EFFECTS OF UNDERDEVELOPED EQUITY MARKET ON INVESTMENT

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Abstract

This paper uses a variant of the Allen, Bernardo, and Welch (2000) model in an open market economy to analyze the effects of equity market development on investment. A country's underdeveloped equity market may discourage investors from investing in the country. Consequently, an underdeveloped equity market may contribute to home equity bias. Asset prices in a less developed equity market tend to be lower. The results suggest that a government may need to facilitate the development of its equity market to attract investment.

Keywords: asset prices, deadweight costs, home equity bias

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1. Introduction

Government policies, market demand and business practices largely fuel the growth in emerging capital markets. Reforms in government policies, the labor market and banking and corporate sectors are seen as necessary. Bakaert and Urias (1996) report a 300 percent increase in mutual fund assets invested in emerging equity markets from 1991 through 1993. Dahlquist and Robertsson (2001) report an increase in foreign investment in Sweden, which is a developed country, from 126 billion Swedish Krona 1991 to 280 billion Swedish Krona in 1997. We seek to provide theoretical insights about the effects of equity market development on investment. We base our analysis on deadweight costs associated with equity market development. We use a variant of the Allen, Bernardo, and Welch (2000) model in an open market to perform our analysis. Our investors are institutional investors such as mutual funds, pension funds, and insurance companies. Small investors may invest in the equity markets through mutual funds. There is one bond market which facilitates lending and borrowing among investors. We subsume currency risk in the risk on returns in the equity markets. We denominate monetary returns in terms of the United States dollar. There is one composite good for consumption. Investors use the United States dollar to purchase units of the consumption good. Investors invest in the equity markets and in the bond market to maximize their expected utility of consumption. We determine equilibrium shareholdings in firms and equilibrium asset prices from first order conditions associated with investors' utility maximization problems and the market clearing conditions for the equity markets and the bond market.

We show that if domestic investors and foreign investors face equal deadweight costs in the equity markets, then investors' equilibrium shareholdings in the firms depend on their risk aversion and diversification needs. If foreign investors face higher deadweight costs in the domestic equity market than domestic investors do, then foreign investors hold a lower fraction of shares of the domestic firms than domestic investors hold, all other things being constant. Consequently, domestic investors hold a higher fraction of shares of the domestic firms. These equilibrium shareholdings may manifest home equity bias. This is a phenomenon where investors tend to invest more in the home equity market than is implied by the benefits of diversification in the international equity markets.

We show that deadweight costs tend to contribute to low asset prices. The reason is that deadweight costs make it costly for investors to acquire shares of firms. Investors who face higher deadweight costs in an equity market may decrease their fraction of shares of firms in the equity market. These investors may increase their supply of funds

to an equity market with lower deadweight costs. The investors may also increase the supply of funds to the bond market. An increased supply of funds to the bond market may cause interest rates to fall. This behavior asset prices in the capital markets is consistent with the evidence that stocks and bonds are substitutes.

The results suggest that a government may need to facilitate the development of its equity market to attract investment. For instance, the government may reduce bureaucratic red tape to reduce the cost of entry by investors to the country's equity market. The government may establish regulatory requirements for participants in the equity market. These regulatory requirements include, among others, the prohibition of insider trading and the protection of minority shareholders. A large volume of investment in the equity market would facilitate the growth of the country's economy. A liquid equity market would enable investors to smooth their consumption.

We provide numerical examples in the appendix to illustrate the insights derived from the model. We choose plausible range of parameter values. For example, the deadweight costs in the equity markets range from zero percent to five percent.

We provide examples to illustrate the effects of deadweight costs on investors' shareholdings in firms and on asset prices. As deadweight costs in a country's equity market increase, investors who face higher deadweight costs decrease their fraction of shareholdings in the firms.

Consequently, investors who face lower deadweight costs hold a higher fraction of shares of the firms. For certain parameter values investors' shareholdings may exhibit home equity bias. As deadweight costs in a country's equity market increase, asset prices tend to fall. This is because deadweight costs make it costly for investors to acquire shares of firms. The fall in share prices compensates investors for the risk they bear by investing in the equity market. This enables investors to earn high returns which are commensurate with their risky investment. We provide examples to illustrate the effects of risk on investors' shareholdings in firms and on asset prices. As risk in a country's equity market increases, domestic investors decrease their fraction of shares of the domestic firms. The reason is that domestic investors seek to lower their risk exposure by diversifying their investments. Foreign investors increase their shareholdings in the domestic firms partly due to diversification needs. We find that share prices of the domestic firms tend to fall. This is due to a decrease in demand for shares of the domestic firms. Interest rates fall because investors may increase the supply of funds the to the bond market to diversify their investments.

