ON THE UTILITY OF THE HURST EXPONENT IN PREDICTING FUTURE CRISES

Tom Coen*, Gianluca Piovani*, Giuseppe Torluccio**

Abstract

The aim of this article is to ascertain whether and to what extent the Hurst exponent can be used to forecast future crises. The first and second sections focus on the Hurst exponent, giving theoretical insights and a summary of its uses in finance. The analysis of a dataset of 35 indices and stocks representing various geographical areas and economic sectors is presented in Section 3, while in the last section the conclusion is drawn that in fact the Hurst exponent has, after all, no usefulness in predicting future crises.

Keywords: Crises, Hurst Exponent, Crisis Detection

** University of Bologna, Collegio Superiore*
*** Corresponding Author. University of Bologna, Department of Management*
Tel: +39-051-2098085
Fax: +39-051-3764180
Email: giuseppe.torluccio@unibo.it

1 Introduction

1.1 Uses of the Hurst Exponent

The Hurst exponent is a measure of the autocorrelation of the data which are part of a certain time series. The concept of autocorrelation, as the word itself suggests, is connected to the influence that a datum of position, say x, in a time series has on a successive datum of position, say x+1. The effects of such a property can be effectively explained in terms of comparison with the mean: if a value somewhat higher than the mean is usually followed by another high one (or in other words "forces" the following to be high, too), then we can say that the data are correlated in a positive way. Conversely, in the case of high value being followed by low ones, negative correlation occurs, while random data should have no correlation.

As the mathematical process to evaluate the Hurst exponent H will be discussed in Section 2, it is sufficient for now to know the following:

$H<0.5$: data are negatively correlated
$H=0.5$: data have no correlation
$H>0.5$: data are positively correlated

The name "Hurst exponent" derives from Harold Edwin Hurst (1880-1978), who first used it for studying the River Nile's cycles of heavy rains and droughts. This hydrological issue aimed at approaching the practical problem of optimizing the size of dams, while from then on the Hurst exponent became increasingly more used in many scientific fields, including physics, DNA research, and economics. As far as economics is concerned, the Hurst exponent has been mainly used in finance and this branch of studies has been classified in the econophysics area.

Examples of the use of the Hurst exponent for financial issues are its applications to the study areas of high frequency trading and market size. In (Di Matteo, 2007), the author notices how the Hurst exponent seems to assume different values in developed and emerging markets, and stresses the importance of such results for portfolio management evaluations. While the Hurst exponent calculated for indices such as the NASDAQ (USA), NIKKEI (Japan) and CAC (France) does not cross the 0.5 value, on the other hand it seems to be consistently superior to this value for the IBEX (Spain) and Hang Seng (Hong Kong). Many other markets are nevertheless quite vaguely sited with small fluctuations in the 0.5 belt: these include the FTSE (United Kingdom), DAX (Germany) and AEX (Netherlands).

Hurst exponent analysis has also proved itself to be of significant value in high frequency trading market investigations. (Bartolozzi et al., 2007) find that Hurst exponent values for different but small time horizons differ noticeably from 0.5, thus contradicting Efficient Market Hypothesis, which states that they should cluster near 0.5 as prices should not be predictable but follow a random walk distribution. This last result could have been caused by high frequency trading itself, as (Smith, 2010) suggests. The author first set a pre and post high frequency trading period and then analyzed Hurst exponent values for these different periods. The date chosen in the article is June 2005, which is the date of approval of Reg NMS whose Rule 611 obliges the automatic execution of trades at the best quote possible: this automatization of the market is considered to be the decisive factor that made it possible to develop high frequency trading on a massive scale. Once the different Hurst exponent values were found for these two periods the author suggested possible causes for this result, which are the breaking of big orders into smaller and reiterated ones, and the feedback-driven method of many high frequency trading techniques.

1.2 Forecasting crises

The most suggestive use of the Hurst exponent is, however, the possibility of anticipating the future. The concept of correlation is indeed the idea of connection between past and future data, which can therefore be forecast with some precision. This idea has been proposed by many scholars in regard to the chance of predicting future crises and abrupt market movements. Fascinating as it is, however, this hypothesis has been the subject of works concentrating only on a narrow set of indices or markets, and even these limited investigations have frequently ended up with different and contrasting results. Our work aims therefore to broaden the range of investigation and testing the results obtained so far on a wider dataset.

A very clear work about the ability of the Hurst exponent to predict crises is that of (Czarnecki, 2008). In this paper the authors evaluate the Hurst exponent for the Polish market index and find evidence that just before a crisis period the Hurst exponent diminishes noticeably. A simple method to verify whether the crash in the Hurst exponent is the prelude to a crisis is also provided. Denoting the simple mean of all Hurst exponent values in x days preceding the crash in the market as H-x, if the period considered is really the prelude to a crash in the market, then:

- The Hurst exponent should display a decreasing trend
- $H\text{-}21 \leq 0.5$
- $H\text{-}5 \leq 0.45$
- The minimum value of the Hurst exponent in the period just before the crisis should be lower than 0.4 (as the Hurst exponent trend is decreasing, the minimum value should be the nearest to the moment of the market crash)

Even though this method is very satisfying, it should be noted that practically it could be quite difficult to identify exactly the 21 or 5 trading days before the crisis while still being in the period before the crisis.

In (Krzisztof, 2010) the author analyzes the values of the Hurst exponent for 126 societies listed on the Warsaw stock exchange. This work suggests that the fall of the Hurst exponent under the 0.4 threshold preludes a crash in the prices of that stock. The noteworthy quantity of data considered makes this work a cornerstone in Hurst exponent analysis, while on the other hand the same author in the conclusion to his paper calls for more work to be done on markets other than the Polish one.

Another work finding correspondence between the fall of the Hurst exponent and its correlated index is (Grech, 2004). This work uniquely considers the Dow Jones index and the crises of 1929 and 1987-88, finding that the Hurst exponent forecasts more effectively future crashes in cases where the quotations of the index are in a clear increasing trend. The same observation is made in (Kristoufek, 2010), where the crashes of the Prague stock exchange of the years 2000, 2005, 2006 and 2007 are studied. In the crises of 2000, 2005 and 2007 the crash is preceded by strong increases (+38.66% in the four months before the 2000 crisis, +46% in the three months before the crisis of 2005, and +30% in pre-2006 crisis values leading into the 2007 crisis), and the Hurst exponent starts decreasing noticeably in the 1-3 month period before the crash. The case of the 2006 crisis is, on the other hand, not detected as there is no such clear pattern before the crash. Next a random set of data generated from the shuffled logarithmic returns of the index which were cumulated to form the new time series is tested with the Hurst exponent. As in the 2006 case, the Hurst exponent does not forecast crashes because no clear preceding trends are identified.

Some years later Kristoufeck published another article, (Kristoufek, 2012), in which the NASDAQ, Dow Jones and S&P indices are analyzed. As all of the three indices are extremely similar, the results obtained with the computation of the Hurst exponent can also be synthesized into one single result, which is the detection of a fall in the Hurst exponent about a year before the 2007 crisis. This result is actually quite unexpected, as by now presented papers have shown that the Hurst exponent anticipates crises no more than three months in advance, and, more often than not, only a single month in advance.

An even more puzzling result is that found in (Morales, 2012). In this work the authors find evidence that companies that were about to be bailed out by USA authorities show a long-running increase in the values of the Hurst exponent. As bailed out companies were presumably those worst hit by the crisis, we would conclude from this that an increase in the Hurst exponent would be the signal for an imminent crash in the prices of a certain stock, which is exactly the opposite of what was suggested by all previously cited papers. The authors moreover detect a decreasing trend in the Hurst exponent in sectors that were less strongly hit by the crisis, such as the Basic Materials sector.

Now that the current state of Hurst exponent academic investigation has been clarified, the next step is to explain the data processing used to obtain Hurst exponent values: this is the topic of Section 2, while the concrete results obtained will be exhibited in Section 3. In the final section the conclusions of the authors are presented.

2 The Hurst exponent and Detrended Fluctuation Analysis

2.1 Theoretical introduction

The Hurst exponent is a coefficient that arises naturally in the study of self-similar stochastic processes. The following definition is taken from (Ebrechts, 2001).

Definition. A stochastic process $\{X(t) : t \geq 0\}$ is said to be self-similar if for any $a > 0$, there exists $b > 0$ such that $\{X(at)\} \triangleq \{bX(t)\}$.

With the symbol $\triangleq$ we denote the equality of all joint distributions for stochastic processes. It is possible to prove (Embrechts, 2001) that for stochastic processes that are nontrivial, stochastically continuous at $t = 0$ and self-similar there exists a unique $H \geq 0$ such that $b = a^H$. In this case, H is called the Hurst exponent of the stochastic process.

A variety of self-similar stochastic processes that admit a Hurst exponent have been studied. Among them, fractional Brownian motion, fractional Gaussian noise and fractional ARIMA (also called ARFIMA) also have an autocorrelation function that depends on the value of H. The autocorrelation function, too, allows for a probabilistic treatment of long-range dependence. In general, values of H strictly higher than $\frac{1}{2}$ indicate a long-term positive autocorrelation, whereas values of H strictly lower than $\frac{1}{2}$ indicate a long-term negative autocorrelation.

It should be noted, however, (Bassler, 2007) that not every self-similar process with $H \neq \frac{1}{2}$ exhibits long-term autocorrelations, as is sometimes erroneously asserted in the literature, so analysis of long-range dependence should not be based on the Hurst exponent alone.

There is evidence (Bhardwaj, 2006) that the behavior of prices of financial assets can at least be approximated by one of the aforementioned stochastic processes, specifically the versatile ARFIMA model, which even allows for non-stationarities. Various techniques for estimating the Hurst exponent of the underlying stochastic process, given a discrete time series, have been proposed in the literature.

