**SECTION 1
ACADEMIC
INVESTIGATIONS
& CONCEPTS**

# THE IMPORTANCE OF ACCURATE RISK MODELLING TECHNIQUES FOR CORPORATE OWNERS AND MANAGERS: AN APPLICATION OF DISTRIBUTION FITTING TO ILLIQUID SECURITIES

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### Abstract

The Normal distribution is both the most commonly cited and highly parameterised of all the known probability distribution functions. This research highlights the importance of probing beyond standard textbook theory which assumes, for risk modelling purposes, that an asset's return should follow a Normal distribution. Methods of modelling the stochastic price process of two illiquid securities, in order to manage price risk within a simple GARCH Value-at-Risk framework are examined. This analysis was developed using Microsoft Excel, IHS's EVIews and Palisade's Decision Tools Suite. These widely used tools are chosen to allow for ease of replication of this analysis for any interested market participant and can be expanded to portfolios of liquid and illiquid assets. By ensuring a strict and efficient risk modelling template owners and managers alike are in turn held accountable to all company stakeholders.

**Keywords:** Illiquid Securities, Risk Modelling, Corporate Accountability, GARCH, Monte Carlo Simulation, Value-At-Risk (VAR), Basel AMA Framework

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## 1. Introduction

The purpose of this paper is to highlight the importance of going beyond standard textbook theory and assuming for risk modelling purposes that an asset's return should follow the Normal distribution simply because it is the most well-known and best parameterised of all the known probability distribution functions. Company managers and owners are sometimes divorced from the reality of methodologies applied by quantitative risk modellers. This paper addresses key foundations assumed by risk modellers which if

questioned and understood may lead to more efficient outcomes for the corporate entity.

According to Tan and Chu (2012) the normal distribution is one of the most widely applied distributions. From the late 1960's it became apparent that empirical studies failed to find confidence in the normality assumption when calculating returns distributions for financial data. Rachev (2003) notes that modern finance theory puts a strong emphasis on the idea that observed random variables are represented by a normal distribution. The distributional assumption is crucial to risk managers. Observable time series in finance often do not follow a Gaussian process. Distributions are often characterised by being fat-tailed and asymmetric. Thus financial modellers would be prudent to question the common assumption of normality. Agrawal (2009) stated that when examining test statistics based on the normality assumption erroneous inferences can eventuate if this assumption fails. This occurs when data which does not follow a normal distribution results in incorrect standard errors.

Anecdotal evidence from risk managers suggests that participants in the Australian financial services and utilities sectors continue to apply this assumption despite basic evidence provided by summary statistics (skewness, kurtosis, Jarque-Bera etc.) clearly indicating some type of non-Normal distribution being representative of returns. This practice possibly persists in these sectors due to the widespread use of Microsoft Excel as a development environment for risk modelling as it conveniently has up to 12 probability distributions to choose from in any analysis. Directly testing whether the empirical data fits a theoretical distribution is, however, problematic, time consuming and prone to error. Without any speciality third-party add-ins, perhaps the best that can be hoped for with Excel is to be able to comprehensively test the data against departures from Normality.

Palisade's @Risk add-in to Excel, a part of the Decision Tools Suite, has over 40 in-built distribution functions[1] and allows the user to fit these to a selected data set and rank the fit according to a statistical test (e.g. Chi-square, Kolmogorov-Smirnoff, and Anderson-Darling). IHS's EVIews is a stand-alone econometric package and doesn't necessarily offer any advantages in terms of the number of probability distributions available for analytical use. Nonetheless its value lies in being able to apply the Empirical Distribution Test in combination with a null hypothesis which posits that the data follows a user-specified theoretical distribution. This is chosen from a palette of 10 distributions according to both a selection of best fit criteria (Anderson-Darling, Lilliefors, Watson and Cramer-von Mises) and confidence level. What is important to note is that even if the null hypothesis is rejected, it is still useful to adopt the better fitting theoretical distribution suggested by @Risk because simulation results will be far more accurate than falling back on the Normality assumptions in the absence of these complimentary tools.