We provide examples to illustrate the effects of risk aversion on investors' shareholdings in firms and on asset prices. As domestic investors' risk aversion increases, domestic investors decrease the fraction of their shareholdings in domestic firms. Consequently, they decrease the fraction of their funds allocated to domestic firms. On other hand, foreign investors increase their fraction of shareholdings in domestic firms. Consequently, they increase the fraction of their funds allocated to domestic firms. Similar reasoning applies to the case where we vary foreign investors' risk aversion.

The remainder of the paper is organized as follows. Section 2 contains literature review and descries our contribution to the financial economics literature. Section 3 describes the mode and our results. Section 4 concludes the paper.

2. Literature Review

The financial economics literature emphasizes the benefits of diversification in the international equity markets. But investors tend to invest more of the funds in home equity markets than would seem to be implied by diversification in the international equity markets. This phenomenon is called home equity bias. Empirical evidence suggests that home equity bias is widespread across developed and developing countries (Chan, Covrig, and Ng, 2005). Our primary contribution is to provide theoretical insights about the effects of equity market development on investment. One measure of equity market development is the turnover ratio, which is the ratio of total value of stocks traded to the average market capitalization in a country. A second measure of equity market development is the market capitalization as a percentage of a country's gross domestic product (GDP). For example, in China the capital market has grown at a very high speed after its economic reform. The total market capitalization reached some three trillion Chinese Yuan in 2000, but the liquid market capitalization is about a third of the total market capitalization. This is because two thirds of the shares of the 649 companies listed on these exchanges are unlisted and may not be traded. Compared to G-10 countries, the market capitalization as a percentage of the country's GDP is much smaller (Neoh, Anthony, 2000).

Foreign investors face liquidity constraint and high discount rate of investment (Chen and Xiong, 2001). A third measure of equity market development is transaction costs (bid-ask spreads) associated with trading securities.

We base our theoretical analysis on deadweight costs associated with equity market development. We make three contributions to the financial economics literature. First, we find that a country's underdeveloped equity market may discourage investors from investing in the country. If foreign

investors face higher deadweight costs in the domestic equity market than domestic investors do, then foreign investors tend to hold a smaller fraction of shares of the domestic firms than domestic investors hold. Consequently, investors' equilibrium shareholdings in firms may exhibit home equity bias.

Second, we determine equilibrium asset prices and find that asset prices in less developed equity markets tend to be lower, all other things being constant. Specifically, we find that as deadweight costs increase, asset prices tend to fall. The reason is that higher deadweight costs make it more costly for investors to acquire shares of firms. Interest rates decrease because investors may increase the supply of funds to the bond market to diversify their investments. This behavior in the capital markets is consistent with evidence that stocks and bonds are (imperfect) substitutes.

Third, the results imply that a government may need to facilitate the development of its equity market to attract investment. For instance, the government may reduce bureaucratic red tape to reduce the cost of entry to its equity market. The government may establish regulatory requirements for market participants. These regulatory requirements include, among others, the prohibition of insider trading and the protection of minority shareholders. Scholars provide several non-mutually exclusive explanations for home equity bias. Some scholars argue that a country's equity market may be a good hedge against inflation and non-traded goods (Adler and Dumas, 1983; Stockman and Dumas, 1998; Tsar, 1993). Some scholars argue that Taxes, costs of cross border trade may contribute to home equity bias (Black, 1974; Cole and Obstfeld, 1991). Some scholars argue that information asymmetry may contribute to home equity bias (Merton, 1989; Brennan and Cao, 1997).

3. The Economy

We use a variant of the Allen, Bernardo, and Welch (2000) model in an open market economy to analyze the effects of equity market development on investment. For simplicity, we assume a two-country open market economy. We assume that one country is developed, for example the United States. The other country is a developing country, which has a less developed equity market.

There is a bond market which facilitates lending and borrowing among investors. Transactions in the bond market are denominated in the United States dollar. The lending and borrowing interests are equal. We shall determine the equilibrium interest rate in the model. Investors are institutional investors such as mutual funds, pension funds, and insurance companies. Small investors may invest in the equity markets through mutual funds. Empirical evidence suggests that institutional investors have similar investment strategies. They prefer large and liquid stocks (Gompers and Merrick, 2001; He, Ng, and Wang, 2004).

We assume a continuum of investors in the open market economy. The investors are uniformly distributed over the unit interval [0,1]. Thus investors have mass equal to one. The proportions of investors are given by

$$\theta_i \in (0,1), \text{ where } \theta_1 + \theta_2 = 1.$$

We assume that one unit of currency can be exchanged for one unit of consumption. This is the same as saying that the price of consumption has been normalized to one. Thus an investor's wealth enters directly into the utility function. We assume that investors' preferences are represented by a negative exponential utility function of the form

$$u(W) = -\exp(-\gamma W).$$

The variable W denotes an investor's wealth and the parameter γ denotes the investor's coefficient of absolute risk aversion. This type of utility function is common in the financial economics literature. Grossman (1976) and Allen, Bernardo and Welch (2000), among others, use a negative exponential utility function in their models.

