CAVEAT WACC: PITFALLS IN THE USE OF THE WEIGHTED AVERAGE COST OF CAPITAL

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Abstract

In Discounted Cash Flow valuations, the WACC approach is very popular. Therefore, knowing which limitations the concept inherits is essential. The objective of this paper is thus twofold: First, it is clarified that a constant WACC rate must fail if the implied leverage ratio is time-varying. This seems to be the rationale for defining a nonlinear WACC (NLWACC). However, the NLWACC appears to be rather artificial when allowing for time-varying WACCs. Second, although the NLWACC approach is further amplified in this paper, it must be emphasized that this approach is, even then, applicable only under specific conditions while a time-varying WACC is still able to provide reliable results. In conclusion, the WACC approach is a valid workhorse whose results can be economically interpreted.

Keywords: WACC, cost of capital, capital structure

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1. Introduction

Methodological issues on valuation have experienced a remarkable renaissance in the financial literature over the last years. Fernandez (2004) initiated a provocative discussion claiming that the value of tax shields is not equal to the present value of tax shields. This claim is indeed provocative as it implies inter alia that the principle of value additivity is not working and that the seminal propositions of Modigliani and Miller are flawed. The subsequent discussion led in several journals revealed that the claim was not well substantiated. For example, Arzac and Glosten (2005) reconsider tax shield valuation looking at a value-based debt policy in the spirit of Miles and Ezzell (1980), and prove the validity of the respective valuation formula. Comments to Fernandez (2004) are given in a comparable vein also by Fieten *et al.* (2005), Cooper and Nyborg (2006), (2007), and Massari *et al.* (2008).

The advanced textbook by Kruschwitz and Löffler (2006) offers a rigorous and authoritative perspective on Discounted Cash Flow valuation leading to many new insights. Cooper and Nyborg (2008) analyze the case of tax-adjusted discount rates with investor taxes and risky debt.

Ruback (2002) advocates the Capital Cash Flows methodology as a simple approach to incorporate the value of the debt tax shield in valuation formulae. His methodology is founded on a value-based debt policy. Booth (2007) makes the case that the Capital Cash Flows and the Adjusted Present Value methodology is not easy to handle in specific valuation scenarios while the weighted average cost of capital (WACC) is a more flexible methodology.

For valuing companies or projects, WACC is the dominant Discounted Cash Flow approach in practice. The idea of this long-lived concept is that the total market value (debt and equity) is calculated by discounting the unlevered cash flows with the weighted returns for shareholders r_e and bondholders r_d.[1] The tax rate t adjusts the return to bondholders downward to reflect the interest tax shield.

$$WACC = w_d r_d (1-t) + w_e r_e$$

Recent survey evidence from the US, UK, and Germany supports the WACC's dominance in practice.[2] Three reasons could support this success: First, the input parameters are rather easily estimated from market data. Second, applying WACC does not require a commitment to judge how risky debt tax shields are. In other words, the debt policy does not have to be specified. This is not an appealing constellation as it is unknown how much the debt tax shield contributes to the company value. However, given today's knowledge, only a vague idea exists which debt policies companies actually apply. Therefore, putting a valid value on the debt tax shield given these estimation problems is not such an easy task. Third, the WACC is computationally elegant. An iterative or recursive procedure is generally not necessary. However, the WACC also has its known shortcomings. It implies by definition that a periodic rebalancing of debt takes place to maintain the capital structure set forth in the WACC formula. If this

[1] For ease of exposition the notation of Miller (2007) is adopted. Expectation operators are therefore also dropped.

[2] For the US, see Bruner, Eades, Harris, & Higgins (1998), and Graham, & Harvey (2001), for the UK, see Arnold, & Hatzopoulos (2000), and for Germany, see Lobe, Niermeier, Essler, & Röder (2008).

inherent mechanism is not acknowledged, WACC is prone to errors.

(Miller, 2007) challenges WACC with a nonlinear WACC (NLWACC). My concern is to evaluate the validity of his assertion that WACC is not quite right, and to examine the potential contribution of NLWACC to the literature.

The remainder of the paper is organized as follows. In section 2, the newly introduced NLWACC is motivated and applied. Also, the NLWACC is generalized to allow for annuities with growth rates g ≠ 0%. In section 3, the concept of the NLWACC will be revisited from the perspective of rebalancing the capital structure using WACC before taxes. The merits of WACC and NLWACC are discussed. Taxes being crucial in this context as shown in the seminal work by Modigliani and Miller (1958), section 4 analyzes the after tax-case. Finally, section 5 summarizes shortly the findings and offers an outlook.