Implementation of such packages, some of which are integrated with Excel, do not require a large outlay in capital or training, nor do they require a PhD in the mathematical sciences to set up and interpret results but they do provide the means to easily achieve a much higher degree of precision within the risk modelling framework. This leads to better understanding of the risk profile of a portfolio of financial and energy assets that is more sensitive to changes in external volatility, better able to anticipate variations in risk profile, and be more acceptable under the market risk stream of the Basel framework potentially resulting in capital charge relief.

This research road-tests Palisade's @Risk, which performs risk analysis using high precision Monte Carlo simulation, to show the possible outcomes and their respective likelihood, on two illiquid securities within a VaR framework using Bollerslev's (1986) Generalised Auto Regressive Conditional Heteroscedacity (GARCH) model. The absence of an active options market precludes the use of implied volatility in the modelling process so any VaR framework must rely on a volatility input derived from historical returns only. The addition of Monte Carlo techniques within the analyses ensures these models, rather than being deterministic, capture the uncertainty in future prices (Alexander, 2008a). This research makes no distributional assumptions – these are determined by the chosen software algorithms.

It is shown that by selecting a probability density function (PDF) more aligned to the portfolio's true but unknown distribution and according to some predefined "best-fit" criteria that the number and independence of violations correspond to the expected level at some significance level. Indeed, the extra precision achieved in terms of violations obtained from choosing better fitting distributions as opposed to relying on the Normal distribution present a compelling case for the rejection of standard textbook theory.

This introduction is followed by a précis on the reasoning behind asset selection. A description of the chosen assets under examination and the data sampling process follows. A modelling approach employing a GARCH methodology is then presented. A model examination process using Coverage Testing is then shown. This leads to an empirical discussion of results followed by relevant conclusions.

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[1] Appendix 1 presents a table that lists all the distributions available for analysis in each of the three packages.

## 2. Asset Selection

One of the defining characteristics of the recent global financial crisis (GFC) was the almost instantaneous evaporation of liquidity and the convergence of correlations across asset classes (Super Review, 2010). Even now current troubles affecting European sovereign debt, significant residual volatility remains and liquidity concerns persist and widespread positive asset price correlations are still present within the global financial system (Citibank, 2010).

This research has been directed towards key issues facing the Australian financial and utilities sectors. The authors have personally observed the persistence of the Normality assumption in practice around the risk modelling of certain illiquid securities. The Australian market for equity, in particular, is small by global standards, representing a mere 2 per cent of global market capitalisation (Trading Economics 2011). Time and again, when financial crises occur no matter where they originate in the world, the illiquidity of the Australian securities market becomes painfully apparent.

Liquidity, or lack thereof, heavily influences the shape and structure of financial asset prices and returns, and understanding this influence is the key to developing and optimising risk models so that they continue to supply relevant early warning signals that facilitate the decision making process. As such the authors have chosen two highly illiquid, some might argue obscure, securities to demonstrate the hypothesis that it is more beneficial to model price risk when one chooses a PDF better suited to the actual returns rather than relying on the Normal distribution simply for computational ease.

The two securities examined in this paper are the "penny" stock PIE Networks traded on the Australian Stock Exchange (ASX) and the renewable energy certificate (REC), the unit of currency underpinning the amended Renewable Energy (Electricity) Act 2000 (Commonwealth) and traded through over-the-counter (OTC) channels. Whilst both assets share no common correlation or are related in any way, they do share some purely technical similarities concerning liquidity: both came into being around the same time and their price histories show long periods of time when prices didn't change or volume rose above zero. Their shared *illiquidity* makes then ideal candidates for this study.

### 2.1 PIE Networks

Australia possesses an embryonic information technology & communications (IT&C) sector that is growing, in fits and starts, but lacks the scale and scope of Europe and the US, with most effort focussed on domestic market solutions by replicating overseas trends (IDG, 2011). The ASX lists a number of GICS (general industry classification standard) dedicated to the IT sector, these include: Semiconductors & Semiconductor Equipment; Software & Services; Technology Hardware & Equipment; and Telecommunication Services.

PIE Networks, nestled in the Hardware & Equipment sector of the ASX, describes itself as a manager of WiFi services and public Internet solutions, marketed to a wide range of customers, from small business through to large corporates across many industry sectors including government (PIE Networks, 2011). The company is led by experienced telecommunications industry professionals, whose vision is to significantly expand the growth of wireless Internet, enabled by the take-up of smart wireless devices that can then be leveraged by businesses to deliver better customer experiences (PIE Networks, 2011).