This negative exponential utility function is bounded from above by zero if consumption grows arbitrarily large. The constant coefficient of absolute risk aversion means that there is no change to an investor's demand for risky assets with respect to changes in the investor's initial wealth. Instead, changes in an investor's initial wealth are absorbed by risk-free lending or borrowing in the bond market.

The open market economy is indexed by dates 0 and 1. We assume that the end-of-period returns V_j are normally distributed with mean μ_j and volatility σ_j. Grossman (1976) and Allen Bernardo, and Welch (2000) make a similar assumption in their models. We subsume currency risk in the risk of returns in a country's equity market. We express this multivariate normal distribution in vector form as:

$$V \sim n(\mu, \Sigma) \tag{1}$$

where

$$V = \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}, \ \mu = \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}, \ \Sigma = \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix}.$$

The matrix Σ is a covariance matrix whose diagonal elements represent variances of the returns and whose off diagonal elements represent the covariance between the returns. We assume that the determinant of the covariance matrix is positive. The joint probability density of the returns is of the form

$$g(y,z) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}} \exp\left(-\frac{1}{2(1-\rho^2)} \left(\left(\frac{y-\mu_1}{\sigma_1}\right)^2 - 2\rho\left(\frac{y-\mu_1}{\sigma_1}\right)\left(\frac{z-\mu_2}{\sigma_2}\right) + \left(\frac{z-\mu_2}{\sigma_2}\right)^2 \right) \right),$$

where $-\infty < y,\ z < \infty,\ \sigma_1 > 0,\ \sigma_2 > 0,\ -1 < \rho < 1$.
The variables *y* and *z* represent realized values of the end-of-period returns. With a multivariate normal distribution, the associated variables are uncorrelated if and only if they are independent, since the joint probability density function only factors when the correlation coefficients are zero $(\rho = 0)$ We only consider this case in our paper. The other cases do not provide further economic insights in our analysis.

They would contribute to diversification in investment. When there is a positive correlation between the returns then investing in both firms offers less diversification than the case of zero correlation. Similarly, when there is negative correlation then there is more diversification benefit. We assume 100 percent equity in the capital structure of firms in our model. Allen, Bernardo, and Welch (2000) make a similar assumption in their model. We assume this for simplicity because our focus is on shareholders.
We state an investor's utility maximization problem.

Problem Investor *i* makes investment allocations *x ji* to firms $j \in \{1, 2\}$, to maximize the investor's expected utility of wealth

$$E\left[-\exp\left(-\gamma_i W_i\right)\right] = -\exp\left(-\gamma_i E\left[W_i\right] + \frac{\gamma_i^2}{2} Var\left(W_i\right)\right)$$

where

$$W_i = \sum_{j=1}^{2} x_{ji}\left(V_j - c_{ji}\right) + \left(\omega_i - \sum_{j=1}^{2} x_{ji} p_j\right)(1+r). \tag{2}$$

We let *E* denote the expectation operator. We let *Var* denote the variance operator. We let ω_i denote date 0 cash endowment for investor *i*. We let c_{ji} denote the deadweight costs investor *i* incurs by investing in firm j. We let p_j denote date 0 share price for firm *j*. We let *r* denote the borrowing and lending interest rate. The end-of-period wealth W_i for investor *i* is the total return on investment in the stock and bond markets. The return on investment in the bond market is positive for a lender but negative for a borrower. From a standard result in probability theory the investor's end-of-period wealth is normally distributed (Muirhead, 1982). We complete squares in the integral associated with expected utility. Consequently, we obtain the right hand side of the investor's expected utility function. We record equilibrium shareholdings and asset prices in the following proposition.