2. WACC and modified WACC: an example

The notion of the modified WACC shall be covered briefly here:

1) Looking at a levered project with given outlays IC_0, cost of capital WACC, and duration N, which break-even unlevered cash flows CF has the project to deliver to be acceptable? Financial acceptance is measured with the net present value NPV. To derive a unique solution for this question, an annuity structure (allowing for geometric growth) is imposed:[3]

$$NPV = 0 = -IC_0 + \frac{CF\left[1-\frac{(1+g)^N}{(1+WACC)^N}\right]}{WACC-g} \Leftrightarrow CF = \frac{IC_0\cdot(WACC-g)}{1-\frac{(1+g)^N}{(1+WACC)^N}} \tag{2}$$

Adopting the numerical example of Miller (2007) which assumes g = 0%, IC_0 = \$200,000, t = 0%, $WACC = w_d r_d(1-t) + w_e r_e =$ $0.25\cdot 0.06 + 0.75\cdot 0.12 = 0.105$, and N = 8, leads to

$$CF = \frac{\$200,000\cdot 0.105}{1-(1.105)^{-8}} \approx \$38,173.86.$$

This threshold operating cash flow CF belongs to the shareholders and bondholders of the company.

2) Having performed this exercise Miller (2007) further asks what the equivalent annuity for shareholders CF_e and bondholders CF_d is? Equivalence here again is achieved by equating the NPV with zero at time T = 0 respectively. This leads in analogy to Eq. (2) to (assuming g = 0%):

$$CF_e = \frac{w_e\cdot IC_0\cdot r_e}{1-(1+r_e)^{-N}} = \frac{0.75\cdot\$200,000\cdot 0.12}{1-(1+0.12)^{-8}} \approx \$30,195.43\text{, and}$$

$$CF_d = \frac{w_d\cdot IC_0\cdot r_d}{1-(1+r_d)^{-N}} = \frac{0.25\cdot\$200,000\cdot 0.06}{1-(1+0.06)^{-8}} \approx \$8,051.80$$

Obviously, the sum of both annuities differs from the annuity of the sum of both flows:

$$\underbrace{CF_e\ (\$30,195.43) + CF_d\ (\$8,051.80)}_{CF(\ \$38,247.23)} \neq CF\ (\$38,173.86)$$

3) Discounting the sum of both annuities (\$38,247.23) with the textbook WACC of 10.5% will overestimate the project value in this numerical example.[4] To overcome this misvaluation, Miller (2007) suggests discounting this cash flow with a modified version of the WACC dubbing it NLWACC (nonlinear WACC). This modified WACC is an internal rate of return r which can be expressed as an implicit function when incorporating also the possibility of geometrically growing annuities.[5]

$$\frac{r-g}{1-\frac{(1+g)^N}{(1+r)^N}} = \frac{w_e(r_e-g)}{1-\frac{(1+g)^N}{(1+r_e)^N}} + \frac{w_d(r_d-g)}{1-\frac{(1+g)^N}{(1+r_d)^N}}$$

In the numerical example with g = 0%, interpolating for r yields according to Eq. (3):

$$\frac{r}{1-(1+r)^{-8}} = \underbrace{\frac{0.75\cdot 0.12}{1-1.12^{-8}} + \frac{0.25\cdot 0.06}{1-1.06^{-8}}}_{=0.191236} \Leftrightarrow r \approx 0.105553$$

Discounting with r now leads to a zero-NPV according to Eq. (2):

$$NPV = -IC_0 + \frac{CF\left[1-\frac{(1+g)^N}{(1+r)^N}\right]}{r-g} = -\$200,000 + \frac{\$38,247.23\left[1-(1+0.105553)^{-8}\right]}{0.105553} \approx \$0$$

The final deduction of Miller (2007) is that WACC is more or less flawed and that NLWACC is needed.

In the following section 3, I put the NLWACC in perspective using the initial example. The analysis reveals why a traditional WACC can not work in this scenario. Also, I show how the challenger, the NLWACC, can be interpreted, and I demonstrate that the NLWACC is a rather narrow concept which is not superior to the WACC.

3. WACC and modified WACC revisited – before taxes

First, it is helpful to remember that in this before tax-setting and under the assumptions set forth by Modigliani and Miller (1958) the capital structure irrelevance theorems hold. (To be absolutely clear, the just mentioned Nobel Prize laureate Merton H. Miller is not to be confused with Richard A. Miller who proposed NLWACC.) This implies that no additional value can be created while dividing the financing funds in a different manner. In other words, the

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3 The following geometric series is evaluated: CF = CF_1, CF (1+g) = CF_2, … , CF $(1+g)^{N-1}$ = CF_N.

4 The present value then is \$200,384.42 which is marginally higher than the true present value of \$200,000.

5 See (Miller, 2007), p. 8, Eq. (23) for g = 0%. The derivation incorporating g is straightforward, and thus needs not be shown here.

weighted average cost of capital equal the unlevered cost of equity. Secondly, it is necessary to introduce a general return definition. A straightforward and commonly applied definition incurs the total return R_T in period T[6]

$$R_T = \frac{D_T + V_T - V_{T-1}}{V_{T-1}}, \tag{4}$$

where D_T are inflows/outflows in period T (dividends, etc.), and V_T is the market value at the end of period T. The total return R can be defined as a return for shareholders of levered (unlevered) projects r_e (r_u), bondholders r_d and for both claimholders combined as WACC. WACC with time-varying input parameters can be written down more generally than in Eq. (1) as follows:[7]

$$WACC_T = w_{d,T-1} \cdot r_{d,T} \cdot (1-t) + w_{e,T-1} \cdot r_{e,T} \tag{5}$$

