# RISK METRICS: ASSESSING EXECUTIVE STOCK OPTIONS PLANS

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**Abstract**

This paper considers deriving measures for assessing the benefits to firms as a result of granting executive stock option plans. The metrics developed relate to assessing the expected total earnings of the company attributed to executives due to executive stock option award. The paper derives metrics based on number of shares as well as on total value of assets. The values of these metrics can be used to compare and asses the benefits to the company in awarding stock option grants by comparing the metrics with actual realized changes in total earnings. The research work in the paper complements the empirical research of Murphy (1999) and others who found the pay-performance sensitivities due to executive stock option awards. Illustrations of the metrics are carried out to show their properties and in particular for the firm WAL-MART.

**Keywords:** executives, stock options, earnings, WAL-MART

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## 1. Introduction

Stock option grants have become the major component of the executive compensation as they reward the value creation better than other forms of compensation such as bonuses tied to earnings. Unlike traded stock options in the stock exchange where customers deal with already issued shares of the company, the executive stock option awards require the issue of new stocks by the company when they are exercised. Thus executive stock options (ESO) awards create a dilution of the company assets and earnings unless there is an improvement in them as a result of better executive management and performance. Issues related to executive stock options were not examined by regulatory authorities and the firms until recently when perceived wind falls of wealth to executives were publicized. Although such awards are justified on economic grounds, shareholders, financial analysts and politicians were critical of the absence of their disclosure to the public. As a result of criticisms, Financial Accounting Standard Board, FASB (FASB 123 (R)), now requires the ESO grants to be valued and included in the financial statement of the company. The issue of ESO is a risk to the firm and shareholders. A measure of such risk either on historical ground to a firm or across the firms is useful for executive compensation boards, shareholders and financial analysts. Using historical data, researchers have estimated the pay-performance sensitivities and elasticities of shareholder wealth due to executive compensation; for example in the papers by Hall and Liebman (1998), Jensen and Murphy(1990b), (Murphy (1999). In this paper we establish formulae called metrics to estimates the expected total earnings attributed to executives due to the initiation of ESO awards. Formulae are based on earnings and the stock price process prior to ESO grant initiation. Thus subsequent to ESO award if the company earnings improves over and above that estimated by metrics derived, one could infer that the compensation provided has been beneficial to the firm.

Thus metrics derived are useful to shareholders, financial analysts and compensation boards. We illustrate the behavior of these metrics for several cases of selected parameter values and also use the data available in the public domain for the company WAL-MART and discuss the use of the metrics.

## 2. ESO Risk Metrics

*Case 1:* A risk measure per share basis.

Let N= number of issued shares of the company and S(t)=stock price at time t, t$\geq 0$.

We assume that stock price has a geometric Brownian process and is represented by $S(t) = S(0)\exp\{X(t)\}$ where X(t) is normal with mean and variance per unit time $\mu$ and $\sigma^2$.

Let $Y^*(t)$ denote the total earnings of the company up to the year t. The random variable $Y^*(t)$ can be negative and we assume, as it is a sum of random variables, that it is normal with mean $\mu_*(t)$ and variance $\sigma_*^2(t)$. Suppose that the company has a executive stock option plan with a series of executive stock option grants exercisable at times $\tau_1, \tau_2, ....., \tau_m (0 < \tau_1 < \tau_2 < .... < \tau_m)$ with the respective exercise prices $K_1, K_2, ...., K_m$.

Then the proportion of shares the executives hold at the end of the grant period $\tau_m$ is

$$M_1 = \frac{\sum_1^m n_j I(S(\tau_j) > K_j)}{N + \sum_1^m n_j I(S(\tau_j) > K_j)} \qquad (1)$$

where $n_j$ represents the number of stock options to be exercised at time $\tau_j; j = 1,2...,m.$ under the award. We may replace the denominator of (1) by its expectation under risk neutral measure so that it reduces to

$$M_1 = \frac{\sum_1^m n_j I(S(\tau_j) > K_j)}{E_Q[\ N + \sum_1^m n_j \Pr(X(\tau_j) > k_j)]} \qquad (2)$$

where $k_j = K_j / S(0)$ and Q denotes the risk neutral measure.

Suppose that the company pays dividends at a compounding rate $\theta$ per unit time.

The effect of the management of the company will be reflected in the total earnings.

