 2009	August 2009	Sept 2009	Oct 2009	Nov 2009	Dec 2009	Jan 2010	Feb 2010	March 2010
ACC	7.74	14.41	-730	9.12	-13.74	-4.19	3.25	1.05	3.90	-6.25	-5.44	-2.52
B.Airtel	23.78	13.44	1.91	-44.77	7.5	2.59	-26.01	6.44	13.76	-2.73	-4.84	8.96
BHEL	5.45	27.27	-3.00	-3.27	-0.46	-3.9	-9.00	-3.11	2.85	-4.34	-6.6	-1.91
DLF	30.16	66.72	-30.88	19.52	-0.86	-4.69	-23.46	-12.91	-5.23	-15.78	-18.65	-3.99
Grasim Indus.	7.32	12.95	4.23	13.13	-7.58	-2.25	-26.71	3.73	-1.41	-0.77	-1.48	-1.16
HDFC	16.57	20.11	1.17	1.78	-8.35	6.07	-10.45	-2.24	-9.11	-17.16	-1.19	2.38
HDFC Bank	7.38	24.69	-2.93	-5.82	-8.37	5.42	-7.63	2.98	-10.42	-10.44	-1.83	7.02
Hero Honda	5.23	7.8	-1.18	9.42	-11.29	5.04	-11.65	4.48	-5.69	-14.62	8.26	4.19
Hindalco Ind.	-8.28	45.25	-9.97	3.87	-6.4	9.69	-17.4	1.16	4.4	-20.44	-2.53	0.64
Hindustan Lever	-1.79	-1.73	15.32	8.76	-11.03	0.89	7.39	0.55	-7.45	-8.06	-3.68	0.99
ICICI Bank	32.98	44.38	-13.18	-5.53	-11.92	10.06	-23.39	-1.2	-9.34	-15.83	-5.67	-1.39
Infosys	7.81	0.25	4.89	10.12	-2.72	2.23	-10.49	2.07	3.25	-10.96	-0.99	-5.51
ITC Ltd.	2.08	-1.55	3.45	31.04	-47.78	0.05	9.78	0.79	-2.95	-0.49	-7.52	13.15
Jaiprakash asso.	57.07	41.72	-8.96	9.68	-13.47	-3.33	-19.2	-0.4	-42.68	-14.04	-11.74	5.34
L& T	21.73	50.78	2.55	-12.96	-4.98	-1.66	-15.92	-6.03	-4.99	-24.18	0.92	-5.23
M&M Ltd.	21.61	33.54	-2.69	18.45	-4.9	-2.82	-0.66	6.25	-0.17	-10.96	-6.46	-51.16
Maruti Suzuki	-0.78	19.22	-1.72	26.62	-4.36	12.23	-23.44	5.29	-6.15	-16.89	-0.74	-0.26

NTPC	4.09	11.88	-10.9	9.11	-2.8	-0.94	-2.51	-2.21	10.94	-10.53	-6.68	0.54
ONGC	7.55	32.41	-12.7	5.68	-1.67	-4.62	-6.75	2.41	-5.24	-10.05	-1.88	-5.11
Reliance comm.	21.21	40.37	-7.1	-6.82	-7.4	16.33	-44.77	-4.14	-1.35	-3.66	-9.26	6.58
Reliance Inds.	19.42	27.41	-10.09	-2.2	3.47	10.9	-11.19	-43.9	3.57	-2.86	-5.48	10.95
Reliance Infra.	26.92	75.62	1.84	-7.42	-12.95	-1.46	-21.41	-9.00	1.36	-17.94	-10.91	-8.61
SBI	13.07	39.56	-13.53	-2.6	-10.64	19.24	-6.94	-4.58	-5.33	-16.05	-10.72	-1.51
Sterlite Inds.	6.57	43.79	-10.82	-2.34	-3.73	6.38	-8.93	2.71	-7.97	-21.11	-4.65	0.07
Sun Pharma.	10.28	-9.5	-14.18	3.36	-2.82	13.23	-5.62	1.09	-0.92	-6.86	0.5	11.78
Tata Consult.	10.39	7.26	-49.33	30.06	-4.91	12.5	-3.91	4.72	4.08	-6.93	-1.55	-2.42
Tata Moters	20.22	24.74	-27.72	30.6	1.89	6.65	-18.65	2.78	5.74	-26.59	-11.78	-7.91
Tata Power	11.5	14.42	2.11	7.94	-4.79	-4.5	-3.51	-4.88	-3.16	-10.55	-12.42	7.85
Tata Steel	4.08	59.2	-15.29	6.92	-19.78	8.8	-19.07	10.53	-4.15	-19.35	-10.66	-1.19
Wipro	24.9	5.67	-10.8	20.14	2.47	-0.52	-8.8	-6.28	-1.75	-14.49	-5.25	-5.33