In particular, Detrended Fluctuation Analysis (often abbreviated DFA), first proposed by (Peng, 1994), and designed specifically for nonstationary processes, provides an estimator of the Hurst exponent H that characterizes the underlying stochastic process. A theoretical justification for the use of DFA in the case of fractional Gaussian noise or fractional ARIMA processes can be found in (Taqqu, 1995).

The initial step of the most basic version of DFA consists of breaking up the time series into blocks of size s. Then, for each block, the partial sums of the series, $\{Y_i\}$, are calculated. A straight line is fitted to $\{Y_i\}$ with the method of least squares, and the sample variance of the residuals is computed. The process is repeated for all the blocks and the average of all the variances for all the blocks of the same size is then computed. This number, for a large enough s, is asymptotically proportional to s^H, as was proved in the appendix of (Taqqu, 1995).

2.2 Estimation of the local Hurst exponent

Our data consisted of financial time series representing the daily closure price of 21 stocks and 14 stock indices for thousands of trading days. The time series that were believed to be generated by some process akin to fractional Gaussian noise or fractional ARIMA were the logarithmic returns, or log returns, defined as $l_i = \ln p_i - \ln p_{i-1}$, where p_i represents the closure price of the asset on the i^{th} trading day. In order to gain an insight into the market dynamics, the local Hurst exponent was calculated. The local Hurst exponent is defined (Kristoufek, 2012), for each point j of a time series where it is applicable, as the DFA estimation of the Hurst exponent for the sample comprising points $j - L + 1$ to j of the original time series, where L is the sliding window length.

The algorithm we employed included several steps and is described here in detail. We denote by $\{p_i\}_{1 \leq i \leq N}$ the sequence of prices of an asset for N trading days.

1. We start with $j = L$. Then the series $\{Y_i\}_{1 \leq i \leq L}$, representing the partial sums of the log returns, is constructed as $Y_i = \ln p_{i+j-L}$.

2. The series $\{Y_i\}_{1 \leq i \leq L}$ is divided into $\lfloor L/s \rfloor$ consecutive non-overlapping blocks of size L starting from the beginning and in addition $\lfloor L/s \rfloor$ starting from the end. Therefore no data is neglected even if L is not a multiple of s .

3. For each block k we denote its subseries of length s by $\{y_i^s\}_{1 \leq i \leq s}$. A linear least square fit is performed for the data in $\{y_i^s\}_{1 \leq i \leq s}$, obtaining a straight line in the form$f_k(x) = m_k x + b_k$. Then, the (squared) detrended fluctuation F_k^2 is calculated for each block as

$$F_k^2 = \frac{1}{s} \sum_{i=1}^{s} \left(y_i^k - f_k(i)\right)^2$$

4. The squared detrended fluctuations for all the blocks are averaged, giving a number that is a function of s, the length of the blocks; and we denote that by $\langle F^2 \rangle(s)$.

5. Steps 2-4 are repeated for all the values of s between some minimum s_{min} and some maximum s_{max}.

6. $\sqrt{\langle F^2 \rangle(s)}$ is plotted on a log-log graph for all the considered values of s. The slope of the linear fit to the data is taken as H_j the estimate of the Hurst exponent for the current value of j .

7. The procedure in steps 1-6 is repeated for $j = (L + 1)$, then for $j = (L + 2)$, and so on, until $j = N$.

Finally we obtain a time series of estimated Hurst exponents $\{H_i\}_{L\leq i\leq N}$ that we may compare with $\{p_i\}_{1\leq i\leq N}$. For $L \leq i \leq N$, H_i represents the Hurst exponent estimated with DFA on the "sliding window of length L" encompassing the prices from p_{i+1-L} to p_i.

For our analysis we selected the same parameters as (Kristoufek, 2012), therefore we chose a sliding window of $L = 500$ trading days (corresponding roughly to two years) and we considered values of s between $s_{min} = 10$ and $s_{max} = 50$.

3 Data processing

3.1 The data

All the data to be used were taken from historical time series available from the website of Yahoo! Finance (http://finance.yahoo.com/). The procedure described in Section 2 was applied to the 35 indices and stocks listed in Table 1, which were chosen in such a way as to have a wide and varied sample representing both different geographical areas and different economical sectors. In Table 1 we also list the country to which each index refers, and certain abbreviations of company names that will be used below. For the above analyses only adjusted close values have been used.

Table 1. Stocks/indices considered

Index/Stock	Abbreviation	Country
AEX	AEX	Netherlands
ATHEX	ATHEX	Greece
ATX	ATX	Austria
CAC	CAC	France
DAX	DAX	Germany
EXCH	EXCH	Norway
FTSE	FTSE	United Kingdom
IBEX	IBEX	Spain
ISEQ	ISEQ	Ireland
NASDAQ	NASDAQ	USA
NIKKEI	NIKKEI	Japan
OMXS	OMXS	Sweden
S&P/TSX	S&P/TSX	Canada
SMI	SMI	Switzerland
Assicurazioni Generali	AG	Italy
Apple	Apple	USA
Barclays	Barclays	United Kingdom
Bayer	Bayer	Germany
Banco Comercial Português	BCP	Portugal
Coca Cola	Coca Cola	USA
Électricité De France	EDF	France
Ente Nazionale per l'energia Elettrica	ENEL	Italy
Ente Nazionale Idrocarburi	ENI	Italy
Exxon Mobil	Exxon	USA
France Télécom	FT	France
IBM	IBM	USA
Microsoft	Microsoft	USA
PSA Peugeot Citroen	PSA	France
Royal Dutch Shell	RDS	Netherlands
Renault	Renault	France
Grupo Santander	Santander	Spain
Société Générale	SG	France
Toyota Motor Corporation	Toyota	Japan
Volkswagen	Vow	Germany
Xstrata	Xstrata	United Kingdom

The results of the calculations of the Hurst exponent are reported in Appendix A. For each index/stock adjusted two graphs are present. Each graph shows in the x axis the date each piece of data refers to; the upper graph shows in the y axis the price of its object in a log scale while the other one shows the values of the Hurst exponent. It should be noted that each piece of data corresponds to a trading day, and that for the first 500 data the Hurst exponent was not evalued because the procedure requires a time window of 500 data. As 500 data represents approximately two years (500 trading days equals two years), each Hurst exponent graph therefore has no values for this first period.

The Hurst exponent values obtained were then studied in order to verify whether or not they could be a useful indicator to forecast the 2007 crisis on indices and stocks analyzed. Once this was done, the procedure described in (Czarnecki, 2008) and summarized in 1.2 above was applied to the dataset to find out if it provided consistent outcomes. The results obtained are discussed below.

3.2 Crisis detection with the Hurst exponent

To ascertain whether the Hurst exponent could forecast a crisis it was first of all necessary to identify the period of the crisis. For each stock and index the date when the value of the price reached its relative maximum in the period from 1st January 2007 to 31st December 2009 was therefore found (from here on we refer to this date as max date). This date was considered to separate the crisis period from the pre-crisis period; if a Hurst exponent decrease really anticipates crises, a crash in its values should be present in the days before max date.

In Table 2, in the column "Before max date", the values of the minimum of the Hurst exponent in the 21 days preceding max date are reported in subcolumn Min. The two following subcolumns, H21 and H5, contain the average value of the Hurst exponent in the 21 and 5 day periods preceding this same date. These values are reported for each period noted in the header row of the table. These same calculations are repeated for different time spans, whose initial date is reported in the following three columns. These different time spans consist of two randomly chosen ones (12th May 2006 and 15th March 2012), and finally a day identified as that which minimizes the mean value of H21 of all 35 indices/stocks (20th November 2008). In the last row of the table, for each column, the number of data superior to their "Before max date" peer are counted. Where data were unavailable, a --- symbol was marked in the appropriate boxes.

A first test on the ability of the Hurst index to foresee crises can be performed by comparing the values in the three columns comprising "Before max date" with those of the following two columns, obtained by choosing two dates randomly, and applying the same procedure. If the Hurst exponent decreased before the 2007 crisis, the values of "Before max date" should be noticeably lower than the others and therefore the majority of the data of these last two columns should be higher than those of "Before max date". This prediction is slightly verified in the case of 12th May 2006, when 19 data from "Before max date" out of a total of 34 are less than their corresponding ones. On the other hand, this hypothesis is clearly contradicted by the columns 15th March 2012, when the number of data of "Before max date" less than their corresponding ones decreases to 9, and 20th November 2008, where this last number falls to as low as 5.

The results of identically conducted investigations for the periods of 21 and 5 days after max date and other two randomly chosen dates (27th November 2004 and 1st September 2009) are presented in Table 3. As the results of these analyses are very similar to previously shown ones, it could be argued that the Hurst exponent does not seem to suffer any anticipated fall forecasting crises.

The hypothesis presented in (Czarnecki, 2008) was then tested. This hypothesis stated that the Hurst exponent indicates a coming crisis in cases where the following conditions are verified:

- The Hurst exponent is in a decreasing trend
- H-21≤0.5
- H-5≤0.45
- The minimum value of the Hurst exponent in the period just before the crisis is inferior to 0.4 (as the Hurst exponent trend is decreasing the minimum should be the nearest to the moment of the market crash)

To verify the first condition all periods of 21 successive trading days for all indices/stocks were considered. Once the coefficient indicating the slope of the line coming from a linear regression of the Hurst exponent values for each period was calculated, only the days presenting a negative value of this coefficient were taken into consideration for the next steps. This procedure selected only periods verifying the first condition, that is to say, decreasing Hurst exponent values. Each period of 21 days was then labeled with the date of the last day, that is the one from which it would have been concretely possible to detect the crisis, as in the previous days it would not have been possible to have any idea of how the Hurst exponents value could evolve afterwards. The next two conditions were then applied (H-21<0.5 and H-5<0.45) and finally a value of the Hurst exponent of 0.4 was looked for in each remaining 21 day period. A final and additional condition requested was that each crisis detected in this way was at least 30 days far from the following one. This was done to avoid having many following days all identified as crisis periods, because the value of primary interest is only the first day from which it was possible to detect each crisis, while other immediately following values are redundant.