We now calculate a risk metric by evaluating the proportion of the total earnings attributable to option awards after the end of exercising the series of ESOs. Let $V(t) = Y_*(t) - Y_*(\tau_m); t > \tau_m$ . Then assuming yearly earnings are independent, $V(t)$ is also normal with mean $\mu_v(t) = \mu_*(t) - \mu_*(\tau_m)$ and variance $\sigma^2{}_v(t) = \sigma^2{}_*(t) - \sigma^2{}_*(\tau_m).$ Hence the total earnings attributed to executives for the period t-$\tau_m$ as a result of ESO awards can be evaluated. Using (2) it is given by

$$E(W_1\ ) = E[V(t)M_1] = \frac{\sum_{j=1}^m p_j E\{V(t)I(X(\tau_j) > k_j)\}}{1 + \sum_{j=1}^m p_j \Phi\{\frac{(r-\theta-\sigma^2/2)\tau_j - k_j}{\sigma\sqrt{\tau_j}}\}} \qquad (3)$$

Using the appendix result and computing the expectation in (3), we have

$$E(W_1\ ) = \frac{\{\mu_*(t) - \mu_*(\tau_m)\}\sum_{j=1}^m p_j \Phi\{\frac{(r-\theta-\sigma^2/2)\tau_j - k_j}{\sigma\sqrt{\tau_j}}\} + \rho^*{}_j (r - \sigma^2/2)\phi(\frac{k_j - (r-\sigma^2/2)\tau_j}{\sigma\sqrt{\tau_j}}}{1 + \sum_{j=1}^m p_j \Phi\{\frac{(r-\theta-\sigma^2/2)\tau_j - k_j}{\sigma\sqrt{\tau_j}}\}}.$$

(4)

where $\rho_j^*$ is the coefficient of correlation between $X(\tau_j)$ and V(t).

The impact on the company earnings due to the series of ESO awards is given in (4) and unless the change in actual observed total earnings of the company exceeds $E(W_1\ )$ sometime after the end of the time $\tau_m$ the awards have not been beneficial to the company.

*Case 2:* A risk measure on dollar assets basis

In this case we are attempting to evaluate components of the total earnings of the firm on dollar assets basis due to the ESO awards for a period commencing from the exercise of all stock options of the series of grants; the proportion used is evaluated based on the value of the company assets rather than per share as in case 1. Then the proportion of dollar assets at time t after the end of the exercise period attributable to executives due to stock options is given by

$$M_2 = \frac{\sum_{j=1}^m n_j e^{r(t-\tau_j)}(S(\tau_j) > K_j)}{[NS(t\ ) + \sum_{j=1}^m n_j e^{r(t-\tau_j)}(S(\tau_j) > K_j)\ ]}; t > \tau_m. \qquad (5)$$

Again, for convenience we replace the denominator of (5) by its expectation under risk neutral measure. Hence the proportion of total earnings for the period t-$\tau_m$ of the company to be shared with ESO holders on dollar assets basis is given by

$$W_2 = \frac{V(t)\{\sum_{j=1}^m n_j e^{r(t-\tau_j)}(S(\tau_j) - K_j)_+\}}{E_Q[NS(t) + \sum_{j=1}^m n_j e^{r(t-\tau_j)}(S(\tau_j) - K_j)_+]}$$

Now we derive the expected value of the expression for $W_2$ assuming risk neutral measure for X(t).

$$E[W_2] = \frac{\sum_{j=1}^m p_j e^{r(t-\tau_j)} E[V(t).(S(\tau_j) - K_j) I(X(\tau_j) > k_j)]}{E_Q[S(t) + \sum_{j=1}^m p_j e^{r(t-\tau_j)}(S(\tau_j) - K_j)_+]}; t \geq \tau_m. \qquad (6)$$

Now using Black-Scholes formula the expectation in the denominator of (6) is given by

$$G(t) = E_Q[S(t) + \sum_{j=1}^m p_j e^{r(t_m-\tau_j)}(S(\tau_j) - K_j)_+] = S(0)e^{rt} + \sum_{j=1}^m p_j e^{rt}\ [\{S(0)\Phi(\frac{(r-\theta+\sigma^2/2)\tau_j - k_j}{\sigma\sqrt{\tau_j}})$$
$$- K_j e^{-r\tau_j}\Phi(\frac{(r-\theta-\sigma^2/2)\tau_j - k_j}{\sigma\sqrt{\tau_j}})\}].$$

(7)

The expectation in numerator component in (9) reduces to

$$E_Q[V(t)(S(\tau_j) - K_j)_+] = S(0)E_Q[V(t)e^{X(\tau_j)} I(X(\tau_j) > k_j)] - K_j E_Q[V(t)I(X(\tau_j) > k_j)].$$

(8)

Again we assume that $X(\tau_j)$ and $V(t)$ is bivariate normal with coefficient of correlation $\rho_j^*$.