Proposition Equilibrium shareholdings of firm $j \in \{1, 2\}$ are given by

$$\theta_1 x_{j1} = \frac{\theta_1\gamma_2}{\theta_1\gamma_2 + \theta_2\gamma_1} + \frac{\theta_1\theta_2\left(c_{j2} - c_{j1}\right)}{\left(\theta_1\gamma_2 + \theta_2\gamma_1\right)\sigma_j^2}$$

$$\theta_2 x_{j2} = \frac{\theta_2\gamma_1}{\theta_1\gamma_2 + \theta_2\gamma_1} + \frac{\theta_1\theta_2\left(c_{j1} - c_{j2}\right)}{\left(\theta_1\gamma_2 + \theta_2\gamma_1\right)\sigma_j^2}. \tag{3}$$

Equilibrium asset prices are given by

$$p_1 = \frac{\left(\theta_1\omega_1 + \theta_2\omega_2\right)\left(\theta_1\gamma_2 + \theta_2\gamma_1\right)\mu_1}{\left(\theta_1\gamma_2 + \theta_2\gamma_1\right)\left(\mu_1 + \mu_2\right) - B - \gamma_1\gamma_2\left(\sigma_1^2 + \sigma_2^2\right)}$$

$$- \frac{\left(\theta_1\omega_1 + \theta_2\omega_2\right)\left(\theta_1\gamma_2 c_{11} + \theta_2\gamma_1 c_{12} + \gamma_1\gamma_2\sigma_1^2\right)}{\left(\theta_1\gamma_2 + \theta_2\gamma_1\right)\left(\mu_1 + \mu_2\right) - B - \gamma_1\gamma_2\left(\sigma_1^2 + \sigma_2^2\right)}$$

$$p_2 = \frac{\left(\theta_1\omega_1 + \theta_2\omega_2\right)\left(\theta_1\gamma_2 + \theta_2\gamma_1\right)\mu_2}{\left(\theta_1\gamma_2 + \theta_2\gamma_1\right)\left(\mu_1 + \mu_2\right) - B - \gamma_1\gamma_2\left(\sigma_1^2 + \sigma_2^2\right)} \tag{4}$$

$$- \frac{\left(\theta_1\omega_1 + \theta_2\omega_2\right)\left(\theta_1\gamma_2 c_{21} + \theta_2\gamma_1 c_{22} + \gamma_1\gamma_2\sigma_2^2\right)}{\left(\theta_1\gamma_2 + \theta_2\gamma_1\right)\left(\mu_1 + \mu_2\right) - B - \gamma_1\gamma_2\left(\sigma_1^2 + \sigma_2^2\right)}$$

$$1 + r = \frac{\left(\theta_1\gamma_2 + \theta_2\gamma_1\right)\left(\mu_1 + \mu_2\right) - B - \gamma_1\gamma_2\left(\sigma_1^2 + \sigma_2^2\right)}{\left(\theta_1\gamma_2 + \theta_2\gamma_{12}\right)\left(\theta_1\omega_1 + \theta_2\omega_2\right)}$$

where

$$B = \theta_1\gamma_2\left(c_{11} + c_{21}\right) + \theta_2\gamma_1\left(c_{12} + c_{22}\right).$$

From (3) we see that if foreign investors incur higher deadweight costs in the domestic equity market than domestic investors do, then they tend to hold lower fraction of shares of the domestic firms than domestic investors hold. Indeed, if the deadweight costs that domestic investors and foreign investors incur are equal, then investors' shareholdings in firms are determined by their risk aversion and diversification needs. To see this, we substitute $c_{j1} = c_{j2} = c_j$ into the shareholdings in (3). Then investors' equilibrium shareholdings for firm $j \in \{1, 2\}$ are given by

$$\theta_1 x_{j1} = \frac{\theta_1\gamma_2}{\theta_1\gamma_2 + \theta_2\gamma_1}$$

$$\theta_2 x_{j2} = \frac{\theta_2\gamma_1}{\theta_1\gamma_2 + \theta_2\gamma_1}.$$

Deadweight costs induce low share prices for firms because these costs make it costly for investors to acquire shares of firms. Investors seek price discount to compensate them for the risk they bear by investing in these firms. The low share prices enable investors to earn high expected returns from their investments. Equilibrium share prices are dependent on investors' cash endowment in the open market economy. If investors' cash endowment is small, then share prices tend to be low. This is because low cash endowment implies that the demand for shares of firms may be low. Interest rates tend to be high to induce investors to supply funds to the bond market. This is because investors may not have large quantities of funds to supply to the bond market.

Conversely, if investors have large cash endowment in the open market economy, then share prices tend to be high. The reason is that investors

have large quantities of funds to invest in the equity markets. The increased demand for shares of firms causes their share prices to go up. Interest rates tend to be low because investors have large quantities of funds to invest in the bond market.

4. Conclusion

We use a variant of the Allen, Bernardo, Welch (2000) model in an open market economy to analyze the effects of a country's equity market development on investment. We base our analysis on deadweight costs associated with a country's equity market development.

We show that a country's underdeveloped equity market may discourage investors from investing in the country. Consequently, investors' equilibrium shareholdings may manifest home equity bias. We find that asset prices in a less developed equity market tend to be lower, all other things being constant. The results suggest that a government may need to facilitate the development of its equity market to attract investment. Our numerical examples in the appendix illustrate the insights derived from our model.