Table 2. Dataset Hurst exponent averages and minima over different periods

	Before *max date*			12/05/06			15/03/12			20/11/08		
	Min	H21	H5	Min	H21	H5	Min	H21	H5	Min	H21	H5
AEX	0.47463	0.49142	0.48589	0.48678	0.50247	0.49619	0.43319	0.44761	0.45315	0.43392	0.46149	0.48582
ATHEX	0.48111	0.49541	0.49985	0.42141	0.47006	0.43486	0.42992	0.45417	0.45920	0.44913	0.46912	0.48761
ATX	0.52144	0.53334	0.52681	0.47495	0.53237	0.48654	0.39812	0.42562	0.45954	0.38202	0.40425	0.42773
DAX	0.40266	0.42426	0.41576	0.41226	0.43405	0.42222	0.41582	0.42578	0.43430	0.39747	0.42669	0.45043
CAC	0.41811	0.43809	0.44426	0.38636	0.40889	0.39516	0.39318	0.40725	0.41371	0.34700	0.37131	0.38838
EXCH	0.48416	0.49253	0.48885	0.47852	0.51136	0.52261	0.41025	0.43114	0.44522	0.39785	0.42301	0.44638
FTSE	0.47517	0.49002	0.49073	0.49000	0.52382	0.51442	0.41127	0.42660	0.43535	0.36059	0.38632	0.41104
IBEX	0.45784	0.47263	0.46233	0.47663	0.49606	0.48582	0.37333	0.37986	0.38049	0.31092	0.32970	0.32863
ISEQ	0.42648	0.45940	0.44228	0.54275	0.59030	0.59495	0.38136	0.40252	0.40584	0.36645	0.37800	0.38236
NASDAQ	0.37403	0.39444	0.39365	0.43680	0.46868	0.47992	0.44470	0.45942	0.47260	0.34544	0.37180	0.38222
NIKKEI	0.46679	0.47567	0.48088	0.43522	0.45244	0.44112	0.44717	0.46426	0.46508	0.36515	0.39474	0.41615
OMXS	0.41642	0.43520	0.43053	---	---	---	0.36317	0.37730	0.38087	0.37099	0.39290	0.41397
S&P/TSX	0.46659	0.48609	0.48908	0.47986	0.50504	0.51833	0.41788	0.43203	0.42983	0.37266	0.40034	0.42895
SMI	0.46359	0.48083	0.49130	0.46501	0.49897	0.49818	0.42444	0.44279	0.45111	0.31375	0.33146	0.33324
AG	0.46944	0.48752	0.47816	0.54736	0.56494	0.56254	0.41191	0.43020	0.42802	0.34957	0.37213	0.38249
Apple	0.48157	0.50198	0.50634	0.43955	0.44958	0.44289	0.38888	0.41253	0.43385	0.51222	0.52522	0.53702
Barclays	0.49486	0.50304	0.50235	0.50382	0.55321	0.57516	0.34770	0.35957	0.36044	0.33789	0.36584	0.34872
Bayer	0.41796	0.43297	0.42282	0.52086	0.52859	0.52711	0.40764	0.42613	0.43921	0.44849	0.45958	0.46545
BCP	0.41273	0.43279	0.41956	0.38466	0.40723	0.40800	---	---	---	0.43275	0.46621	0.44826
Coca Cola	0.40854	0.42068	0.41979	0.55680	0.57021	0.58521	0.42695	0.44156	0.43401	0.30734	0.33861	0.35016
EDF	---	---	---	---	---	---	0.47913	0.48769	0.49404	0.46532	0.48105	0.49809
ENEL	0.40564	0.41755	0.40833	0.53053	0.53958	0.54421	0.42920	0.44557	0.45116	0.35529	0.36964	0.36980
ENI	0.49245	0.52707	0.50013	0.45609	0.47912	0.46837	0.47416	0.48813	0.49081	0.40553	0.44785	0.47891
Exxon	0.52316	0.53402	0.53957	0.48317	0.49463	0.49349	0.46699	0.48616	0.50244	0.31114	0.33574	0.34974
FT	0.45442	0.47181	0.46458	0.37571	0.38831	0.38952	0.39921	0.41586	0.42645	0.32849	0.34194	0.34895
IBM	0.41937	0.43123	0.42409	0.59445	0.60536	0.60802	0.39819	0.40714	0.41094	0.39300	0.42321	0.43093
Microsoft	0.42473	0.45857	0.44293	0.45918	0.49338	0.51825	0.48969	0.51829	0.53524	0.32207	0.33251	0.33840
PSA	0.54434	0.56969	0.55286	0.53596	0.54854	0.55693	0.44618	0.46654	0.47158	0.34442	0.36053	0.36661
RDS	0.35886	0.52145	0.51724	---	---	---	0.48794	0.50106	0.50409	0.41152	0.43972	0.46802
Renault	0.40403	0.41833	0.41223	0.51715	0.53099	0.53323	0.41174	0.43014	0.41957	0.35355	0.39471	0.37211
Santander	0.41708	0.44204	0.42474	0.46370	0.50826	0.47200	0.35561	0.36790	0.36212	0.30670	0.38632	0.36681
SG	0.48148	0.50084	0.50577	0.40161	0.42559	0.44063	0.36069	0.38669	0.37505	0.32274	0.34865	0.34221
Toyota	0.43511	0.45336	0.47083	0.41519	0.44690	0.43100	0.46939	0.48881	0.49323	0.35419	0.38816	0.39142
Vow	0.08343	0.40321	0.25776	0.52732	0.56125	0.57236	0.47078	0.48655	0.48860	0.08549	0.17360	0.12964
Xstrata	0.43532	0.44745	0.44404	0.36115	0.40231	0.37978	0.41765	0.43057	0.43689	0.32978	0.35617	0.40163
Number of data superior than "Before *max date*" column				18	19	20	9	8	9	5	4	5

Table 3. Other tests on Hurst exponent values

	Before *max date*			After Max			27-nov-04			01-set-09		
	Min	H21	H5	Min	H21	H5	Min	H21	H5	Min	H21	H5
AEX	0.47463	0.49142	0.48589	0.46926	0.49197	0.47905	0.36976	0.40987	0.44188	0.47140	0.48278	0.49418
ATHEX	0.48111	0.49541	0.49985	0.48676	0.50470	0.50064	---	---	---	0.48621	0.50233	0.51674
ATX	0.52144	0.53334	0.52681	0.51455	0.52907	0.52695	0.52727	0.54277	0.53951	0.42952	0.44117	0.44894
CAC	0.41811	0.43809	0.44426	0.42709	0.44689	0.45036	0.31868	0.34933	0.37183	0.40805	0.42034	0.43049
DAX	0.40266	0.42426	0.41576	0.39807	0.42322	0.40280	0.39657	0.41149	0.41888	0.46905	0.48695	0.50065
EXCH	0.48416	0.49253	0.48885	0.48246	0.50746	0.49587	0.55012	0.56235	0.56054	0.41708	0.43611	0.44315
FTSE	0.47517	0.49002	0.49073	0.47498	0.48525	0.48344	0.37405	0.41413	0.42193	0.43533	0.44616	0.45900
IBEX	0.45784	0.47263	0.46233	0.45479	0.47447	0.46394	0.48174	0.49967	0.49915	0.38485	0.39531	0.40400
ISEQ	0.42648	0.45940	0.44228	0.42986	0.45469	0.44662	0.46661	0.48473	0.49242	0.40862	0.42033	0.42782
NASDAQ	0.37403	0.39444	0.39365	0.38705	0.40592	0.39058	0.42936	0.44793	0.44794	0.41527	0.43644	0.44806
NIKKEI	0.46679	0.47567	0.48088	0.46772	0.48229	0.47216	0.44614	0.46308	0.46618	0.43115	0.44502	0.44842
OMXS	0.41642	0.43520	0.43053	0.41977	0.43497	0.43208	---	---	---	0.39832	0.41222	0.42007
S&P/TSX	0.46659	0.48609	0.48908	0.48171	0.49972	0.49020	0.40223	0.41785	0.42409	0.41336	0.42451	0.42876
SMI	0.46359	0.48083	0.49130	0.45750	0.46782	0.47360	0.37081	0.40745	0.42463	0.36338	0.38401	0.39632
AG	0.46944	0.48752	0.47816	0.48192	0.52121	0.49398	0.48925	0.53033	0.54233	0.51068	0.53072	0.54118
Apple	0.48157	0.50198	0.50634	0.48845	0.50253	0.50588	0.50254	0.52137	0.51920	0.51947	0.54124	0.55174
Barclays	0.49486	0.50304	0.50235	0.47168	0.50122	0.49489	---	---	---	0.45102	0.47889	0.49069
Bayer	0.41796	0.43297	0.42282	0.40730	0.45179	0.41442	0.32851	0.36012	0.36373	0.47191	0.49371	0.50803
BCP	0.41273	0.43279	0.41956	0.39259	0.42797	0.42287	---	---	---	0.44851	0.46064	0.46584
Coca Cola	0.40854	0.42068	0.41979	0.38963	0.41759	0.40496	0.54658	0.56174	0.55967	0.40691	0.41870	0.41528
EDF	---	---	---	0.50268	0.52813	0.52383	---	---	---	0.44716	0.46928	0.47167
ENEL	0.40564	0.41755	0.40833	0.38535	0.40626	0.40505	---	---	---	0.47198	0.48316	0.48616
ENI	0.49245	0.52707	0.50013	0.50950	0.53876	0.51281	---	---	---	0.46121	0.47008	0.47226
Exxon	0.52316	0.53402	0.53957	0.49147	0.50558	0.51186	0.47555	0.48861	0.48422	0.33407	0.34321	0.34702
FT	0.45442	0.47181	0.46458	0.43155	0.44535	0.45033	---	---	---	0.29273	0.30601	0.30188
IBM	0.41937	0.43123	0.42409	0.41870	0.43251	0.43250	0.42216	0.44823	0.45279	0.41885	0.43644	0.44378
Microsoft	0.42473	0.45857	0.44293	0.40871	0.44751	0.44927	0.45564	0.48379	0.48776	0.35348	0.36559	0.36835
PSA	0.54434	0.56969	0.55286	0.51759	0.55042	0.54986	---	---	---	0.43750	0.44966	0.46019
RDS	0.35886	0.52145	0.51724	0.51637	0.54910	0.52704	---	---	---	0.45386	0.47008	0.47046
Renault	0.40403	0.41833	0.41223	0.40189	0.42286	0.41543	---	---	---	0.49619	0.51225	0.52799
Santander	0.41708	0.44204	0.42474	0.41650	0.42815	0.41921	---	---	---	0.46574	0.48458	0.49412
SG	0.48148	0.50084	0.50577	0.48681	0.51558	0.49165	---	---	---	0.42524	0.43803	0.44056
Toyota	0.43511	0.45336	0.47083	0.44770	0.46019	0.46160	0.48228	0.50205	0.49732	0.41637	0.43267	0.44105
Vow	0.08343	0.40321	0.25776	0.02920	0.08112	0.09688	---	---	---	0.23876	0.27328	0.27832
Xstrata	0.43532	0.44745	0.44404	0.44704	0.46125	0.44898	0.36359	0.38320	0.39684	0.37173	0.38289	0.38916
Number of data superior than "Before *max date*" column				14	18	16	11	11	12	12	11	13