Table: 3. List of invested amount in form of Rs. 1.0×10^9 of 30 stocks for one year time period

	April 2009	May 2009	June 2009	July 2009	August 2009	Sept 2009	Oct 2009	Nov 2009	Dec 2009	Jan 2010	Feb 2010	March 2010
ACC	2.01	2.11	2.53	3.02	3.19	2.07	1.36	1.6	1.84	2.05	1.4	1.96
B.Airtel	6.29	14.46	13.68	8.31	10.75	9.41	31.00	12.34	11.00	5.35	8.67	6.3
BHEL	9.04	9.59	8.98	9.72	6.49	5.9	64.41	4.82	6.19	4.78	3.2	4.28
DLF	2.13	8.41	36.42	39.23	27.32	29.04	27.94	25.05	20.63	15.08	13.61	12.3
Grasim Indus.	1.90	1.4	2.28	3.32	1.85	1.64	3.74	2.39	1.56	2.42	1.84	1.64
HDFC	7.04	9.76	6.22	9.17	5.81	4.86	4.64	3.89	3.75	4.42	2.96	3.62
HDFC Bank	4.94	2.05	4.26	5.61	3.61	3.01	2.55	8.17	3.15	4.19	3.96	3.28
Hero Honda	2.63	1.6	1.51	2.5	2.66	2.11	1.50	1.96	3.14	3.78	1.35	2.83
Hindalco Ind.	3.08	5.03	9.46	4.82	7.65	8.13	5.59	5.03	6.77	6.6	7.93	1.5
Hindustan Lever	2.43	2.82	1.72	4.92	2.63	1.77	1.55	1.56	1.47	2.43	1.58	1.87
ICICI Bank	5.94	6.77	6.41	1.07	6.09	6.15	20.21	17.99	69.78	8.94	10.88	5.86
Infosys	34.8	43.24	32.17	3.76	25.52	18.42	7.88	10.55	17.04	14.27	4.89	12.78
ITC Ltd.	2.22	2.89	5.7	5.68	2.62	2.63	2.92	2.58	1.93	2.52	1.9	3.07
Jaiprakash asso.	1.93	19.27	28.97	2.22	16.87	21.5	15.35	12.61	5.8	7.19	7.18	5.51
L& T	1.74	20.6	25.15	2.14	1.4	11.21	13.25	17.23	9.09	13.1	9.58	6.43
M&M Ltd.	1.61	2.14	3.18	4.64	5.28	4.14	4.39	3.99	3.15	4.06	2.63	3.62
Maruti Suzuki	3.68	2.97	1.92	3.9	4.87	5.73	5.24	4.38	3.49	5.35	3.16	3.28
NTPC	6.13	5.9	10.97	5.5	3.76	2.42	2.67	1.94	4.04	4.28	2.88	2.34
ONGC	5.65	9.94	7.88	12.78	7.34	6.4	3.64	3.53	2.25	3.3	2.33	3.46
Reliance comm.	15.87	21.36	28.7	18.98	10.74	13.13	20.21	9.07	6.24	5.71	3.09	2.83
Reliance Inds.	4.43	4.71	5.52	5.61	3.88	6.37	38.91	35.39	18.62	37.1	17.28	17.36
Reliance Infra.	3.19	3.2	2.83	3.2	1.6	1.06	13.77	13.22	8.54	7.35	5.52	5.88
SBI	22.2	2.71	2.42	2.33	1.54	1.9	35.26	31.1	26.17	21.09	18.46	14.98
Sterlite Inds.	6.53	1.08	1.01	1.64	1.24	9.62	87.49	75.15	57.98	6.56	5.95	5.77
Sun Pharma.	5.93	1.05	1.99	1.5	1.01	7.6	9.89	7.08	7.92	0.64	4.38	0.66
Tata Consult.	3.88	4.71	5.26	8.88	6.79	6.84	6.93	4.71	3.85	6.42	2.58	4.69
Tata Moters	8.04	6.64	9.49	10.74	18.45	16.49	13.21	11.99	13.25	10.04	8.45	51.25
Tata Power	1.83	1.89	21.15	2.42	2.22	1.42	2.08	2.15	1.22	1.73	1.52	4.46
Tata Steel	23.38	30.4	43.07	42.75	33.75	29.93	22.43	28.32	28.84	24.66	22.4	14.35
Wipro	2.25	2.64	2.18	2.86	2.42	1.91	2.98	2.22	2.76	4.36	1.38	2.22