The company's key product is the Hotspot Webphone – a 21$^{st}$ Century payphone that also provides internet access and WiFi Hotspot connectivity. It is designed to be a "telco" gateway with customer access (WiFi & fixed), 3G network offload, a retail, payment and advertising channels. The product is envisaged as a replacement for traditional indoor payphones in high traffic retail locations, such as shopping malls, airports and banks. PIE is currently partnered with Telstra – the government-owned, dominant Australian telco – to conduct a trial deployment of Webphones into Australian airports, with a view to a more comprehensive rollout. The revenue model is based on hardware sales (i.e. the Webphone), recurring software and service fees (PIE Networks, 2011).

PIE's official stock market listing date was 7 April 2000: the market low and high prices since then respectively are $0.007 and $0.118. The daily turnover of shares transacted has averaged 35,000 during this period and there have been numerous periods of its history when no shares have traded particularly in the earlier years. PIE is also one of the only small capitalisation, or 'penny' stocks listed on the ASX that has not had a reconstruction of capital since its original listing date (ASX, 2011).

### 2.1.1 Data

The PIE Networks weekly price and volume data was sourced from SIRCA for the period 27 November 2002 to 6 January 2011, and is aligned to the data for the second asset, discussed below. It is interesting to note that the period chosen saw the bulk of volume in traded shares. Prior to this period, share turnover was low even by small capitalisation standards. Figure 1 shows how the price of PIE has varied since November 2002. In

terms of liquidity, on 5 September 2007 when the stock hit a high of $0.118, 1,188,863 shares traded

hands equating to a marketable parcel of just AUD$140,285.
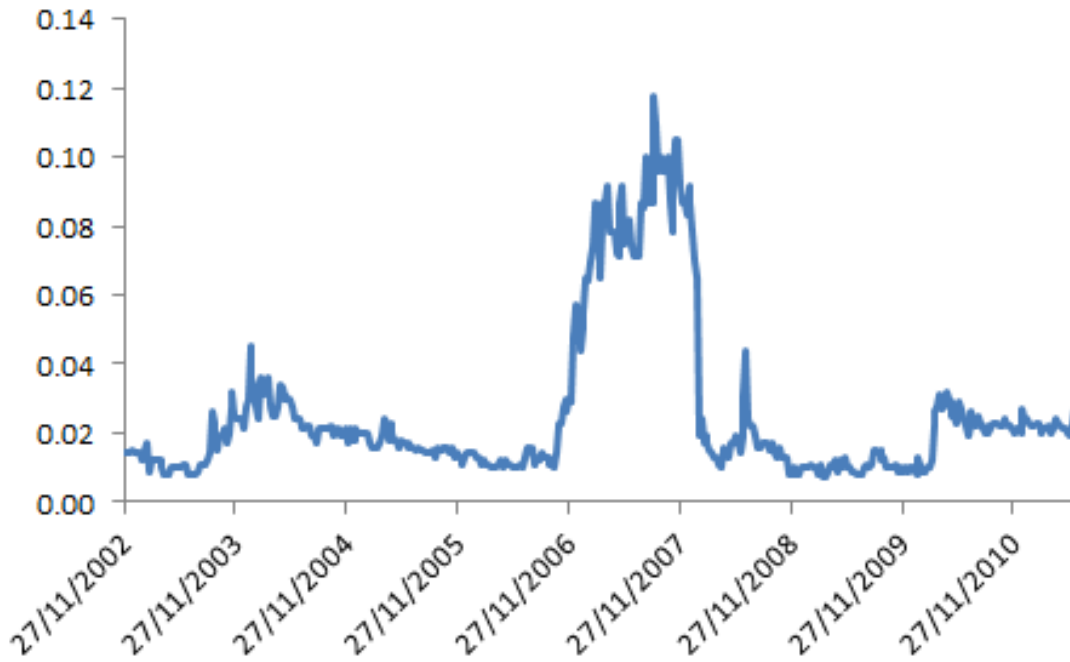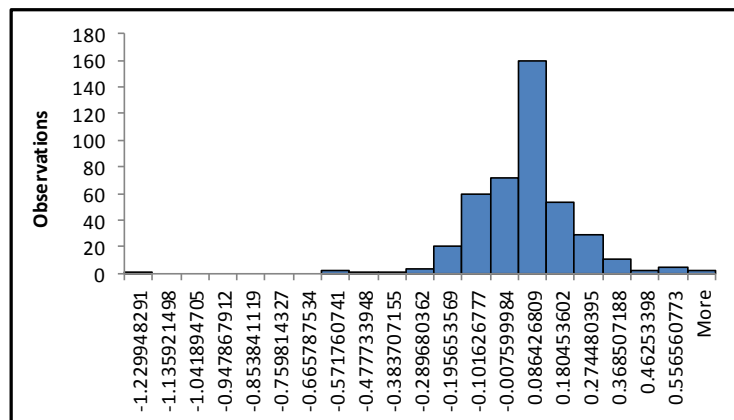
**Figure 1.** PIE Price Chart