Then we can derive (see Appendix)

$$E_Q[V(t)e^{X(\tau_j\ )} I(X(\tau_j) > k_j)] = \exp\{r\tau_j\}[(\mu_v(t) + \rho_j^*\sigma_v(t)\sigma\sqrt{\tau_j})\Phi\{\frac{(r+.5\sigma^2)\tau_j - k_j}{\sigma\sqrt{\tau_j}}\}$$
$$+ \rho_j^*\sigma_v(t)\phi\{\frac{(k_j - (r+.5\sigma^2)\tau_j}{\sigma\sqrt{\tau_j}}\}].$$

(9)

Hence the expression in (6) reduces to

$$E[W_2] = \frac{1}{G(t)}\{S(0)e^{rt}\sum_{j=1}^{m} p_j[\{\mu_v(t) + \rho_j \sigma\sqrt{\tau_j}\sigma_v(t)\}\Phi\{\frac{(r-\theta+\sigma^2/2)\tau_j - k_j}{\sigma\sqrt{\tau_j}}\} +$$
$$\rho_j^{*}\sigma_v(t)\phi(\frac{k_j-(r-\theta+\sigma^2/2)\tau_j}{\sigma\sqrt{\tau_j}}] - \sum_{j=}^{m} p_j K_j e^{r(t-\tau_j)}[\mu_v(t)\Phi\{\frac{(r-\theta-\sigma^2/2)\tau_j - k_j}{\sigma\sqrt{\tau_j}}\} +$$
$$\rho_j^{*}\sigma_v(t)\phi\{\frac{k_j-(r-\theta-\sigma^2/2)\tau_j}{\sigma\sqrt{\tau_j}}\}]\}. \quad (10)$$

One can conclude that, post exercising of ESO, the realized change in actual total earnings per dollar assets of the company must exceed $E[W_2]$ computed in (10) for the ESO awards to be beneficial to the firm. The company directors and shareholders can now have a measure to monitor the effectiveness of the ESO grants in improving the earnings of the company.

The sensitivity of this metric with respect to share price at award is the derivative of $E[W_2]$ at t= $\tau_m$ with respect to S(0) (similar to Greeks ) and is given by

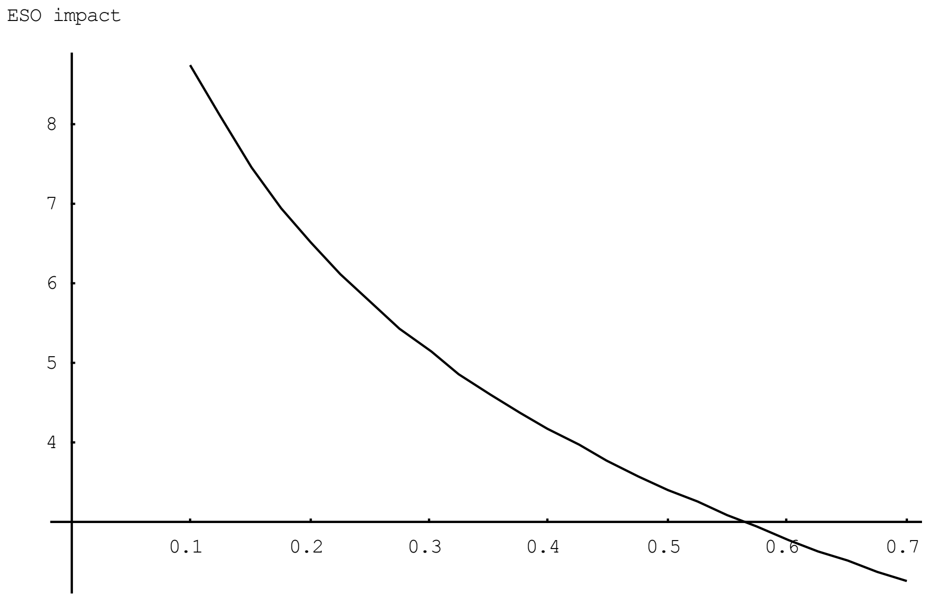
$$\frac{\partial}{\partial S}E[W_2] = \frac{e^{r\tau_m}}{G}[\sum_{j=1}^{m} p_j\{\mu_v(\tau_m) + t\rho_j^{*}\sigma\sqrt{\tau_j}\sigma_v(\tau_m)$$
$$\Phi\{\frac{(r-\theta+\sigma^2/2)\tau_j - k_j}{\sigma\sqrt{\tau_j}}\} + \rho_j^{*}\sigma_v(\tau_m)\phi\{\frac{k_j-(r-\theta+\sigma^2/2)\tau_j}{\sigma\sqrt{\tau_j}}\}]$$
$$-\frac{E[W_2]}{G}[1+\sum_{j=1}^{m} p_j\Phi\{\frac{(r-\theta+\sigma^2/2)\tau_j - k_j}{\sigma\sqrt{\tau_j}}\}]. \quad (11)$$

In the granting of ESO awards, the firm might place restrictions on exercising options based on firm's total earnings. This means that exercising of options is permitted provided that the total annual earnings or the share price exceed a pre-assigned value. Formulae for such cases can also be derived using similar computations as in deriving (4) and (10).