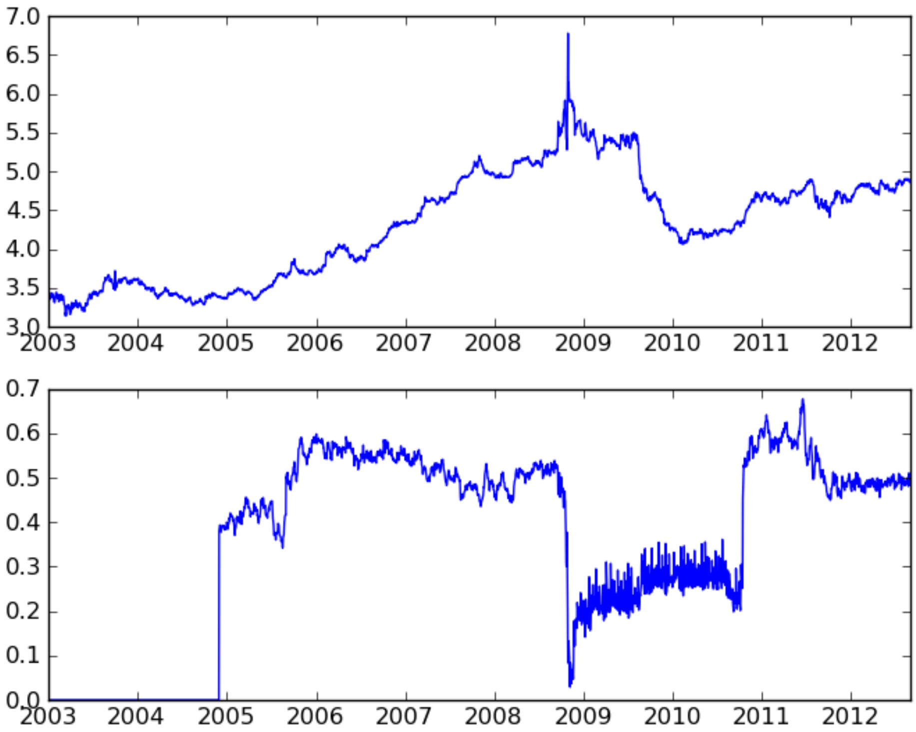
The results of the analysis described in the previous paragraph are reported in Appendix B. First, the name of each index/stock is followed by the date when the price hit its relative maximum in the period from 1st January 2007 to 31st December 2009, until now called max date. In the same row, the minimum price registered in the same time period is also reported, corresponding to the hardest time in the crisis. The following dates are the ones when the adopted procedure indicates a future fall in the quotation of the index/stock. Finally the dates when future crises are detected follow.

The next step was to count how many times our procedure detected a crisis in the period between max date and three months before this day, that is to say how many times Hurst exponent analysis effectively forecast the 2007 crisis. Out of 35 indices/stocks only in 3 cases was the coming crisis forecast. It was also noted that this procedure identified many dates that were not followed by any fall in the index/stock considered. This result is clearly in conflict with the hypothesis that the Hurst exponent decreases considerably before a crisis, thus this hypothesis has to be rejected. In short, Hurst exponent analysis does not seem able to forecast crises.

3.3 Discussion

Let us now consider the quite curious case of Volkswagen. Its Hurst exponent, as shown by the following graph, decreases considerably after an abrupt movement in its price:

Figure 1. Prices and local Hurst exponent values of Volkswagen stock



A sudden movement in the prices, as in the case of Volkswagen at the end of 2008, increasing their mean value, shows, when we compare the data with their average value, first a period of low prices (the ones occurring in the normal market period) followed by a high one (that is the peak) and then other low prices. As the couple of high-low values indicate anticorrelation, this causes the Hurst exponent to fall. The same effect is not necessarily caused by a peak followed by other normal values, but could also be produced by one single considerable movement. Let us consider a sudden decrease in a stock quotation: this movement would cause a fall in the mean value so that quotations before the turning point would reveal high values followed by low ones. All this is to say that the crisis could be considered as a quick and abrupt movement in the prices, causing the Hurst exponent to fall. Moreover, the same effect could be magnified by the increase in volatility which usually follows a crisis, mimicking the large movement effect on a smaller scale.

It was then hypothesized that a fall in the Hurst exponent was not a sign of a coming crash in the prices, but vice versa it was a fall in the prices that then caused the Hurst exponent to fall too. In this case the periods where the values of the Hurst exponent reaches their minimum should be somewhat after max date, that is to say after the start of the crisis, and not before. Using the mean of H21 of all the analyzed indices/stocks as an indicator of the general state of the Hurst exponent in the dataset, it was found that this value reaches its minimum on the 20th November 2008. In 32 cases out of 35 it was found that this date is after max date confirming this hypothesis.

4 Interpreting the results obtained

4.1 Interpreting our results

Considering our results, why have many authors found correspondence between Hurst exponent crashes and future crises? The first possibility is the chance factor: many papers concentrating only on one single index might have had bad luck, and this could be the case with (Kristoufek, 2012). In this article the author only considers the NASDAQ, Dow Jones and S&P indices, which because of their similarity present a case similar to the analysis of one single index. Unfortunately the NASDAQ is in addition one of the 3 indices out of 35 that were found to react positively to the procedure described above which was used to detect crises.

Another issue could have affected many other research projects. Some authors indeed underline that the Hurst exponent does not work properly if the crisis period is not preceded by a clear and very strong increasing trend in prices. This trend could be considered as a kind of abrupt movement similar to that which caused the Hurst exponent crash in the case of Volkswagen. The Hurst exponent crash would not therefore detect the coming crisis but only a sudden movement in prices. As many economic crashes are preceded by speculative bubbles, this could be a reason for this phenomenon.

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4.2 Conclusions

The analysis of the dataset reported in Table 1 does not seem to give any confirmation to the hypothesis of a *connection between future crises and falls in the Hurst exponent. Both the comparison of beofre max date values with other randomly chosen ones and the study* of the dataset using the described procedure did not give indeed any positive result. This hypothesis was therefore abandoned and considered erroneous.

A closer look at the Hurst exponent suggests that this apparent correlation is a consequence of its property of decreasing in cases of abrupt movements and very volatile market conditions. These conditions are certainly typical of a crisis period while on the other hand could have sometimes anticipated it because of speculative bubbles anticipating the crisis.

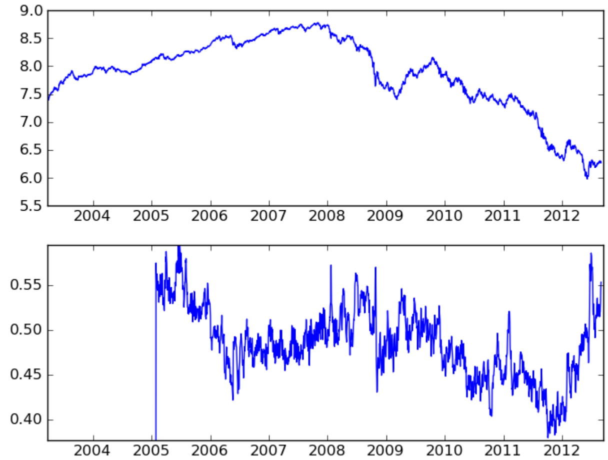
References

1. M. Bartolozzi, C. Mellen, F. Chan, D. Oliver, T. Di Matteo, T. Aste, 2007, Application of Physical Methods in High-Frequency Futures Markets, *European Physical Journal* B 58 207-220
2. K. E. Bassler, J. L. Mc Cauley, G. H. Gunaratne, 2007, Hurst Exponents, Markov Processes, and Fractional Brownian Motion, *Physica A: Statistical Mechanics and its Applications* 379
3. G. Bhardwaj and N. R. Swanson, 2006, An Empirical Investigation of the Usefulness of ARFIMA Models for Predicting Macroeconomic and Financial Time Series, *Journal of Econometrics* 131 539--578
4. N. Crato, R. Linhares, S. Lopes, 2010, Statistical Properties of Detrended Fluctuation Analysis, *Journal of Statistical Computation and Simulation* 80 625-641
5. Ł. Czarnecki, D. Grech, G. Pamuła, 2008, Comparison Study of Global and Local Approaches Describing Critical Phenomena on the Polish Stock Exchange Market, *Physica* A 387, 6801-6811.
6. T. Di Matteo, T. Aste, Michel M. Dacorogna, 2005, Long Term Memories of Developed and Emerging Markets: Using the Scaling Analysis to Characterize their Stage of Developement, *Journal of Banking & Finance* 29 827-851
7. T. Di Matteo, 2007, Multi-Scaling in Finance, *Quantitative Finance*, Vol. 7, No.1