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Appendix A: Proofs

Proof of Proposition. An investor's utility maximization problem is equivalent to the following problem. Investor *i* makes the portfolio allocations *ji x* to minimize

$$-\gamma_i E[W_i]+\frac{\gamma_i^2}{2}Var(W_i)=-\gamma_i\left(x_{1i}(\mu_1-c_{1i})+x_{2i}(\mu_2-c_{2i})\right)$$
$$-\gamma_i(\omega_i-x_{1i}p_1-x_{2i}p_2)(1+r)+\frac{\gamma_i^2}{2}\left(x_{1i}^2\sigma_1^2+x_{2i}^2\sigma_2^2\right). \quad (5)$$

The left hand side of this problem is equivalent to the right hand side because we assume that the returns for the firms are uncorrelated. Investors are price takers because individual investors are too small to affect asset prices. But their aggregate demand for assets does affect asset prices. Thus the partial derivates of share prices and interest rate with respect to portfolio allocations are zero. From the first order conditions associated with the problem defined by (5) we obtain investors' shareholdings in the equity market of country $i\in\{1,2\}$These are given by

$$x_{1i} = \frac{\mu_1 - c_{1i} - (1+r)p_1}{\gamma_i \sigma_1^2}$$

$$x_{2i} = \frac{\mu_2 - c_{2i} - (1+r)p_2}{\gamma_i \sigma_2^2}. \quad (6)$$

From the second order conditions we conclude that these shareholdings yield a minimum value to the problem defined by (5). This implies that the corresponding solution to the utility maximization problem yields maximum utility. The fraction of investors in country *i* is given by ϑ_i. Therefore multiplying an investor's allocations by ϑ_i yields the aggregate allocations for investors in country *i*. Multiplying an investor's wealth by ϑ_i yields the aggregate wealth for this country's investors. We determine the equilibrium shareholdings and share prices. The market clearing condition for firm $j \in \{1,2\}$ is given by

$$\theta_1 x_{j1} + \theta_2 x_{j2} = 1. \quad (7)$$

Thus we have

$$\frac{\theta_1 \left(\mu_j - c_{j1} - (1+r)p_j\right)}{\gamma_1 \sigma_j^2} + \frac{\theta_2 \left(\mu_j - c_{j2} - (1+r)p_j\right)}{\gamma_2 \sigma_j^2} = 1.$$

The product of the gross risk-free return and this firm's share price is given by

$$(1+r)p_j = \mu_j - \frac{\theta_1 \gamma_2 c_{j1} + \theta_2 \gamma_1 c_{j2} + \gamma_1 \gamma_2 \sigma_j^2}{\theta_1 \gamma_2 + \theta_2 \gamma_1}. \quad (8)$$

We substitute the relation in (8) into the investors' shareholdings given by (6). Thus the investors' shareholdings in firm $j \in \{1,2,3\}$ are given by

$$\theta_1 x_{j1} = \frac{\theta_1 \gamma_2}{\theta_1 \gamma_2 + \theta_2 \gamma_1} + \frac{\theta_1 \theta_2 \left(c_{j2} - c_{j1}\right)}{(\theta_1 \gamma_2 + \theta_2 \gamma_1)\sigma_j^2}$$

$$\theta_2 x_{j2} = \frac{\theta_2 \gamma_1}{\theta_1 \gamma_2 + \theta_2 \gamma_1} + \frac{\theta_1 \theta_2 \left(c_{j1} - c_{j2}\right)}{(\theta_1 \gamma_2 + \theta_2 \gamma_1)\sigma_j^2}. \quad (9)$$

We determine the relations in (8) from the bond market clearing condition

$$\left(\theta_1 \omega_1 - \sum_{j=1}^{2} \theta_1 x_{j1} p_j\right) + \left(\theta_2 \omega_2 - \sum_{j=1}^{2} \theta_2 x_{j2} p_j\right) = 0. \quad (10)$$

The bond market clearing condition says that short (negative) positions held by borrowers are equal to long (positive) positions held by lenders. That is, bonds are held in zero net supply. We then substitute the equity market clearing conditions by (7) into the bond market clearing condition given by (10). We obtain the relation

$$\theta_1 \omega_1 + \theta_2 \omega_2 = \left(\theta_1 x_{11} + \theta_2 x_{12}\right) p_1 + \left(\theta_1 x_{21} + \theta_2 x_{22}\right) p_2 = p_1 + p_2. \quad (11)$$