One of the constituent characteristics of the WACC concept is that returns to shareholders and bondholders are weighted with their respective market value weights of the prior period: $w_{d,T-1} = \frac{V_{d,T-1}}{V_{e,T-1} + V_{d,T-1}}$, and $w_{e,T-1} = 1 - w_{d,T-1}$. It is important to emphasize that the definition of WACC is based on market, and not on book value weights. Thus, discounting multi-period cash flows with a time-constant WACC implies ceteris paribus a constant relative capital structure over time. This is demonstrated for the initial example in Panel A of Table 1 showing the expected levels of equity and debt over time given the information at T = 0.

Based on the calculations in Panel A of Table 1 it is natural to compute the implied cash flows to shareholders and bondholders over the life of the project in Panel B. The results reveal two remarkable points: First, the cash flow when divided between bondholders and shareholders is sufficient to pay each group its necessary cash flow. Discounting these cash flows with their respective returns leads to their implied market values. Second, both flows do not conform to an annuity. In fact, the equity cash flows decrease while the debt cash flows increase over time. Therefore, forcing the cash flows to shareholders and bondholders to be annuities portrays a rather different scenario (see section 2, part 2)). This scenario II is investigated in Table 2.

Panel A of Table 2 now shows that market values (i.e., $V_{e,T}$, $V_{d,T}$, and V_T) are identical with market values given in Panel A of Table 1 only at T = 0. At other valuation dates T, the values differ, hence exhibiting a different scenario than scenario I considered in Table 1. Scenario I (rebalancing at a constant ratio of debt to market value), and scenario II (rebalancing at an implicit time-varying ratio of debt to market value) are obviously not directly comparable.

Panel B of Table 2 analyzes the capital structure ratios over time in scenario II and highlights several implications for the returns. First, the debt ratio is expected to shrink over time. Appendix B shows that under plausible conditions this is generally true. It is only the same as in Panel A of Table 1 at T = 0. Second, this observation has an impact on the WACC returns according to Eq. (5). Under plausible conditions, WACC increases over time. Third, because the return to equity is supposed to be constant in this valuation exercise a very specific behaviour of the operating returns over time is implied when the capital structure is changing. Building on the (Modigliani and Miller, 1958) proposition 2, Eq. (6) postulates that the operating return $r_{u,T}$ is expected to behave over time as follows.

$$r_e = r_{u,T} + (r_{u,T} - r_d)\frac{w_{d,T-1}}{w_{e,T-1}} \Leftrightarrow r_{u,T} = \frac{r_e + r_d \frac{w_{d,T-1}}{w_{e,T-1}}}{1 + \frac{w_{d,T-1}}{w_{e,T-1}}} \tag{6}$$

This built-in feature seems not very appealing. Confirming the irrelevance theorem again, $WACC_T = r_{u,T}$ proves to be true.

Panel B of Table 2 also explains why discounting with a time-constant WACC has to fail. The WACC is simply time-varying in scenario II. Textbooks usually do not emphasize that the WACC can also be time-varying.[8] However, the use of a time-varying WACC has the advantage of an easy economic interpretation. A changing WACC points at a changing capital structure over time. The NLWACC does not provide this information. Thus, discounting debt and equity annuity cash flows ($38,247.23) with time-varying WACCs leads to consistent results shown in Panel A of Table 2.

$$V_{\tau-1} = \sum_{T=\tau}^{N} \frac{CF_T}{\prod_{j=\tau}^{N}\left(1 + WACC_j\right)}$$

How can the modified WACC which was employed in section 2 be interpreted now? Instead of using time-varying WACCs, the NLWACC allows to calculate the present value at T = 0

- with a single (amalgamated) discount rate
- if the cash flows to shareholders and bondholders exhibit an annuity structure, and
- if the operating cost of capital incidentally follow a specific structure to ensure a constant equity return over time (as in the no tax-case shown by Eq. (6)).

[6] See e.g. (Campbell *et al.*, 1997), p. 12.

[7] A derivation of Eq. (5) can be found in Appendix A.

[8] See, for example, (Brealey *et al.*, 2008), (Copeland *et al.*, 2005), (Daves *et al.*, 2004), (Lundholm and Sloan, 2004), (Stowe *et al.*, 2007), (Titman and Martin, 2008), (Koller *et al.*, 2005).