8. P. Embrechts, M. Maejima, 2001, *Selfsimilar Processes*, Princeton University Press
9. D. Grech, Z. Mazur, 2004, Can One Make Any Crash Prediction in Finance Using the Local Hurst Exponent Idea?, *Physica A: Statistical Mechanics and its Applications* 336 133-145
10. J. W. Kantelhardt, E. Koscielny-Bunde, H. H. A. Rego, S. Havlin, A. Bunde, 2001, Detecting Long-Range Correlations with Detrended Fluctuation Analysis, *Physica A: Statistical Mechanics and its Applications* 295 441-454
11. L. Kristoufek, 2012, Fractal Markets Hypothesis and the Global Financial Crisis: Scaling, Investment Horizons and Liquidity, *Advances in Complex Systems* 15
12. L. Kristoufek, 2010, Local Scaling Properties and Market Turning Points at Prague Stock Exchange, *Acta Physica Polonica* B 41(6), 1223-1336,
13. D. Krzisztof, 2010, The use of the Hurst exponent to predict changes in trends on the Warsaw Stock Exchange, *Physica A* 390, 98-109.
14. D. Krzisztof, 2011, The use of the Hurst exponent to investigate the global maximum of the Warsaw Stock Exchange WIG20 index, *Physica A* 391, 156-169.
15. B. B. Mandelbrot , 1968, J. W. Van Ness, Fractional Brownian Motions, Fractional Noises and Applications, *SIAM review* 10 422--437
16. C. K. Peng, S. V. Buldyrev, S. Havlin, M. Simons, H. E. Stanley, A. L. Goldberger, 1994, Mosaic Organization of DNA Nucleotides, *Physical Review E* 49 1685
17. R. Smith, 2010, Is high-frequency trading inducing changes in market microstructure and dynamics?, available at SSRN http://ssrn.com/ abstract=1632077
18. M. S. Taqqu and V. Teverovsky and W. Willinger, 1995, Estimators for Long-Range Dependence: An Empirical Study, *Fractals* 3 785--798
19. Yahoo! Finance website. http://finance.yahoo.com/

Appendix A

Hurst exponent graphs



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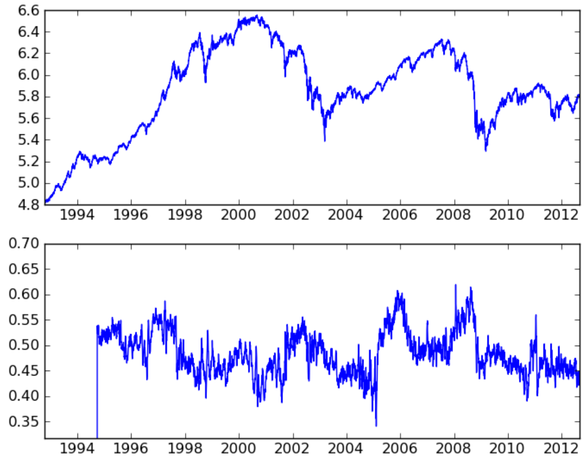

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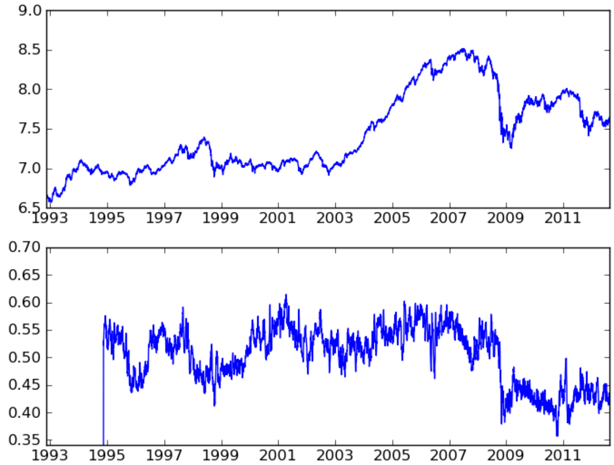

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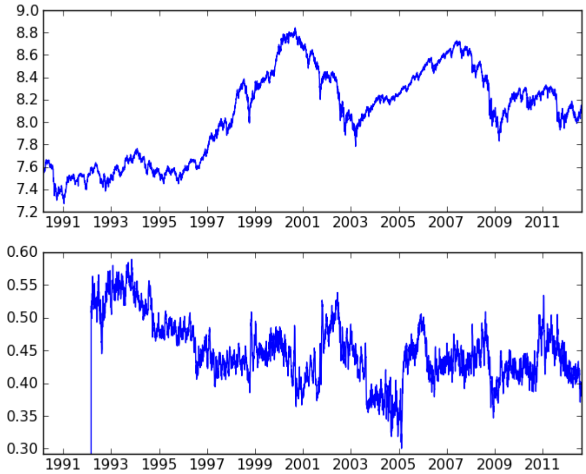

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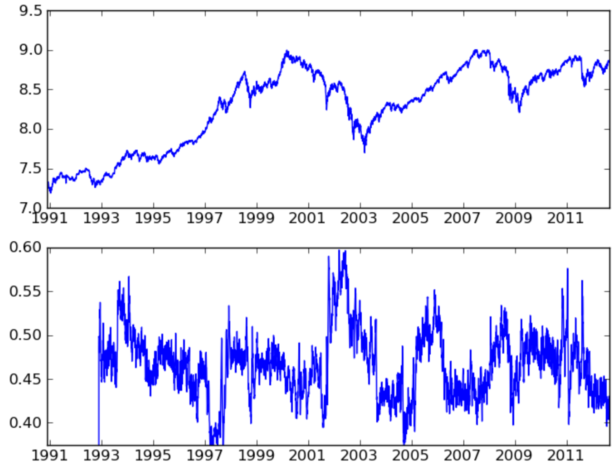

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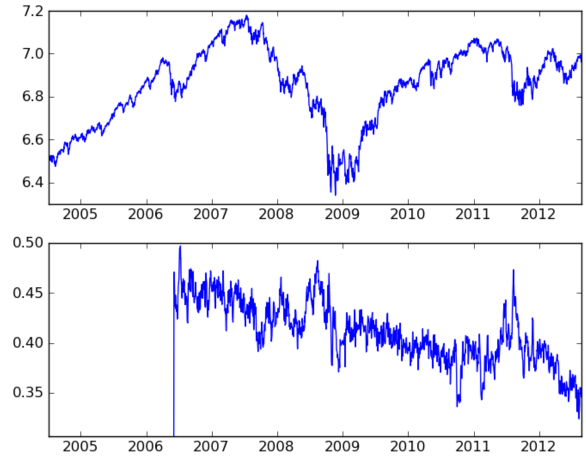
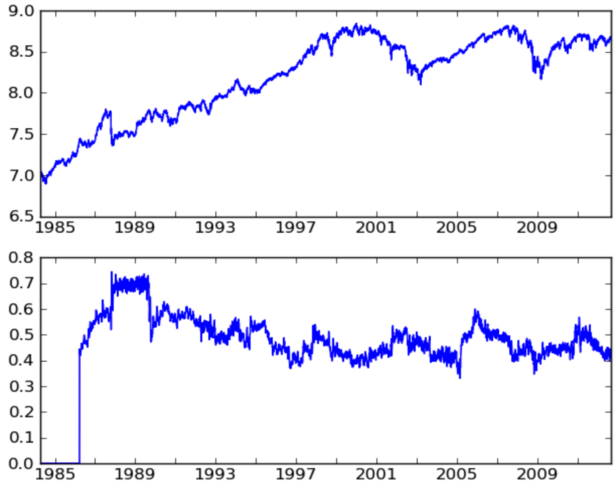


EXCH



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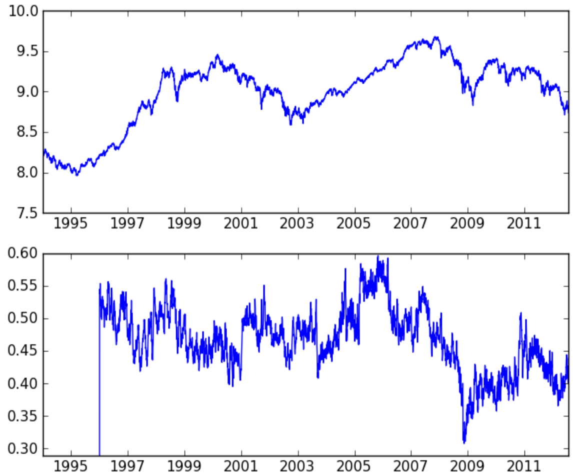

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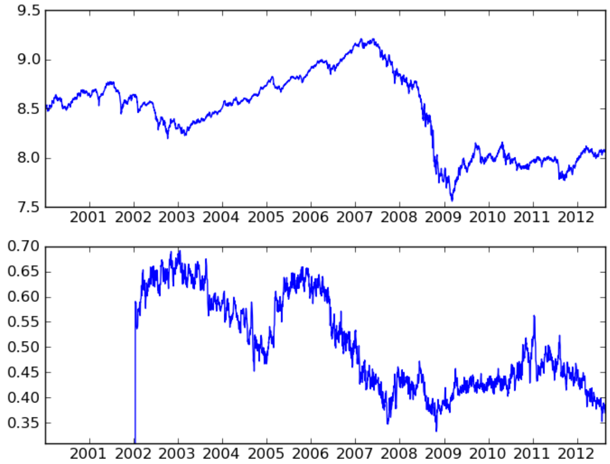