From (8) we can express the share price for firm 2 in terms of the share price for firm 1. We have

$$p_2 = \left(\frac{(\theta_1 \gamma_2 + \theta_2 \gamma_1)\mu_2 - (\theta_1 \gamma_2 c_{21} + \theta_2 \gamma_1 c_{22}) - \gamma_1 \gamma_2 \sigma_2^2}{(\theta_1 \gamma_2 + \theta_2 \gamma_1)\mu_1 - (\theta_1 \gamma_2 c_{11} + \theta_2 \gamma_1 c_{12}) - \gamma_1 \gamma_2 \sigma_1^2} \right) p_1. \quad (12)$$

We substitute the relation in (11) into (12). Thus we obtain the equilibrium share prices

$$p_1 = \frac{(\theta_1 \omega_1 + \theta_2 \omega_2)(\theta_1 \gamma_2 + \theta_2 \gamma_1)\mu_1}{(\theta_1 \gamma_2 + \theta_2 \gamma_1)(\mu_1 + \mu_2) - B - \gamma_1 \gamma_2 \left(\sigma_1^2 + \sigma_2^2\right)}$$

$$- \frac{(\theta_1 \omega_1 + \theta_2 \omega_2)\left(\theta_1 \gamma_2 c_{11} + \theta_2 \gamma_1 c_{12} + \gamma_1 \gamma_2 \sigma_1^2\right)}{(\theta_1 \gamma_2 + \theta_2 \gamma_1)(\mu_1 + \mu_2) - B - \gamma_1 \gamma_2 \left(\sigma_1^2 + \sigma_2^2\right)}$$

$$p_2 = \frac{(\theta_1 \omega_1 + \theta_2 \omega_2)(\theta_1 \gamma_2 + \theta_2 \gamma_1)\mu_2}{(\theta_1 \gamma_2 + \theta_2 \gamma_1)(\mu_1 + \mu_2) - B - \gamma_1 \gamma_2 \left(\sigma_1^2 + \sigma_2^2\right)}$$

$$- \frac{(\theta_1 \omega_1 + \theta_2 \omega_2)\left(\theta_1 \gamma_2 c_{21} + \theta_2 \gamma_1 c_{22} + \gamma_1 \gamma_2 \sigma_2^2\right)}{(\theta_1 \gamma_2 + \theta_2 \gamma_1)(\mu_1 + \mu_2) - B - \gamma_1 \gamma_2 \left(\sigma_1^2 + \sigma_2^2\right)}, \quad (13)$$

where

$$B = \theta_1 \gamma_2 \left(c_{11} + c_{21}\right) + \theta_2 \gamma_1 \left(c_{12} + c_{22}\right).$$

We substitute (13) into (8) and obtain the equilibrium interest rate

$$1 + r = \frac{(\theta_1 \gamma_2 + \theta_2 \gamma_1)(\mu_1 + \mu_2) - B - \gamma_1 \gamma_2 \left(\sigma_1^2 + \sigma_2^2\right)}{(\theta_1 \gamma_2 + \theta_2 \gamma_1)(\theta_1 \omega_1 + \theta_2 \omega_2)}.$$

Appendix B: Numerical Examples

We provide numerical examples to illustrate the insights derived from our model. We use plausible range of parameter values. First, we illustrate the effects of deadweight costs on investment allocations and asset prices. Second, we illustrate the effect of volatility of returns on investment allocations and asset prices. Third, we illustrate the effect of risk aversion on investment allocations and asset prices.

Definitions of variables

c_{ji} = deadweight costs that investor i incurs in firm j

x^{ji} = investors of country i's shareholdings in firm j

ω^{ji} = investors of country i's allocation of funds in firm j

b^i = investors of country i's investment in the bond market.

Effect of deadweight costs on allocation of investment funds

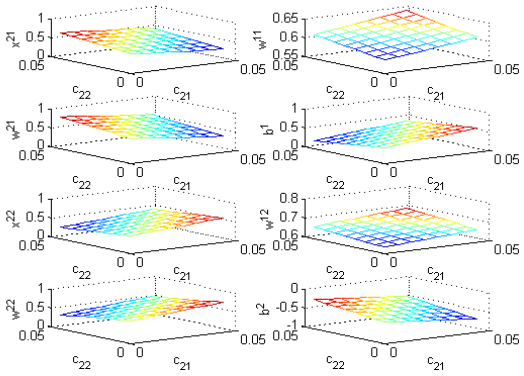

Figure 1: surfaces for shareholdings and investment allocations. The parameters are $\theta_1 = 0.52,\ \theta_2 = 0.48,\ \omega_1 = 3.235,\ \omega_2 = 1.735,\ \gamma_1 = 1.0135,\ \gamma_2 = 1.0125,$ $\mu_1 = 0.30,\ \mu_2 = 0.30,\ \sigma_1 = 0.25,\ \sigma_2 = 0.25,\ c_{11} = 0.0126,\ c_{12} = 0.0125.$

As deadweight costs that foreign investors incur in the domestic firm (c_{21}) increase, foreign investors decrease the fraction of their shareholdings in the domestic firm. Consequently, domestic investors increase the shares they hold in the domestic firm. Foreign investors increase their fraction of shareholdings in the home equity market and increase the supply funds to the bond market. That is, foreign investors are lenders ($b^1 > 0$) and domestic investors are borrowers ($b^2 < 0$). For some parameter values we observe that investors' allocation of funds exhibit home equity bias. That is, investors invest more of their funds in the home equity market than is implied by the benefits of diversification in the equity markets.