Table 4. List of risk in 30 stocks for one year time period

	April 2009	May 2009	June 2009	July 2009	August 2009	Sept 2009	Oct 2009	Nov 2009	Dec 2009	Jan 2010	Feb 2010	March 2010
ACC	20.44	155.89	53.36	51.46	54.08	--	63.78	--	--	36.41	**11.72**	12.98
Bharti Airtel	149.84	379.50	97.87	380.13	92.42	--	818.11	78.71	**141.71**	54.05	42.85	59.12
BHEL	**17.08**	828.55	156.23	172.41	72.08	--	**143.67**	--	--	89.80	23.73	25.87
DLF	64.28	561.69*	1127.32	772.15	**45.06**	133.44	663.08	323.21	101.51	245.90	253.96	51.37
Grasim Indus.	15.52	**50.91**	**22.60**	**51.36**	19.22	--	102.33	**8.47**	--	22.10	3.78	**6.36**
HDFC	**118.70**	437.98	44.40	64.35	54.47	33.16	59.05	7.16	26.01	87.83	6.63	14.49
HDFC Bank	38.85	**71.24**	24.95	46.04	34.55	**12.90**	27.31	24.09	25.82	**59.92**	8.18	25.23
Hero Honda	17.37	79.51	32.21	**38.24**	38.69	--	33.32	7.57	--	54.31	**12.37**	19.58
Hindalco Ind.	27.47	250.35	100.37	46.19	**54.64**	80.20	101.45	**4.30**	**21.77**	141.06	20.88	**13.55**
Hindustan Lever	16.01	145.23	38.33	72.49	35.89	--	**28.66**	--	--	39.01	6.81	8.99
ICICI Bank	**62.61**	1946.44	426.91	25.17*	302.83	**188.03**	489.55	27.09	164.34	235.73	197.18	50.74
Infosys	195.19	498.54	**61.40**	22.27*	91.13	50.65	224.86	39.64	334.75	160.31	27.55	68.11
ITC Ltd.	86.44	217.49	51.58	**20.73***	134.00	34.45	**114.55**	13.85	24.56	**47.83**	19.78	**49.42**
Jaiprakash asso.	112.76	991.64	277.90	30.15*	252.79	100.92	377.76	**3.76**	252.92	148.58	89.83	50.67
L& T	40.12	**1101.20**	79.32	23.15*	**96.30**	42.35	270.49	108.83	**72.76**	339.29	**26.83**	54.23
M&M Ltd.	**47.34**	**102.71**	12.48	95.04	**44.72**	25.18	**79.53**	26.88	23.68	68.71	20.06	186.59
Maruti Suzuki	62.70	134.67	**17.20**	**113.13**	37.38	**72.17**	137.57	**24.20**	34.35	105.11	**11.41**	35.93
NTPC	86.95	233.89	124.36	95.56	39.21	18.17	52.09	7.61	**49.16**	61.75	21.49	19.76
ONGC	131.95	532.96	103.66	167.68	64.99	51.35	144.23	16.04	29.24	58.97	10.85	27.98
Reliance comm.	343.14	884.52	206.78	158.27	96.16	217.06	918.16	40.23	32.76	**57.99**	30.50	**28.96**
Reliance Inds.	100.89	179.66*	73.52*	3.22*	17.21	70.30*	513.04	1547.94	82.24	274.55	122.23	188.16
Reliance Infra.	**93.63**	259.88*	32.06*	13.14*	22.28	6.19*	391.80	--	**58.90**	231.91	100.33	43.98
SBI	33.67	120.52*	37.85*	--	17.41	37.00*	319.80	107.30	147.30	383.49	214.57	--
Sterlite Inds.	51.00	**48.52***	**12.21***	**1.14***	4.86	61.48	**85.40**	**146.38**	469.37	142.00	30.47	**2.47**
Sun Pharma.	72.11	22.65*	13.03*	--	26.74	**31.06**	72.93	--	47.22	**74.85**	**6.87**	--
Tata Consult.	88.07	183.19	273.28	257.38	58.06	96.55	**61.57**	--	31.62	**130.82**	37.16	--
Tata Moters	171.23	**199.96**	268.80	326.56	**44.21**	**111.54**	253.91	**11.87**	**77.61**	274.50	103.50	403.08
Tata Power	94.08	245.47	**117.72**	--	38.27	28.29	80.40	--	23.11	101.74	49.88	27.73
Tata Steel	139.43	1822.63	668.96	284.79	668.68	265.31	438.34	294.21	124.49	502.93	242.81	**8.06**
Wipro	69.71	144.47	47.37	**44.56**	18.22	15.30	59.37	12.45	13.42	102.78	**16.60**	8.10