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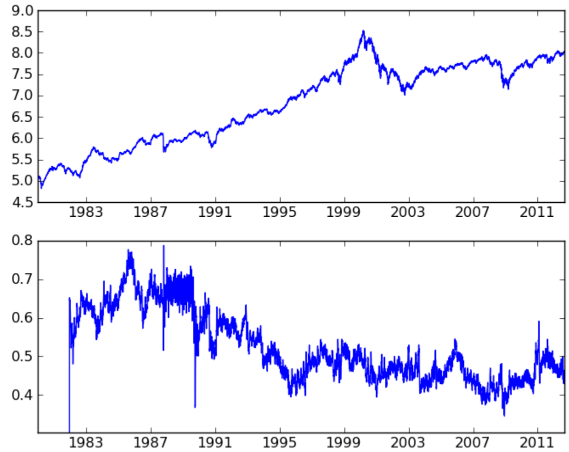
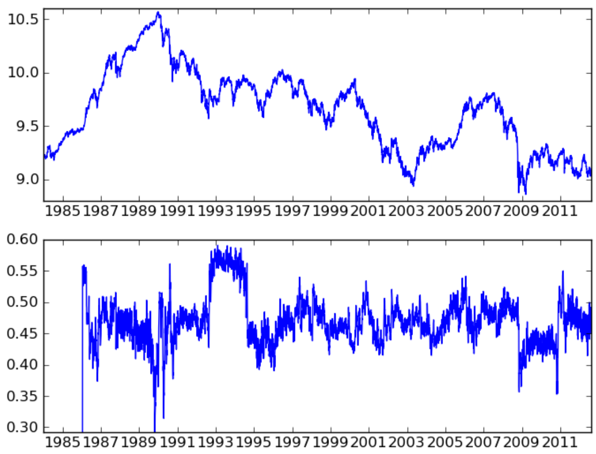


NASDAX



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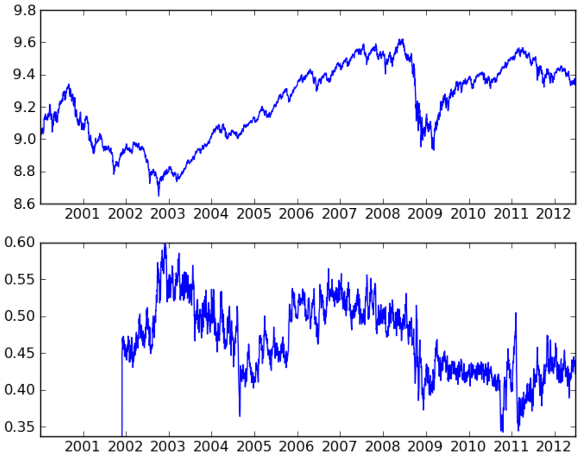
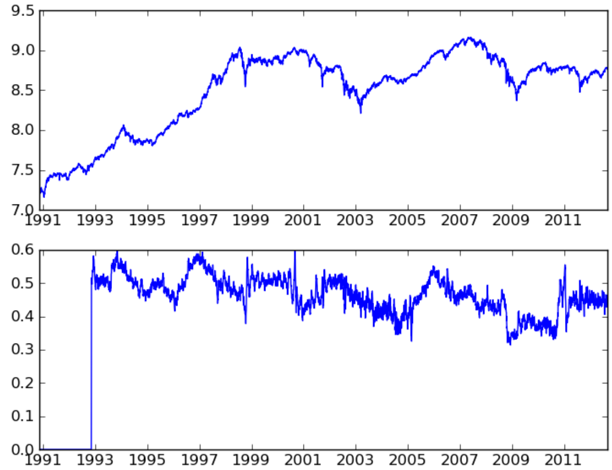


S&P/TSX



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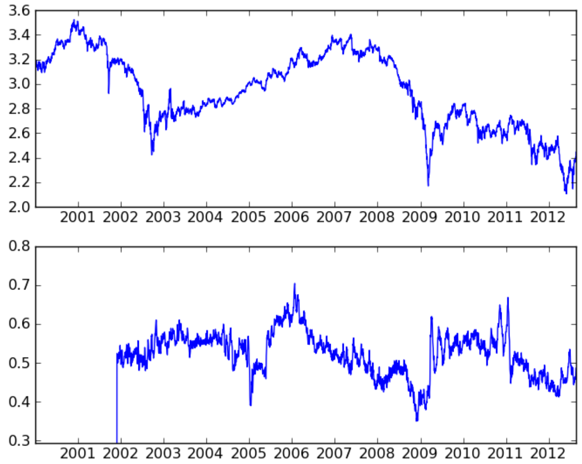

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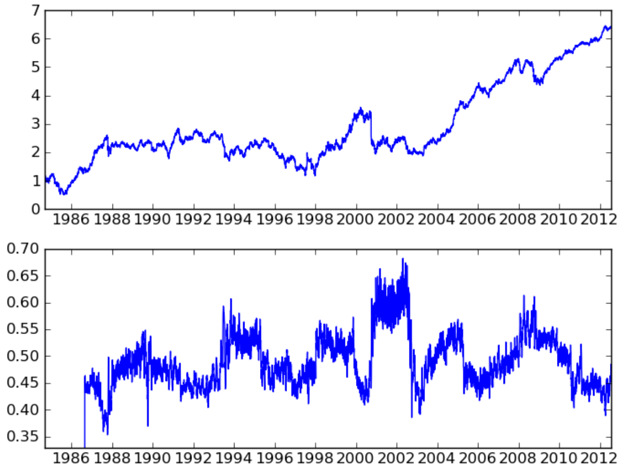

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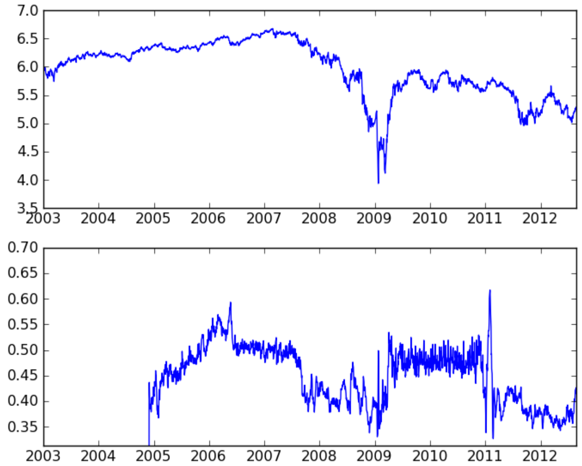
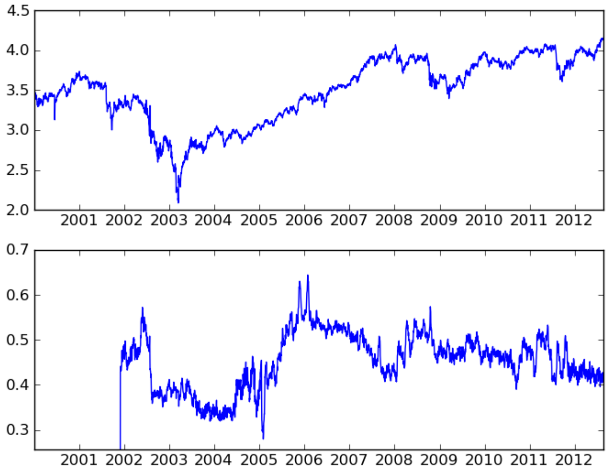
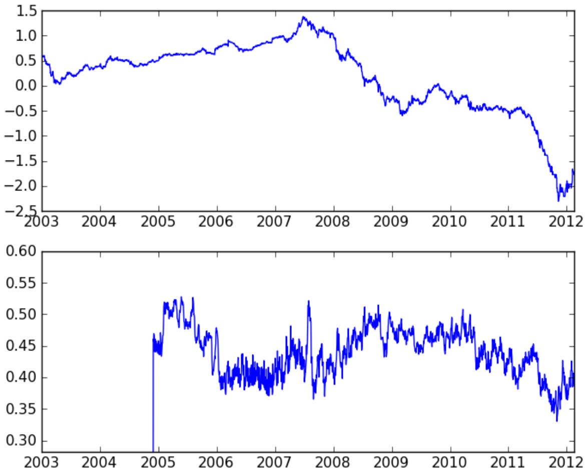


Barclays



Bayer



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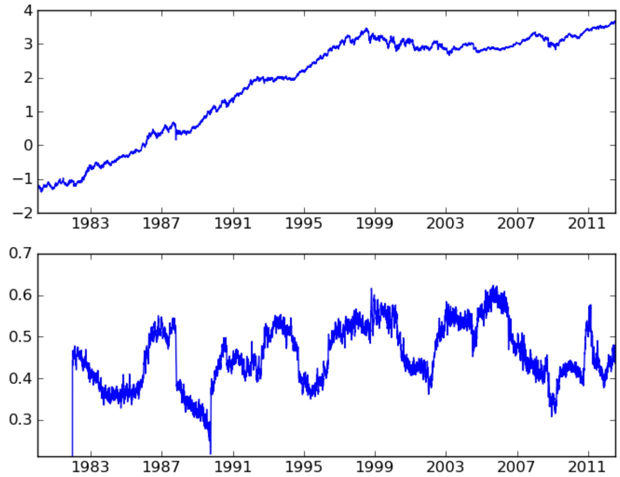