Effect of deadweight costs on asset prices

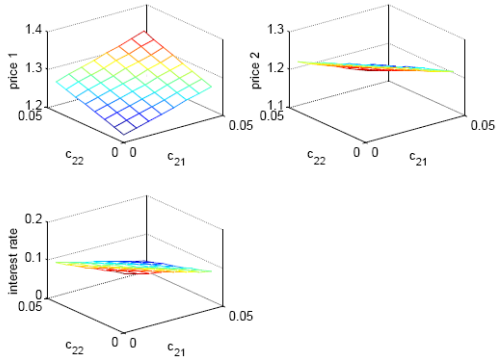

Figure 2: surfaces for asset prices. The parameters are $\theta_1 = 0.52,\ \theta_2 = 0.48,$ $\omega_1 = 3.235,\ \omega_2 = 1.735,\ \gamma_1 = 1.0135,\ \gamma_2 = 1.0125,\ \mu_1 = 0.30,\ \mu_2 = 0.30,$ $\sigma_1 = 0.25,\ \sigma_2 = 0.25,\ c_{11} = 0.0126,\ c_{12} = 0.0125.$

As deadweight costs that foreign investors incur in the domestic firm (c_{21}) increase, investors put less value on shares of the domestic firm. This is because deadweight costs make acquiring shares of the domestic firm costly. As deadweight

costs in the domestic firm increase, the share price of firm 2 decreases. Because the deadweight costs of firms 1 are held constant, investors may put more value on the shares of foreign firm. Consequently, the share price for firm 1 goes up due to increased demand for the shares of this firm. Interest rates fall because foreign investors increase the supply of funds ($b^1 > 0$) to the bond market.

Effect of variance on allocation of investment funds

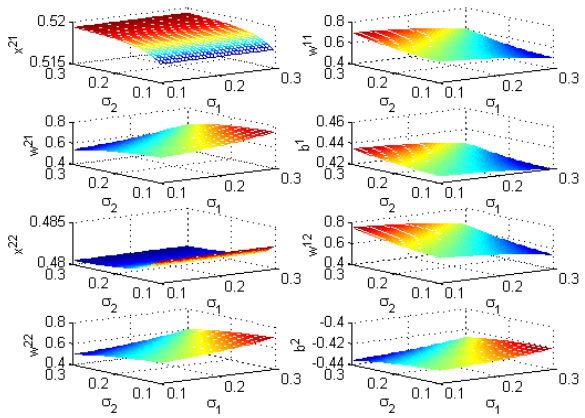

Figure 3: surfaces for shareholdings for firms 2. The parameters are $\theta_1 = 0.52,\ \theta_2 = 0.48,$
$\omega_1 = 3.235,\ \omega_2 = 1.725,\ \gamma_1 = 1.0135,\ \gamma_2 = 1.0125,\ \mu_1 = 0.30,\ \mu_2 = 0.30,$
$c_{11} = 0.0125,\ c_{12} = 0.0126,\ c_{21} = 0.0126,\ c_{22} = 0.0125.$

As the variance of returns of firm 2 (σ_2) increases, domestic investors decrease their fraction of shares of firm 2. Consequently, they decrease the funds they invest in domestic firm and increase the funds they invest in the foreign firm. They borrow in the bond market ($b^2 < 0$) to facilitate the purchase of shares of firm 1. On other hand, foreign investors increase their fraction of shares of firm 2. Thus they increase the funds they invest in the domestic firm and increase the supply of funds to the bond market. We have similar reasoning if we vary the variance of returns for firm 1.