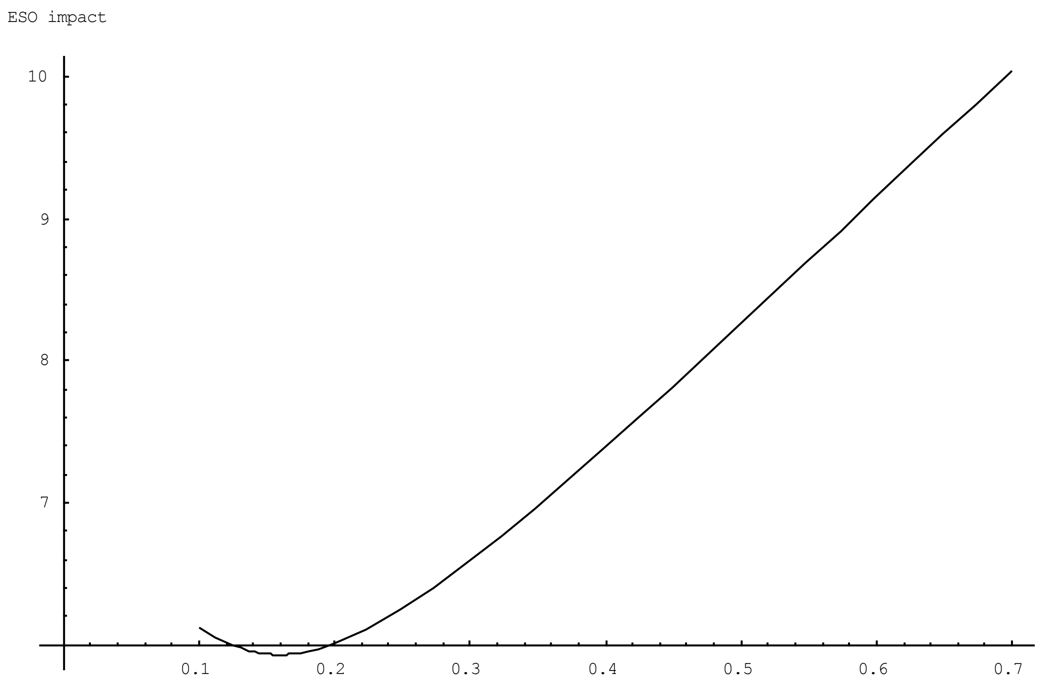
## 3. Numerical values for the Metrics

### (a)Illustrations with hypothetical parameter values

We examine the metrics given by relations (4) and (10) as volatility varies for the values of the parameters : r=0.05, $\mu_e = \$1$, $\theta = 0$ ,m=2, $\tau_1 = 5, \tau_2 = 10$ and $p_j = 0.005$, $\rho^{*}{}_j = 0.5$;j=1,2. The values of $E[W_1]$ and $E[W_2]$ are plotted against volatility. For thousand of stock options, the earnings per share expected to be shared with executives is given by 1000 $E[W_1]$. Given the risk free rate r specified, $E[W_1]$ decreases as the volatility increases as shown in Plot1. In the situation when consideration is based on dollar value of assets as in case2, we compute the expression in (10) for varying values of volatility when the parameter values are : r=0.05, $\mu_v = \$1000$, $\sigma_v = 100$, $\theta = 0$ ,m=1, S(0)= $K_1 = 50$ $p_1 = 0.005$, $\rho^{*}{}_1 = 0.5$ and $t = \tau_1 = 10.$ We observe from Plot 2 that $E[W_2]$ increases with volatility unlike the situations in case 1.On the average it appears that the increase is of the order of \$9 for a firm having total earnings with mean \$1000 and standard deviation \$100. This may be compared with the empirical results obtained by Murphy (1999).




**Plot 1:** 1000E [$W_1$ ] $\sigma$ varies




**Plot 2.** Impact of ESO on total earnings on value of assets basis

### (b) Illustrations with financial values from the firm WAL-MART

The following details from SEC 10K filings in January 2000 by the firm and those from its own web site are used for our illustrations.