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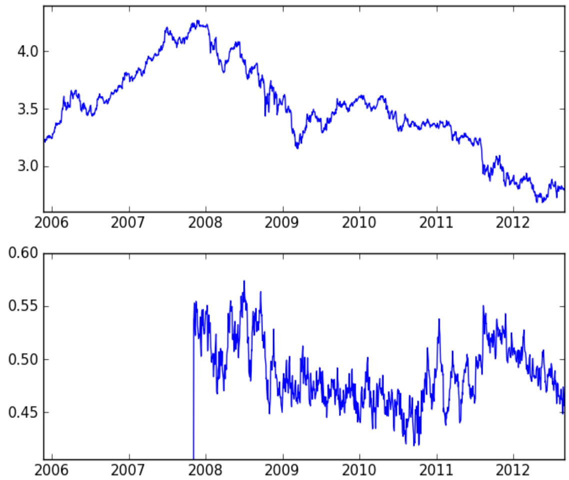

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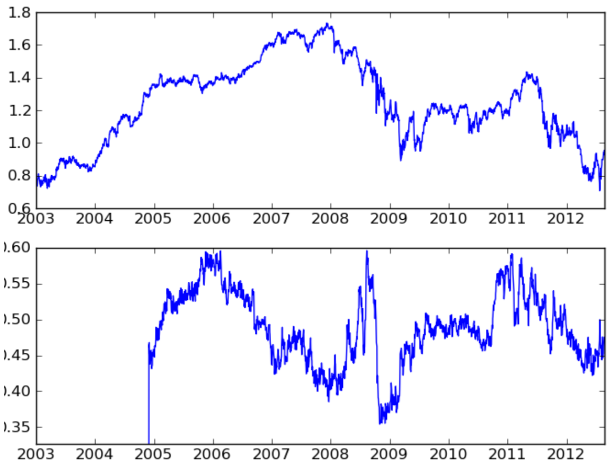

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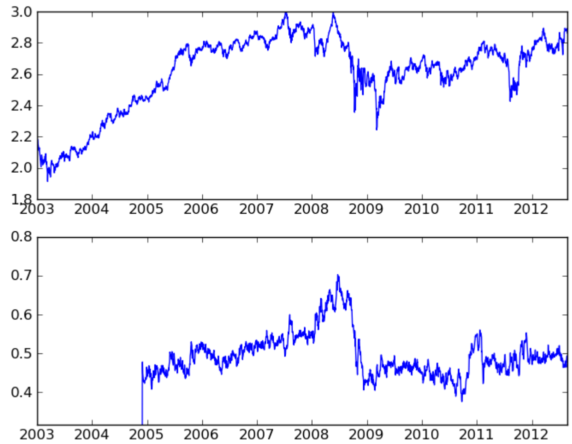

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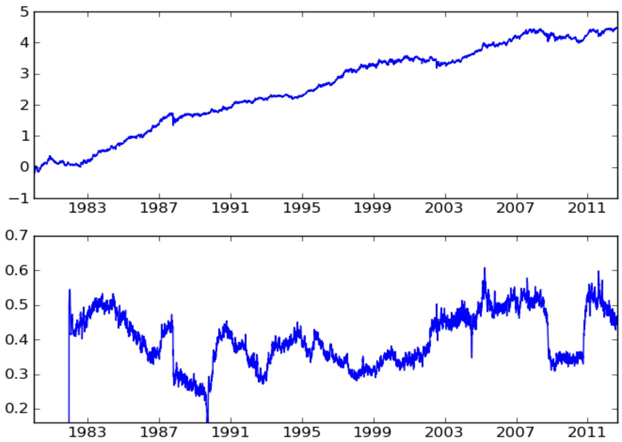

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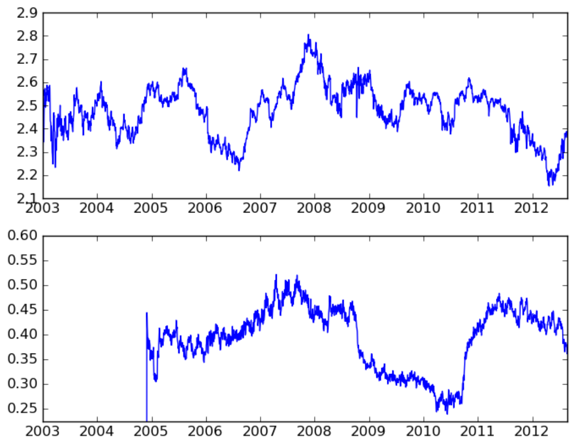

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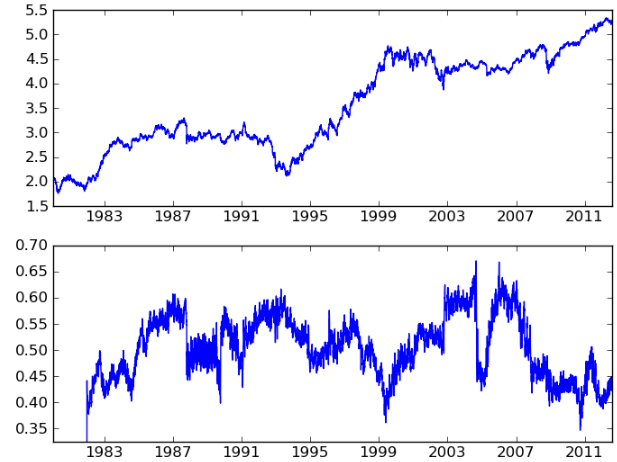

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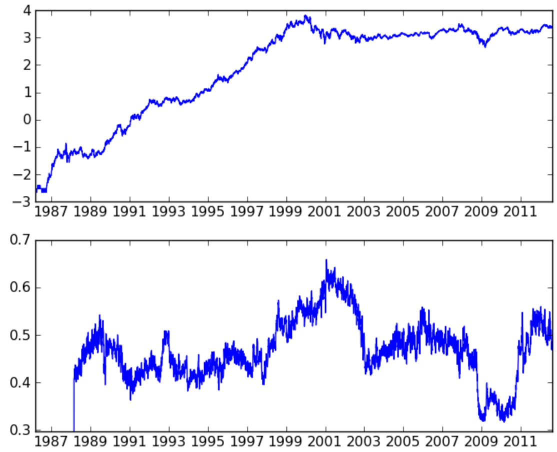

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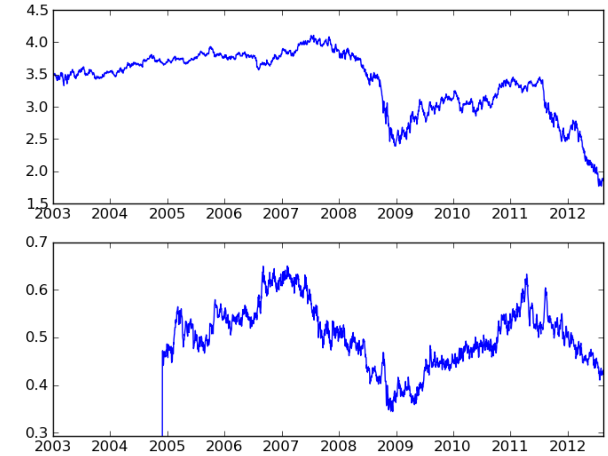

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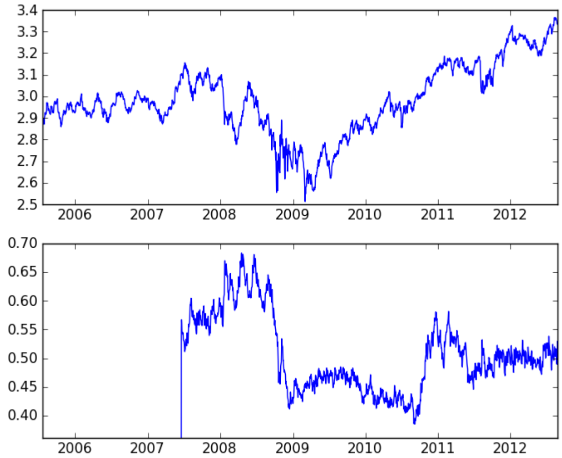

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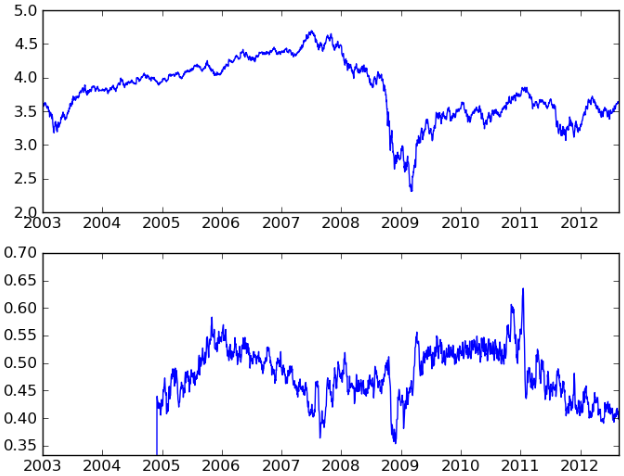

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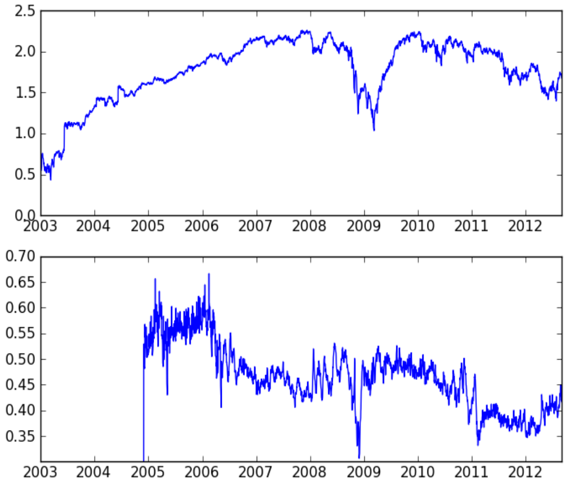