Figure 2 displays the descriptive statistics for PIE price returns over the period November 2002 to 29 June 2011. The average price of PIE was $0.17 with a mean return of 0.13 per cent which, whilst

not shown here, is statistically insignificant from zero. The weekly return volatility was 16.16 per cent - high compared to many Australian small capitalised stocks.

**Figure 2.** PIE Summary Statistics & Histogram



As can be seen from the histogram in Figure 2, the majority of observations are clustered about the mean, the body of the distribution curve is fairly well represented but of interest to the risk manager are those extreme returns in the left hand tail representing a significant loss event. Other summary statistics such as the skew and kurtosis tend to reject the assumption of normality, which is subsequently confirmed by the Jarque-Bera test.

Palisade's @Risk add-in for Excel is employed to estimate the most likely distribution from the sample data. Ranking the efficiency of the fitted distribution by the Anderson-Darling test statistic which attempts to fit the tails (Heiat 2005, 6) it is found that the Logistic distribution (appendix 2) provides the better fit, in terms of tail coverage, to the underlying population as per Table 1 below. It is clear that the fit of the Logistic

distribution is superior to the standard Normality

assumption by a clear margin.

**Table 1.** @Risk Distribution Fit Statistics

| Distribution | A-D Statistic |
|---|---|
| Logistic | 5.6212 |
| Normal | 10.0630 |
| Weibull | 18.0036 |

Using the Empirical Distribution Test in EVIews the null hypothesis that the underlying data follows the Logistic distribution is tested. As can be observed from Figure 3, the p-values for the three

best fit criteria are quite small, less that 0.5 per cent, which would ordinarily result in a rejection using any standard confidence level measurement.

**Figure 3.** Empirical Distribution Test Results for PIE



Empirical Distribution Test for RESID
Hypothesis: Logistic
Date: 06/20/12   Time: 02:43
Sample (adjusted): 12/04/2002 1/05/2011
Included observations: 423 after adjustments

| Method | Value | Adj. Value | Probability |
|---|---|---|---|
| Cramer-von Mises (W2) | 1.235268 | 1.238006 | < 0.005 |
| Watson (U2) | 1.235268 | 1.238006 | < 0.005 |
| Anderson-Darling (A2) | 5.621165 | 5.624488 | < 0.005 |

## 2.2 Renewable Energy Certificates

In 2009, the Australian Mandatory Renewable Energy Target (MRET) scheme was amended calling for the amount of renewable energy to represent a minimum of 20 per cent of the total energy mix, the equivalent of 45,000 GWh, by 2020. The intent behind the legislation is to encourage more investment in sustainable energy technologies and to cut the total carbon output from the energy sector (Anderson and Strate, 2009).

The financial means to achieve the increased MRET target is to be through, at least in the short-term, the interchange between producers and obligors, of renewable energy certificates (RECs). Entities that produce renewable energy are eligible to create one REC for each MWh of output which they can sell to energy retailers and large energy consumers (say a steelworks), who are obligated to surrender RECs in accordance with their total energy purchases made each calendar year (ORER, 2011). This exchange of certificates occurs on the primary market. The event of failure to surrender the correct number of certificates can result in federally mandated fines and reputational damage[2].

Within each calendar year, RECs can be traded in the secondary market through OTC networks, dominated by the major energy utilities and a few specialised financial intermediaries. The

lack of liquidity is a key characteristic of the RECs market but since 2001 anecdotal evidence suggests that because of increases in the number of participants, the volume of RECs traded in both the spot and forward years has increased significantly.