* The risk value will be in 1.0 x 10^{10}

Interpretation

The selection of top six securities from thirty security of BSE index for the month of April has been done on the basis of return and risk analysis by using Markowitz model and selected the ICICI bank, the investment in the ICICI bank is less risky with high return. In the month of May the performance of the HDFC Bank which has given good return with optimal risk among the six best performing securities, hence in the May month the HDFC Bank is the best security for investment. Whereas in the month of June the ratio of risk and return shows that the Hindustan Lever Limited has given best return. From

six portfolio of July month ITC Ltd. performance over the year was good. The maximum return of the ITC Ltd. at the low level of the risk, therefore in the July month the investment in the ITC Ltd. is less risky with high return. From above table it is clear that in the month of August the Tata Motors have a very good combination of Risk and Return, hence in the August month the Tata Motors is the best security for investment. In the month of September the Sun Pharmaceutical Industries has best return paying securities for investment. Through the selection of top six securities from thirty security of October month Hindustan Lever Limited which has best return among these six, hence in the October month the Hindustan Lever Limited is the best security for

investment. In the month of November Hindalco Industries Ltd. has best for investment among the six companies from the point of view of risk and return. Once again the Hindalco Industries Ltd. has good return among other companies; hence in the month of December the Hindalco Industries Ltd. is the best available security for investment. The Grasim Industries, first textile company which has best return among other companies in the month January. In the month of February the ratio of return and risk of the company Hero Honda shows very good for investors. Above table reflect that the Sterlite Industries has best return among others, hence in the March the Sterlite Industries is the best for investment.

Table 5. Month wise top most company on the basis of return and risk

Month	Company	Return	Risk
April	ICICI Bank	43.64	62.61
May	HDFC Bank	31.04	71.24
June	Hindustan Lever Limited	15.58	38.33
July	ITC	29.92	44.56
August	Tata Motors	16.08	44.21
September	Sun Pharmaceutical Industries	17.59	31.06
October	Hindustan Lever Limited	10.03	114.55
November	Hindalco Industries Ltd.	13.20	4.3
December	Hindalco Industries Ltd.	16.44	21.77
January	Grasim Industries	4.70	22.10
February	Hero Honda Motors Ltd.	13.69	12.37
March	Sterlite Industries	8.51	2.47

In the above table the top securities of 2009 to 2010, the security of that month which has the top performance in respect of highest return at the level of the optimum risk. After analysis it was found that the ICICI Banks performance is best among the other companies for the first six months and the next six month the Hindalco Industries Ltd. performance was encouraging for the investors.

Concluding observation

The study present the results that display the difference between the returns and the risk based on Markowitz model. The Markowitz portfolio selections were obtained by solving the portfolio optimization problems for periods from April, 2009 to March, 2010. The basic solution approach to this problem is to implement the Markowitz model in finding an optimal portfolio selection in each month. The basic objectives behind Markowitz model were to achieve high returns or stable returns with low uncertainty. It helps in the detection of the optimum portfolio for getting the maximum return at the level of the minimum risk for invested fund. It is basically calculates the standard deviation and returns for each of the feasible portfolios and identifies the efficient frontier, the boundary of the feasible portfolios of increasing returns. Although the comparison is not

completely accurate because of the large number or data required for calculation of the risk and return of the portfolio and as their portfolio stocks are not wholly identical, the returns data are still useful to reflect economic influence. The financial planners can help the investors/traders to arrive at the risk level that they can assume. If the investor specifies his risk level in terms of standard deviation of the portfolio return, the appropriate portfolio for him can be identified using the efficient frontier. Therefore, process compare to the securities to each month and then from each month to semester which were well performing securities in overall year and select that as a efficient portfolio and take decision for investment in that companies security. In this model, the objective function is to maximize total returns, constrained by maximum allowable risk level. Hence the final portfolio selection for an investor/trader requires the combination of portfolio analysis and financial planning.