Table 1. Scenario I (before taxes)

Panel A: Total market value, equity market and debt market value over the life cycle of the project

T	0	1	2	3	4	5	6	7	8
CF_T (in \$)		38,173.86	38,173.86	38,173.86	38,173.86	38,173.86	38,173.86	38,173.86	38,173.86
V_T (in \$)	200,000.00	182,826.17	163,849.06	142,879.35	119,707.82	94,103.28	65,810.26	34,546.48	0.00
$V_{e,T}$ (in \$)	150,000.00	137,119.63	122,886.79	107,159.51	89,780.86	70,577.46	49,357.70	25,909.86	0.00
$V_{d,T}$ (in \$)	50,000.00	45,706.54	40,962.26	35,719.84	29,926.95	23,525.82	16,452.57	8,636.62	0.00
w_e	0.750	0.750	0.750	0.750	0.750	0.750	0.750	0.750	-
w_d	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	-

CF_T is the aggregated cash flow to shareholders and bondholders during the period T. V_T is the total market value (debt and equity, $V_{d,T}$ + $V_{e,T}$) at date T. $V_{e,T}$ is the equity market value ($w_e \cdot V_T$) at date T, and $V_{d,T}$ is the debt market value ($w_d \cdot V_T$) at date T. No taxes t = 0, WACC = $r_u = w_d \cdot r_d + w_e \cdot r_e = 0.25 \cdot 0.06 + 0.75 \cdot 0.12 = 0.105$, and N = 8. V_T is the present value of CF_T given WACC. w_e is the equity weight (equity/total value), and w_d is the debt weight (debt/total value). The weights are constant over time.

Panel B: Implied cash flows to shareholders and bondholders over the life cycle of the project

T	1	2	3	4	5	6	7	8
$CF_{e,T}$ (in \$)	30,880.40	30,687.19	30,473.70	30,237.79	29,977.11	29,689.06	29,370.76	29,019.04
$CF_{d,T}$ (in \$)	7,293.46	7,486.67	7,700.16	7,936.07	8,196.75	8,484.80	8,803.10	9,154.82
CF_T (in \$)	38,173.86	38,173.86	38,173.86	38,173.86	38,173.86	38,173.86	38,173.86	38,173.86

$CF_{e,T}$ is the implied cash flow to shareholders during the period T: $CF_{e,T} = CF_T - CF_{d,T}$. $CF_{d,T}$ is the implied cash flow to bondholders during the period T based on Panel A: $CF_{d,T} = V_{d,T-1} \cdot r_d + (V_{d,T-1} - V_{d,T})$. The aggregated cash flow to shareholders and bondholders during the period T is $CF_T = CF_{e,T} + CF_{d,T}$. Discounting these flows at $r_e = 0.12$ for $CF_{e,T}$, and $r_d = 0.06$ for $CF_{d,T}$ leads to the same values of $V_{e,T}$ and $V_{d,T}$ as in Panel A.

Table 2. Scenario II (before taxes)

Panel A: Annuity cash flows, and implied market values over the life cycle of the project

T	0	1	2	3	4	5	6	7	8
$CF_{e,T}$ (in \$)		30,195.43	30,195.43	30,195.43	30,195.43	30,195.43	30,195.43	30,195.43	30,195.43
$CF_{d,T}$ (in \$)		8,051.80	8,051.80	8,051.80	8,051.80	8,051.80	8,051.80	8,051.80	8,051.80
$V_{e,T}$ (in \$)	150,000.00	137,804.59	124,145.71	108,847.77	91,714.07	72,524.33	51,031.82	26,960.21	0.00
$V_{d,T}$ (in \$)	50,000.00	44,948.22	39,593.31	33,917.11	27,900.34	21,522.56	14,762.11	7,596.04	0.00
V_T (in \$)	200,000.00	182,752.81	163,739.02	142,764.88	119,614.41	94,046.89	65,793.93	34,556.24	0.00

$CF_{e,T}$ and $CF_{d,T}$ is the annuity cash flow to shareholders and bondholders, respectively. The implied equity market value $V_{e,T}$ at date T is the present value given $CF_{e,T}$ and $r_e = 0.12$. The implied debt market value $V_{d,T}$ at date T is the present value given $CF_{d,T}$ and $r_d = 0.06$.

V_T is the total market value of debt and equity ($V_{d,T} + V_{e,T}$) at date T.

Panel B: Implied capital structure ratios and returns over the life cycle of the project

T	0	1	2	3	4	5	6	7	8
$w_{e,T}$	0.750	0.754	0.758	0.762	0.767	0.771	0.776	0.780	-
$w_{d,T}$	0.250	0.246	0.242	0.238	0.233	0.229	0.224	0.220	-
$WACC_T = r_{u,T}$		0.10500	0.10524	0.10549	0.10575	0.10600	0.10627	0.10654	0.10681

$w_{e,T}$ is the time-varying equity weight (equity/total value), and $w_{d,T}$ is the time-varying debt weight (debt/total value) based on Panel A. The time-varying $WACC_T$ is computed via Eq. (5) as $WACC_T = w_{d,T-1} \cdot r_d + w_{e,T-1} \cdot r_e = w_{d,T-1} \cdot 0.06 + w_{e,T-1} \cdot 0.12$, and is confirmed by Eq. (6) calculating $r_{u,T}$. Applying Eq. (7), the total market values of Panel A are confirmed by discounting CF_T (= $CF_{e,T} + CF_{d,T}$) with $WACC_T$.