The price of RECs is directly correlated to the cost of supplying renewable generation, and it is well known that the differential cost between fossil fuel energy and that produced through renewable channels has always been large, hence the apathy with developing sustainable potential, i.e. the argument exists that if something needs a subsidy to develop then clearly it is not profitable to begin with. Other significant factors that, in Australia in particular, impact on the REC price are the structure of the wholesale energy market, the climate[3], the exercise of market power, regulatory uncertainty[4] and secondary market liquidity which manifest themselves in relatively large swings in both spot and forward prices (IES, 2002). There are two additional features of the MRET scheme that exert some influence on price but whose effects have not been thoroughly studied either academically or by industry[5]. The first is the ability to bank excess certificates from one year to the next (and beyond) and the second feature is the fact that

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[2] The Federal government can and does name and shame non-compliance in parliament each year.

[3] Drought in particular has made its presence known in the REC market particularly during 2007.
[4] This existed for a long time surrounding the passage of a carbon pollution reduction scheme (tax) and the very state of the MRET market.
[5] At least nothing that has appeared to be made public.

the non-compliance penalty is refundable (IES, 2002) and non-indexed, creating an incentive to 'game' the system.

Overall there are a number of factors that impact on the price and availability of RECs, and the fact that the amount of renewable energy in the system must increase by 2020 means that managing the risk of renewable portfolios demands greater analytical resources by industry participants.

### 2.2.1 Data

Price data for RECs is collected from industry participants and published, via subscription, by the Australian Financial Markets Association (The authors gratefully acknowledge AFMA's generosity in supplying the data to us without cost for research purposes in 2011. In particularly, Jacinta Lee went above and beyond the call of duty in providing to us a complete set of prices dating back to 27/11/2002, more than is generally made available via subscription) ("AFMA"). AFMA polls its members each week requesting disclosure as to what they judge to be the prevailing offer and bid prices for the relevant environmental instrument, for the calendar years from spot to five years forward. The survey participants encompass various types of organisations on both sides of the market, citing not firm, but indicative prices only.

As per the PLATTS pricing benchmark most commonly used in the energy industry, it is deemed that the 'Median of Mids' to be the best statistical representation of the data. The Median of Mid is calculated from the midpoint of each bid and offer pair submitted to AFMA. Of the distribution describing all these resulting midpoints, one standard deviation is calculated both sides of the median and data points lying outside of this range are deemed outliers and are removed. Given that the Median of Mids has unrealistic prices removed, the resulting time series data handles skewness better than otherwise would be expected from the raw data collected by poll. Figure 4 shows how the price of RECs has varied since November 2002 when AFMA began publishing weekly prices.

The price of RECs has a financial impact on those that produce them and those that discharge compliance obligations. Certificates confer a revenue stream on those that produce renewable energy and are adversely impacted by falling REC prices. On the other hand, energy consumers, those that surrender RECs, incur an expense and are hurt by rising prices. Therefore, as with fund managers in the financial sector, there is a need to manage the price risk associated with the RECs portfolio and the principles of VaR and portfolio risk management techniques become important within an integrated firm-wide framework for reporting energy market risks.

**Figure 4.** REC Price Chart



Figure 5 sets out the descriptive statistics for REC price returns over the period 27 November 2002 to 6 January 2011 (Note that four weekly observations are missing. This is due to the polling day falling on a public holiday, usually around the

Christmas period, and as such no data was collected). The average price per REC was $36.21 with a mean return of -0.05 per cent which, whilst not shown here, is statistically insignificant from zero. The weekly return volatility was 4.02 per

cent. The price of RECs fell to a low of $11.94 on 19 October 2006 after a large-scale hydro-electric generator released a large number of certificates onto the market. The price rebounded strongly on the back of the drought in eastern Australia to reach a peak of $53.21 on 25 May 2008 before settling to a price of $29.78, $6.82 below its initial "listing" price, at the end of its life in January 2011.

As can be seen from the histogram in Figure 5, the majority of observations are clustered about the mean, the body of the distribution curve is fairly bare of returns but there are extreme returns evident in both tails, each representing an eight standard deviation event. As with PIE, the skew and kurtosis tend to reject the assumption of normality, which is subsequently confirmed by the Jarque-Bera test.