Effect of variance on asset prices

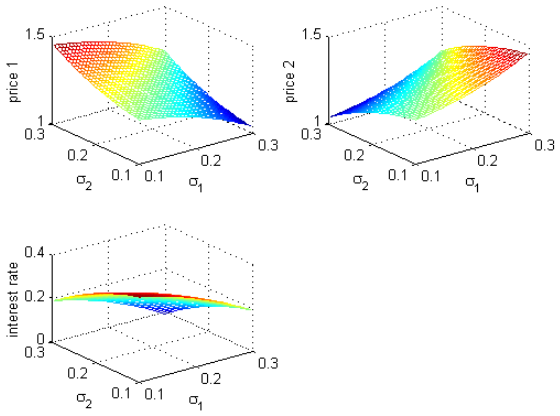

Figure 4: surfaces for asset prices. The parameters are $\theta_1 = 0.52,\ \theta_2 = 0.48,$
$\omega_1 = 3.235,\ \omega_2 = 1.725,\ \gamma_1 = 1.0125,\ \gamma_2 = 1.0126,\ \mu_1 = 0.30,\ \mu_2 = 0.30,$
$c_{11} = 0.0125,\ c_{12} = 0.0126,\ c_{21} = 0.0126,\ c_{22} = 0.0125.$

As the variance of returns of firm 2 (σ_2) increases, domestic investors decrease their shareholdings in firm 2. The share price for firm 2 falls because of decreased demand. The shares of firm 1 are relatively attractive. Thus the share price of firm 1 increases due to increased demand. The interest rate falls because foreign investors increase the supply funds to the bond market. Similar reasoning applies to the case when we vary the variance of returns of firm 1.

Effect of risk aversion on allocation of investment funds

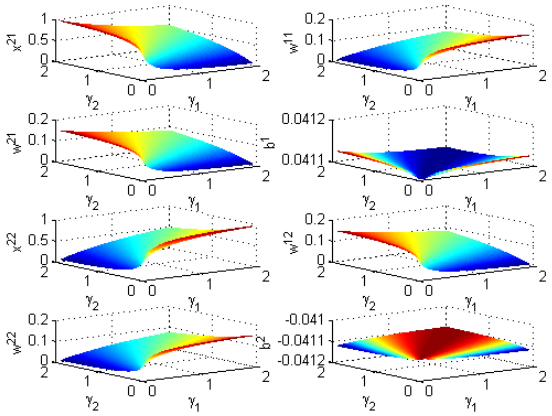

Figure 5: surfaces for shares of firm 2 and allocation of investment funds. The parameters are $\theta_1 = 0.52,\ \theta_2 = 0.48,\ \omega_1 = 0.375,\ \omega_2 = 0.235,\ \mu_1 = 0.30,\ \mu_2 = 0.30,$ $\sigma_1 = 0.25,\ \sigma_2 = 0.25,\ c_{11} = 0.0125,\ c_{12} = 0.0126,\ c_{21} = 0.0126,\ c_{22} = 0.0125.$

As the risk aversion of domestic investors (γ_2) increases, domestic investors decrease the fraction of their shareholdings in the domestic firm. Consequently, they decrease the fraction of their funds allocated to the domestic firm. Foreign investors increase their fraction of shareholdings in the domestic firm. Consequently, they increase the fraction of their funds allocated to the domestic firm. Similar reasoning applies to the case where we vary risk aversion of foreign investors.

Effect of risk aversion on asset prices

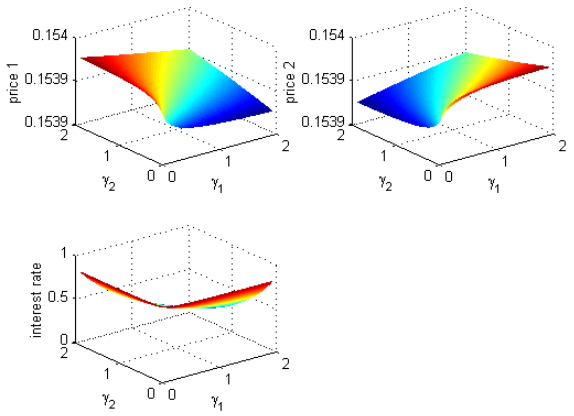

Figure 6: surfaces for asset prices. The parameters are $\theta_1 = 0.52,\ \theta_2 = 0.48,$ $\omega_1 = 0.375,\ \omega_2 = 0.235,\ \mu_1 = 0.30,\ \mu_2 = 0.30,\ \sigma_1 = 0.25,\ \sigma_2 = 0.25,$ $c_{11} = 0.0125,\ c_{12} = 0.0126,\ c_{21} = 0.0126,\ c_{22} = 0.0125.$

As the risk aversion of domestic investors (γ_2) increases, domestic investors decrease the fraction of their shareholdings in the domestic firm. The decrease in demand causes the share price of firm 2 to fall. The increase demand for shares of firm 1 causes the share price of firm 1 to rise. Consequently, they decrease the fraction of their funds allocated to the domestic firm. Foreign investors increase their fraction of shareholdings in the domestic firm. Consequently, they increase the fraction of their funds allocated to the domestic firm. Interest rates may fall due to supply of funds to the bond market. Similar reasoning applies to the case where we vary risk aversion of foreign investors.