The NLWACC certainly is not a return as defined by Eq. (4). It is an amalgamation of time-varying returns into one discount rate, as typically is the case with internal rate of return procedures. This limits its economic interpretation. If one wants to use NLWACC also for prospective valuation at dates T > 0, it has to be updated, that is, N in Eq. (3) has to be updated. The conditions set forth in scenario II are rather limiting (even when allowing for $g \neq 0\%$) and only then the use of NLWACC is admissible. Clearly, the NLWACC does not help in situations where cash flows are not annuities. A time-varying WACC does not share these limitations. Its use leads to consistent results.

However, it is not clear why under scenario II the flow to equity approach is not used instead. This would be the natural choice. The NLWACC (and WACC) seem like a detour.

Under scenario I, WACC is definitely the right choice, and even under scenario II a time-varying WACC (keeping the linear structure), solves the problem as does the NLWACC. WACC obviously is a technique better able to handle more general situations than NLWACC.

4. Analysis after taxes

Modigliani and Miller (1958) have shown in their seminal work that without taxes the WACC concept does not render any additional insights in comparison to the unlevered cost of capital. The after tax-WACC is covered lengthy in Miller's (2007) paper. Thus, to reconcile the argument taxes have to be considered.

Again, in scenario I, an after tax-annuity is discounted with a constant after tax-WACC to arrive at V_0 = \$200,000, which implies ceteris paribus, a constant relative capital structure over time. The after tax-annuity now is \$37,488.80. This is demonstrated for the initial example in Panel A of Table 3 showing the expected levels of equity and debt over time given the information at T = 0.

Based on the calculations in Panel A of Table 3 it is obvious to compute the implied cash flows to shareholders and bondholders over the life of the project in Panel B. The results again reveal that actually both flows do not conform to an annuity. In fact, the equity cash flows decrease while the debt cash flows increase over time. Therefore, forcing the cash flows to shareholders and bondholders to be annuities portrays a rather different scenario which is investigated further in Table 4. To strengthen the argument, an alternative calculation based on the Adjusted Present Value technique (APV) is provided in Panel C. As is well known, the operating value (unlevered value) V_u and the value of the debt tax shield V_t are valued separately. Unlike in the no tax-case, it is now important to specify which financing policy is pursued. Different debt tax policies will require a case-by-case discussion. For the purpose of illustration, I consider a value-based debt policy only.[9] Miles and Ezzell (1980) and Kruschwitz and Löffler (2006) show for this policy:

$$r_e = r_{u,T} + \left(r_{u,T} - r_d\right)\cdot\left(1 - \frac{r_d t}{1+r_d}\right)\cdot\frac{w_{d,T-1}}{w_{e,T-1}} \Leftrightarrow r_{u,T} = \frac{r_e + r_d\cdot\left(1 - \frac{r_d t}{1+r_d}\right)\cdot\frac{w_{d,T-1}}{w_{e,T-1}}}{1 + \left(1 - \frac{r_d t}{1+r_d}\right)\cdot\frac{w_{d,T-1}}{w_{e,T-1}}} \tag{8}$$

The unlevered value is computed with the unlevered cost of capital as follows:

$$V_{u,\tau-1} = \sum\nolimits_{T=\tau}^{N} \frac{CF_T}{\prod_{j=\tau}^{N}\left(1+r_{u,j}\right)}$$

In the special case of a time-constant r_u, the formula is easier:

$$V_{u,\tau-1} = \sum\nolimits_{T=\tau}^{N} \frac{CF_T}{\left(1+r_u\right)^{T-\tau+1}}.$$

The value of the debt tax shield is risky given this financing policy, and is therefore discounted with the unlevered cost of equity apart from the cash flow of the previous period which is certain.

$$V_{t,\tau-1} = \sum\nolimits_{T=\tau}^{N} \frac{V_{d,T-1}\cdot r_d \cdot t}{\prod_{j=\tau}^{N-1}\left(1+r_{u,j}\right)\left(1+r_d\right)} \tag{9}$$

Simplifying with a time-constant r_u leads to

$$V_{t,\tau-1} = \sum\nolimits_{T=\tau}^{N} \frac{V_{d,T-1}\cdot r_d \cdot t}{\left(1+r_u\right)^{T-\tau}\left(1+r_d\right)}.$$

The APV approach confirms the consistency of the calculation in Panel C of Table 3. To arrive at the equity value, the value of debt has to be subtracted from the total market value (V_u + V_t).

Under scenario II (forced annuity structure) Panel A of Table 4 exhibits that market values are identical with market values given in Panel A of Table 3 only at T = 0.[10] For other valuation dates T, values are different, hence exhibiting a different scenario than scenario I considered in Table 3. Scenario I (rebalancing at a constant ratio of debt to market value) and scenario II (rebalancing at an implicit time-varying ratio of debt to market value) are, again, not directly comparable.

Panel B of Table 4 analyzes the capital structure ratios over time in scenario II and highlights several implications for the returns. First, the debt ratio is expected to shrink over time again. It is only the same as in Panel A of Table 3 at T = 0. Second, this observation has an impact on the WACC returns according to Eq. (5). WACC increases over time. Third, because in this valuation exercise the return to equity is supposed to be constant a very specific behaviour of the operating returns over time is implied when the capital structure is changing. The unlevered cost of equity increases over time. Again, this built-in feature seems not very appealing.