References

1. "Dow Jones AIG-Commodity Index." *Dow Jones Indexes.* 2008. Dow Jones. <http://www.djindexes.com/mdsidx/index.cfm?event=showAigDataCharts>.

2. Elton, Edwin J., and Martin J. Gruber. *Modern Portfolio Theory and Investment Analysis*. 3rd ed. John Wiley & Sons, 1987. 95-128.
3. Fourer, Robert, David M. Gay, and Brian W. Kernighan. *AMPL: a Modeling Language for Mathematical Programming*. 2nd ed. Thomson - Brooks/Cole, 2003.
4. French, Kenneth R. "Data Library." *Kenneth R. French*. 2007. <http://mba.tuck.dartmouth.edu/ pages/faculty/ken.french/data_library.html>.
5. Markowitz, Harry M. *Portfolio Selection: Efficient Diversification of Investments*. John Wiley & Sons, New Jersey, 1959.
6. Feinstein, Charles D., and Thapa, Mukund N., *A Reformulation of a Mean-Absolute Deviation Portfolio Optimization Model*, Management Science Vol. 39 (1993) No. 12, pp. 1552-1553.
7. Konno, Hiroshi, and Yamazaki, Hiroaki, *Mean-Absolute Deviation Portfolio Optimization and Its Applications to Tokyo Stock Market*, Management Science Vol. 37 (1991) No. 5, pp.519-531.
8. Luenberger, David G., *Investment Science.* Oxford University Press, New York, 1998.
9. Simaan, Yusif, *Estimation Risk in Portfolio Selection: The Mean Variance Model versus the Mean Absolute Deviation Model*, Management Science Vol. 43 (1997) No. 10, pp. 14
10. Arnold, Steven F. The Theory of Linear Models and Multivariate Analysis, Wiley, 1981.
11. Huang, Chi-Fu. Foundations for Financial Economics, Elsevier Science Publishing Co., 1988.
12. Markowitz, Harry M. Portfolio Selection: Efficient Diversification of Investments, Wiley, 959 Markowitz, Harry M. "Portfolio Selection," Journal of Finance. March 1952. . Chapter 5, finding the efficient set *H. M. Markowitz 287*
13. Blume, M. (1971), "On the assessment of risk", *Journal of Finance,* March. Ederington, L. H. (1986), "Mean-variance as an approximation of expected utility maximization", Working Paper 86 - 5, School of Business Administration, Washington University, St. Louis, Missouri.
14. King, B. F. (1966), "Market and industry factors in stock price behavior", *Journal of Business,* January Supplement.
15. Levy, H. and Markowitz, H.M. (1979), "Approximating expected utility by a function of mean and variance", *American Economic Review,* June.
16. Lintner, J. (1965), "The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets", *Review of Economics and Statistics,* February.
17. Markowitz, H. M. (1952), "Portfolio selection", *The Journal of Finance,* March.
18. Markowitz, H. M. (1959), *Portfolio Selection: Efficient Diversification. of Investments,* Wiley, Yale University Press, 1970, Basil Blackwell, 1991.
19. Markowitz, H. M. (1987), Mean-Variance *Analysis in Portfolio Choice and Capital Markets,* Basil Blackwell, paperback edition, Basil Blackwell, 1990.
20. Sharpe, W. F. (1963), "A simplified model for portfolio analysis", *Management Science,* January.
21. Sharpe, W. F. (1964), "Capital asset prices: a theory of market equilibrium under conditions of risk", *The Journal of Finance,* September.
22. Simaan, Y. (1987), "Portfolio selection and capital asset pricing for a class of nonspherical distributions of assets returns", dissertation, Baruch College, The City University of New York.
A. Wiesenberger and Company, *Investment Companies,* New York, annual editions.
23. Von Neumann, J., and Morgenstern, O. (1944), *Theory of Games and Economic Behavior,* 3rd edition, Princeton University Press, 1953.
24. Williams, J. B. (1938), *The Theory of Investment Value,* Harvard University Press, Cambridge, Massachusetts.