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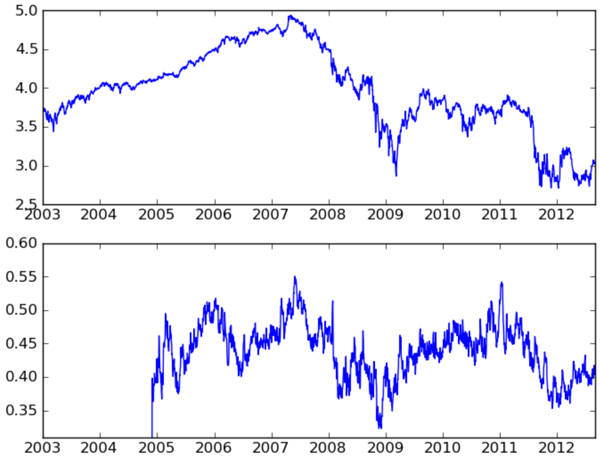

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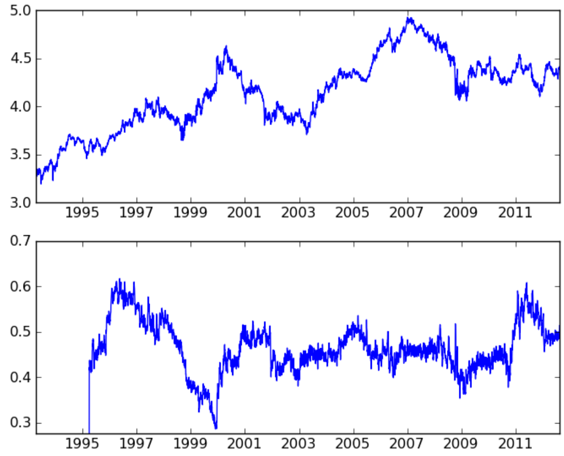
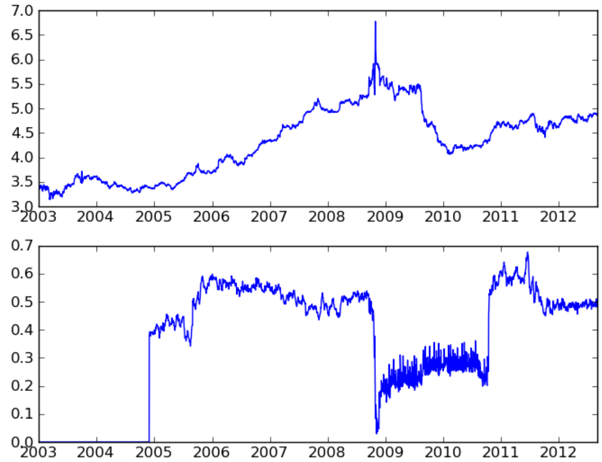
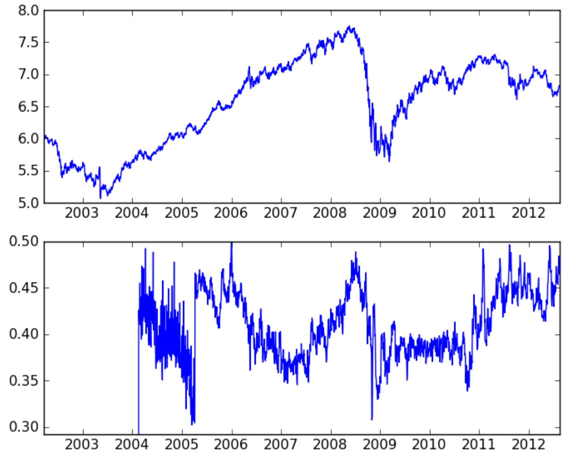


Toyota



Vow



Xstrata

Appendix B

Crises detected

AEX:
Max 2007-07-16, Min 2009-03-09
2004-08-26
2004-12-15
2010-09-17
ATHEX:
Max 2007-10-31, Min 2009-03-09
2011-09-30
ATX:
Max 2007-07-09, Min 2009-03-09
2008-10-30
2009-02-17
2010-05-04
2010-08-24
2011-02-22
2012-04-03
CAC 40:
Max 2007-06-01, Min 2009-03-09
2004-04-22
2004-12-03
2005-02-02
2006-05-12
2007-09-25
2008-10-25
2009-02-12
2009-05-06
2009-07-07
2010-01-01
2010-02-18
2010-06-09
2012-01-25
2012-03-06
2012-07-13
DAX:
Max 2007-07-16, Min 2009-03-06
2004-09-04
2004-12-11
2005-02-10
2007-07-18
2008-12-09
2011-02-04
2012-07-24
EXCH:
Max 2007-07-19, Min 2008-12-05
2008-12-10
2010-01-09
2010-04-28
2012-06-15
FTSE:
Max 2007-06-15, Min 2009-03-03
2004-05-11
2004-10-30
2004-12-23
2007-09-28
2008-10-21
2009-02-11
2012-05-01
2012-07-28
IBEX:
Max 2007-11-08, Min 2009-03-09
2008-09-19
2009-02-13
2009-04-23
2009-07-01
2009-09-03
2010-02-19
2010-04-20
2011-12-03
2012-07-31
ISEQ:
Max 2007-02-20, Min 2009-03-09
2007-08-03
2007-12-01
2008-03-28
2008-07-31
2009-01-21
2012-03-21
NASDAQ:
Max 2007-10-31, Min 2009-03-09
2007-07-04
2008-02-12
2008-05-06
2008-07-19
2008-10-25
NIKKEI 225:
Max 2007-07-09, Min 2009-03-10
2008-10-30
2009-02-18
2010-10-19
OMX:
OMX: Max 2007-07-16, Min
2008-11-21
2007-09-15
2008-04-24
2008-10-29
2009-03-31
2009-05-12
2009-07-09
2010-02-26
2010-04-27
2010-12-09
2011-07-08
2011-09-10
2012-02-10
SMI:
Max 2007-06-01, Min 2009-03-09
2004-05-14
2004-12-14
2005-02-12
2007-10-03
2008-10-14
2009-04-21
2009-06-30
2009-09-10
2010-04-09
2011-01-28
2011-10-08
S&P/TSX:
Max 2008-06-18, Min 2009-03-09
2004-08-21
2008-10-29
2009-01-27
2010-08-21
2010-12-25
2011-02-15
2011-06-04
2011-08-26
2012-05-03
AG:
Max 2007-05-18, Min 2009-03-09
2005-01-12
2008-11-04
Apple:
Max 2009-12-30, Min 2009-01-20
2012-03-29
Barclays:
Max 2007-02-23, Min 2009-01-23
2005-01-22
2007-09-28
2008-03-04
2008-07-01
2008-09-13
2009-01-14
2011-01-01
2011-02-22
2011-07-05
2012-07-13
Bayer:
Max 2008-01-09, Min 2009-03-17
2004-04-16
2004-07-24
2004-10-01
2005-05-07
2010-09-14
2012-01-25
2012-06-14
BCP:
Max 2007-06-26, Min 2009-03-09
2006-01-18
2007-02-09
2007-07-05
2007-08-22
2007-10-31
2008-01-02
2008-02-27
2011-01-29
2011-07-09
2011-12-28

Coca Cola:
Max 2008-01-10, Min 2009-03-05
2008-01-18
2008-10-01
2009-04-22
2009-06-23
2010-07-24
2011-10-12
2012-01-18
EDF:
Max 2007-11-22, Min 2009-03-13
ENEL:
Max 2007-12-06, Min 2009-03-09
2007-10-11
2007-12-12
2008-02-08
2008-10-18
2009-01-29
ENI:
Max 2007-07-09, Min 2009-03-06
2010-08-10
Exxon Mobil:
Max 2008-05-20, Min 2008-10-15
2004-07-01
2008-10-16
2009-02-11
2009-04-24
2009-06-24
2010-07-24
FT:
Max 2007-11-20, Min 2009-07-10
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2005-03-16
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2005-06-25
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2006-03-29
2006-11-03
2008-10-14
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2011-01-25
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2012-06-08
2012-07-24

IBM:
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2008-12-10
2009-02-07
2010-07-24
2011-11-16
Microsoft:
Max 2007-11-01, Min 2009-03-09
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2009-04-24
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2010-04-21
2010-06-16
2010-08-18
PSA:
Max 2007-07-24, Min 2008-12-23
2008-10-28
2009-01-31
2009-03-31
RDS:
Max 2007-07-06, Min 2009-03-03
2010-09-01
Renault:
Max 2007-07-03, Min 2009-03-03
2007-08-22
2008-10-31
2009-01-15
2012-05-05
2012-07-18
Santander:
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2008-11-05
2010-12-07
2011-02-01
2011-05-04
2012-02-28
2012-05-08
2012-07-06
SG:
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2005-01-25
2005-04-28
2008-02-22
2008-07-04
2008-09-13
2009-01-17
2009-04-22
2011-09-10
2012-05-05
2012-07-05
Toyota:
Max 2007-01-04, Min 2009-03-09
2006-06-27
2008-09-30
2009-07-09
2010-09-22
Vow:
Max 2008-10-28, Min 2009-12-21
2004-12-29
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2005-05-11
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2008-10-18
2009-01-07
2009-03-04
2009-05-05
2009-09-18
Xstrata:
Max 2008-05-19, Min 2009-03-09
2004-04-30
2006-05-05
2006-08-18
2007-06-12
2007-09-01
2007-11-10
2008-01-25
2008-09-27
2009-02-03
2009-04-23
2009-06-27
2009-08-28
2010-12-11
2011-02-12