**Table 1.** ESO awards details

| ESOs outstanding | Weighted average exercise price | Weighted average term in years |
|---|---|---|
| 24,000 | 5.33 | 0.6 |
| 686,000 | 7.27 | 1.0 |
| 28,336,000 | 12.00 | 5.6 |
| 10,443,000 | 19.31 | 8.0 |
| 709,000 | 29.60 | 8.6 |
| 6,374,000 | 40.11 | 9.0 |
| 4,742,000 | 46.97 | 4.5 |
| 51,314,000 | 20.39 | 6.4 |

We value the metric $E(W_1)$ as of January 31st, 2000 when its price was $62.34 and the number of shares issued N=4,143,352,994. All the conditions required are satisfied. For the evaluation of $E[W_2]$ we use EBIT (total earnings before interest and tax), in the WAl-MART web site and estimated $\mu_v(t)$ and $\sigma_v(t)$ for 2009 when all ESOs in table 1 are exercised. These values are subsequently used in the evaluation of $E[W_2]$ given by (10) for 2009. The calculated value of $E[W_2]$ is $ 1.3442x$10^6$ showing that its EBIT should increase by this amount to regard the ESO award plan to be beneficial to the company.

For another interpretation, the Black –Scholes (BS) value of all the stock options with r=.05 is evaluated to be $4.88651x$10^9$ on 1st January 2000. It may be useful to compare this cash value of all the executive stock options with some of the metrics evaluated. For example company would expect the earning to increase after all the options are exercised. Taking the case 2, considering $E[W_2]$ is the expected share of the total earnings for executives based on assets when all ESOs are exercised, company should expect the change in actual total earnings realized to increase by this amount at least for the ESO awards to be beneficial to the firm. Thus comparing $E[W_2]$ to the Black-Scholes cash value which is the incentive provided would be useful. Therefore the company should expect the total earnings to increase by at least $E[W_2]$ / BS value per dollar value of the awards. This ratio is 0.28 cents per 1000 dollar in this case and therefore the company may consider the awards to be beneficial provided actual total earnings increase exceed this ratio. It may be compared with the alternative measure developed and evaluated in the empirical study in Jensen and Murphy (1990b) who showed that CEO (chief executive officer) wealth change by $3.25 for every $1000 change in shareholder wealth.

## 4. References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## Appendix

Suppose that the random variables X, Y are bivariate normal with means $\mu_x$ and $\mu_y$ , standard deviations $\sigma_x$ and $\sigma_y$ respectively and coefficient of correlation $\rho$ . Then the joint density is given by

$$f_{X,Y}(x,y) = \frac{1}{2\pi(1-\rho^2)^{0.5}} \exp[-\frac{1}{2(1-\rho^2)}\{(\frac{x-\mu_x}{\sigma_x})^2 + (\frac{y-\mu_y}{\sigma_y})^2 - \rho\ (\frac{x-\mu_x}{\sigma_x})(\frac{y-\mu_y}{\sigma_y})\}]. \tag{A1}$$

We first evaluate $E[XI(Y \in B)]$ which is required in (3) where B is a region of Y.
From standard results on bivariate normal distribution, we have

$$E[X \middle| Y] = \mu_y + \rho\frac{\sigma_x}{\sigma_y}(y-\mu_y)\,. \tag{A2}$$

Hence

$$E[XI(Y \in B)] = \int_{y \in B} \{\mu_x + \rho\frac{\sigma_x}{\sigma_y}(y-\mu_y)\} f_Y(y)dy. \tag{A3}$$

where $f_Y(y)$ is the marginal density of Y.
Integrating the component in (A3), we have

$$E[XI(Y \in B) = E(X)\Pr(B) + \rho\sigma_x \int_{y \in B} (\frac{y-\mu_y}{\sqrt{2\pi}\sigma_y})\exp\{-.5(\frac{y-\mu_y}{\sigma_y})^2\}dy. \tag{A4}$$

Now we complete the integration in (A4) giving for any real k

$$E[XI(Y > k) = E(X)\Pr(B) + \rho\sigma_x\phi(\frac{k-\mu_y}{\sigma_y}). \tag{A5}$$

Again for the expectation component in (11), consider the expression leading to (A4).
Then for any real t ,we establish the relation

$$E[Ye^{tX} I(X > k)] = \exp[\mu_x t + .5\sigma_x{}^2 t^2\}[(\mu_y + \rho t\sigma_x\sigma_y)\Phi(\frac{\mu_x + t\sigma_x{}^2 - k}{\sigma_x}) + \rho\sigma_y\phi(\frac{k-\mu_x-\sigma_x{}^2 t}{\sigma_x}). \tag{A6}$$