Panel B of Table 4 also explains why discounting with a time-constant WACC has to fail. WACC is simply time-varying in scenario II. The NLWACC does not provide any information about the changing capital structure. Thus, discounting unlevered cash flows with time-varying WACCs leads to results consistent with Panel A of Table 4. The arguments already established in section 3 can be analogously repeated. Additionally, Miller's (2007) claim that in the tax-case the integration of the interest tax shield in the WACC formula seems misplaced does not have to be followed. The time-varying WACC approach proves this fact. The APV calculation in Panel C of Table 4 confirms that operating returns are time-varying, indeed, and also confirms the results of Panel A and B. For example, the value of the debt tax shield at the beginning of period 6 is calculated based on Eq. (10) as:

$$V_{t,6} = \frac{295.21}{1.06} + \frac{151.91}{1.10674\cdot 1.06} = 407.99\cdot \tag{10}$$

The analysis underlines that in the tax-case the situation is not getting any better for the NLWACC and the aforementioned results hold in principle.

[9] For other policies see (Kruschwitz and Löffler, 2006).

[10] Panel A of Table 4 is identical with Panel A of Table 2. Therefore, also the weights in Panel B of Tables 2 and 4 are the same. The returns are different, of course, as different tax situations are considered in Table 2 and 4.

Table 3. Scenario I (after taxes)

Panel A: Total market value, equity market and debt market value over the life cycle of the project

T	0	1	2	3	4	5	6	7	8
CF_T (in \$)		37,488.80	37,488.80	37,488.80	37,488.80	37,488.80	37,488.80	37,488.80	37,488.80
V_T (in \$)	200,000.00	182,511.18	163,273.50	142,112.05	118,834.45	93,229.10	65,063.21	34,080.73	0.00
$V_{e,T}$ (in \$)	150,000.00	136,883.38	122,455.12	106,584.04	89,125.84	69,921.82	48,797.40	25,560.55	0.00
$V_{d,T}$ (in \$)	50,000.00	45,627.79	40,818.37	35,528.01	29,708.61	23,307.27	16,265.80	8,520.18	0.00
w_e	0.750	0.750	0.750	0.750	0.750	0.750	0.750	0.750	-
w_d	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250	-

CF_T is the unlevered cash flow during period T. V_T is the total market value (debt and equity, $V_{d,T} + V_{e,T}$) at date T. $V_{e,T}$ is the equity market value ($w_e \cdot V_T$) at date T. $V_{d,T}$ is the debt market value ($w_d \cdot V_T$) at date T. Taxes are considered with t = 0.3333, WACC = $w_d \cdot r_d \cdot (1\text{-}t) + w_e \cdot r_e$ = $0.25 \cdot 0.06 \cdot 0.6667 + 0.75 \cdot 0.12 = 0.10$, and N = 8. V_T is the present value of CF_T given WACC at date T. w_e is the equity weight (equity/total value), and w_d is the debt weight (debt/total value). The weights are constant over time.

Panel B: Implied cash flows to shareholders and bondholders over the life cycle of the project

T	1	2	3	4	5	6	7	8
$CF_{d,T}$ (in \$)	7,372.20	7,547.09	7,739.47	7,951.08	8,183.86	8,439.91	8,721.57	9,031.39
$V_{d,T\text{-}1} \cdot r_d \cdot (1\text{-}t) + (V_{d,T\text{-}1} - V_{d,T})$ (in \$)	6,372.20	6,634.53	6,923.10	7,240.52	7,589.68	7,973.76	8,396.25	8,860.99
$CF_{e,T}$ (in \$)	31,116.60	30,854.27	30,565.70	30,248.28	29,899.12	29,515.04	29,092.55	28,627.81
CF_T (in \$)	37,488.80	37,488.80	37,488.80	37,488.80	37,488.80	37,488.80	37,488.80	37,488.80

$CF_{d,T}$ is the implied cash flow to bondholders during the period T based on Panel A: $CF_{d,T} = V_{d,T\text{-}1}\ r_d + (V_{d,T\text{-}1} - V_{d,T})$. To compute the unlevered cash flow CF_T $V_{d,T\text{-}1} \cdot r_d \cdot (1\text{-}t) + (V_{d,T\text{-}1} - V_{d,T})$ and $CF_{e,T}$ is needed. $CF_{e,T}$ is the implied cash flow to shareholders during the period T: $CF_{e,T} = CF_T - CF_{d,T}$. Discounting $CF_{e,T}$ with $r_e = 0.12$ leads to $V_{e,T}$, and discounting $CF_{d,T}$ with $r_d = 0.06$ leads to $V_{d,T}$.

Panel C: Adjusted Present Value and market values over the life cycle of the project

T	0	1	2	3	4	5	6	7	8
$V_{u,T}$ (in \$)	196,260.03	179,420.39	160,809.00	140,239.44	117,505.69	92,380.04	64,610.85	33,919.97	0.00
$V_{d,T\text{-}1} \cdot r_d \cdot t$ (in \$)		1,000.00	912.56	816.37	710.56	594.17	466.15	325.32	170.40
$V_{t,T}$ (in \$)	3,739.95	3,090.79	2,464.50	1,872.61	1,328.77	849.05	452.36	160.76	0.00
$V_T = V_{u,T} + V_{t,T}$ (in \$)	200,000.00	182,511.18	163,273.50	142,112.05	118,834.45	93,229.10	65,063.21	34,080.73	0.00

For illustration purposes a value-based debt policy is assumed. The unlevered firm value $V_{u,T}$ is computed by discounting CF_T based on Eq. (9) with a time-constant $r_u = 0.10521$ according to Eq. (8) . The debt tax shield $V_{d,T\text{-}1} \cdot r_d \cdot t$ is discounted with r_u and r_d to arrive at the value of the debt tax shield $V_{t,T}$ according to Eq. (10). V_T is the total market value ($V_{u,T} + V_{t,T}$) at date T.

Table 4. Scenario II (after taxes)

Panel A: Annuity cash flows, and implied market values over the life cycle of the project

T	0	1	2	3	4	5	6	7	8
$CF_{e,T}$ (in \$)		30,195.43	30,195.43	30,195.43	30,195.43	30,195.43	30,195.43	30,195.43	30,195.43
$CF_{d,T}$ (in \$)		8,051.80	8,051.80	8,051.80	8,051.80	8,051.80	8,051.80	8,051.80	8,051.80
$V_{e,T}$ (in \$)	150,000.00	137,804.59	124,145.71	108,847.77	91,714.07	72,524.33	51,031.82	26,960.21	0.00
$V_{d,T}$ (in \$)	50,000.00	44,948.20	39,593.30	33,917.10	27,900.33	21,522.55	14,762.11	7,596.04	0.00
V_T (in \$)	200,000.00	182,752.79	163,739.01	142,764.87	119,614.40	94,046.88	65,793.92	34,556.24	0.00

$CF_{e,T}$ and $CF_{d,T}$ is the annuity cash flow to shareholders and bondholders, respectively. The implied equity market value $V_{e,T}$ at date T is the present value given $CF_{e,T}$ and $r_e = 0.12$. The implied debt market value $V_{d,T}$ at date T is the present value given $CF_{d,T}$ and $r_d = 0.06$. V_T is the total market value of debt and equity ($V_{d,T} + V_{e,T}$) at date T.

Panel B: Implied capital structure ratios and returns over the life cycle of the project

T	0	1	2	3	4	5	6	7	8
$w_{e,T}$	0.750	0.754	0.758	0.762	0.767	0.771	0.776	0.780	-
$w_{d,T}$	0.250	0.246	0.242	0.238	0.233	0.229	0.224	0.220	-
$WACC_T$		0.10000	0.10032	0.10066	0.10099	0.10134	0.10169	0.10205	0.10242
$r_{u,T}$		0.10521	0.10545	0.10570	0.10595	0.10621	0.10647	0.10674	0.10701

$w_{e,T}$ is the time-varying equity weight (equity/total value), and $w_{d,T}$ is the time-varying debt weight (debt/total value) based on the results of Panel A. The time-varying $WACC_T$ is computed with Eq. (5) as $WACC_T = w_{d,T\text{-}1} \cdot r_d \cdot (1\text{-}t) + w_{e,T\text{-}1} \cdot r_e = w_{d,T\text{-}1} \cdot 0.06 \cdot 0.6667 + w_{e,T\text{-}1} \cdot 0.12$. The implied unlevered cost of capital $r_{u,T}$ is calculated for illustration purposes in line with a value-based debt policy according to Eq. (8).

Panel C: Adjusted Present Value and market values over the life cycle of the project

T	0	1	2	3	4	5	6	7	8
CF_T (in \$)		37,247.33	37,348.35	37,455.44	37,568.95	37,689.28	37,816.82	37,952.01	38,095.32
$V_{u,T}$ (in \$)	196,383.29	179,798.09	161,410.14	141,015.75	118,387.64	93,272.10	65,385.93	34,412.93	0.00
$V_{d,T-1} \cdot r_d \cdot t$ (in \$)		999.90	898.87	791.79	678.27	557.95	430.41	295.21	151.91
$V_{t,T}$ (in \$)	3,616.73	2,954.71	2,328.87	1,749.11	1,226.76	774.78	407.99	143.31	0.00
$V_T = V_{u,T} + V_{t,T}$ (in \$)	200,000.00	182,752.79	163,739.01	142,764.87	119,614.40	94,046.88	65,793.92	34,556.24	0.00

CF_T is the unlevered cash flow during period T computed as $CF_T = CF_{e,T} + V_{d,T-1} \cdot r_d \cdot (1-t) + (V_{d,T-1} - V_{d,T})$; see Panel A, $r_d = 0.06$, $t = 0.3333$. The unlevered value $V_{u,T}$ is computed according to Eq. (9) with $r_{u,T}$. The debt tax shields $V_{d,T-1} \cdot r_d \cdot t$ are discounted according to Eq. (10) with $r_{u,T}$ and r_d to arrive at the value of the debt tax shield $V_{t,T}$. V_T is the total market value ($V_{u,T} + V_{t,T}$) at date T.

5. Summary and outlook

The claim that the NLWACC is conceptually superior to a constant WACC seems for two reasons rather artificial:

1) A slightly modified WACC which is time-varying is conceptually superior to the NLWACC. It does not seclude itself from an economic interpretation.

2) Given the special valuation scenario for which the NLWACC is motivated, the flow to equity approach is the direct solution while the (NL)WACC is a detour.

Therefore, the foundations of WACC are sound. However, valuation is still a field which has many promising research questions to offer. Just to name a few: Which debt policies can be empirically supported? Given its autarkic nature, this is a question the WACC does not necessarily have to approach. How do more realistic tax regimes with personal taxes influence tax shield valuation? Which terminal value calculations are plausible? These and others seem to be more pressing questions which deserve further attention in future research.

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Appendix A: Derivation of time-varying WACC

The following derivation draws on the proof of a time-constant WACC by Brealey *et al.* (2008), p. 533. Starting with the value in the next to last period: $V_{N-1} = V_{e,N-1} + V_{d,N-1}$. The total cash flow to debt and equity investors is the cash flow CF plus the interest tax shield: $CF_N + t \cdot r_{d,N} \cdot V_{d,N-1}$. This total cash flow can also be written based on returns:

$$V_{N-1}\left(1 + r_{d,N}\frac{V_{d,N-1}}{V_{e,N-1}+V_{d,N-1}} + r_{e,N}\frac{V_{e,N-1}}{V_{e,N-1}+V_{d,N-1}}\right) = V_{N-1}\left(1 + r_{d,N}w_{d,N-1} + r_{e,N}w_{e,N-1}\right)$$

Equate both definitions and solve for V_{N-1}:

$$V_{N-1} = \frac{CF_N}{1 + r_{d,N}(1-t)w_{d,N-1} + r_{e,N}w_{e,N-1}} = \frac{CF_N}{1+WACC_N}$$

The WACC-definition of Eq. (5) shows up. This can be repeated for V_{N-2}. As the return-definition is based on Eq. (4) the next period's payoff includes V_{N-1}:

$$CF_{N-1} + t \cdot r_{d,N-1} \cdot V_{d,N-2} + V_{N-1} = V_{N-2}\left(1 + r_{d,N-1}w_{d,N-2} + r_{e,N-1}w_{e,N-2}\right)$$

Solving for V_{N-2} yields:

$$V_{N-2} = \frac{CF_{N-1}+V_{N-1}}{1 + r_{d,N-1}(1-t)w_{d,N-2} + r_{e,N-1}w_{e,N-2}} = \frac{CF_{N-1}+V_{N-1}}{1+WACC_{N-1}}$$

$$= \frac{CF_{N-1}}{1+WACC_{N-1}} + \frac{CF_N}{\left(1+WACC_{N-1}\right)\left(1+WACC_N\right)}$$

By recursion, one obtains for any arbitrary valuation date $\tau-1$:

$$V_{\tau-1} = \sum_{T=\tau}^{N} \frac{CF_T}{\prod_{j=\tau}^{N}\left(1+WACC_j\right)} \tag{7}$$

Appendix B: Is the debt to total market value ratio falling over time?

To show the conditions for this relationship, I compare growth rates of debt and equity market value, e.g. the equity growth rate is: $g_T(V_e) = \frac{V_{e,T} - V_{e,T-1}}{V_{e,T-1}}$. If debt is not growing as strong as equity, the debt ratio has to shrink. Inserting Eq. (2) yields the following growth rate:

$$g_T(V_e) = \frac{\dfrac{CF_e\left[1 - \dfrac{(1+g)^{N-T}}{(1+r_e)^{N-T}}\right]}{r_e - g} - \dfrac{CF_e\left[1 - \dfrac{(1+g)^{N-T+1}}{(1+r_e)^{N-T+1}}\right]}{r_e - g}}{\dfrac{CF_e\left[1 - \dfrac{(1+g)^{N-T+1}}{(1+r_e)^{N-T+1}}\right]}{r_e - g}} = \frac{1 - \left[\dfrac{1+g}{1+r_e}\right]^{N-T}}{1 - \left[\dfrac{1+g}{1+r_e}\right]^{N-T+1}} - 1$$

For the debt growth rate equivalently:

$$g_T(V_d) = \frac{1 - \left[\dfrac{1+g}{1+r_d}\right]^{N-T}}{1 - \left[\dfrac{1+g}{1+r_d}\right]^{N-T+1}} - 1$$

Under realistic conditions ($r_d < r_e$) equity growth rates are higher than debt growth rates. Therefore, $w_{d,T}$ decreases over time.[11]

[11] Except for $N \to \infty$, and T = N. Then, $g_T(V_d) = g_T(V_e)$.