**Figure 5.** REC Summary Statistics & Histogram
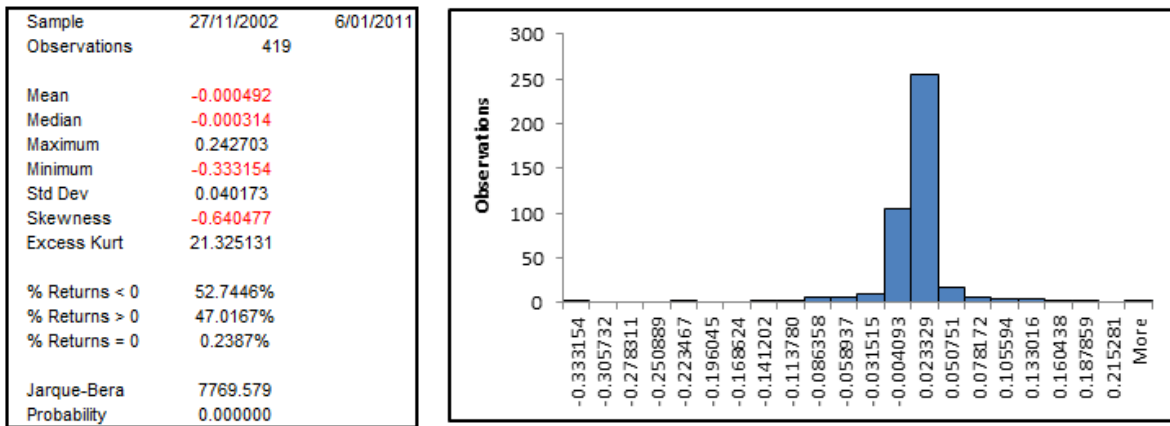


In 2011, the original renewable energy certificate (REC) was split into two component parts to reflect the disparities in scale of renewable plant that create certificates (ORER, 2011): the large-scale generation certificates (LGCs) and the small-scale technology certificates (STC) have subsequently replaced the original REC. As a result, a disconnect now exists between the original certificate price series and the new certificate price series. As there is insufficient history for the new scheme, it has been elected to focus the analysis on the original certificate scheme. This results in a time series of 418 data points for the period 27 November 2002 to 6 January 2011. Thus no further times-series data on this security is available and as a consequence further research analysis on this security is not possible.

As with PIE, the rates of change in the REC prices are determined and @Risk is employed to estimate the most likely distribution of returns. Not surprisingly, the Logistic distribution provides a better fit than the Normal distribution according to the A-D test statistic, as per Table 2 below, although the fit isn't as efficient as the one for PIE (5.9769 (PIE) versus 22.3093 (REC)). This is expected though given the description of the histogram of REC returns above.

**Table 2.** @Risk Distribution Fit Statistics for REC

| Distribution | A-D Statistic |
|---|---|
| Logistic | 22.3093 |
| Normal | 46.7965 |
| Weibull | 63.9391 |

Again the hypothesis that the sample of REC returns tested is drawn from the Logistic distribution is applied and as can be seen from Figure 6 the p-value for the Anderson Darling (A-D) test is between zero and 0.5 per cent leading to the conclusion that the true distribution is something other than Logistic (When the authors tested whether the PIE and REC data came from the Normal distribution in EVIews, the p-values were both zero). This exercise is an important part of the data analysis and consequently highlights the limitations of @Risk: it can only choose from the distributions available in its library and on this basis the Logistic is the best choice but not necessarily the true fit.

**Figure 6.** Empirical Distribution Test Results for REC

Empirical Distribution Test for RESID
Hypothesis: Logistic
Date: 06/20/12   Time: 02:52
Sample (adjusted): 12/04/2002 12/08/2010
Included observations: 419 after adjustments

| Method | Value | Adj. Value | Probability |
|---|---|---|---|
| Cramer-von Mises (W2) | 4.178212 | 4.188017 | < 0.005 |
| Watson (U2) | 4.178212 | 4.188017 | < 0.005 |
| Anderson-Darling (A2) | 22.30926 | 22.32257 | < 0.005 |

### 3. Data Sampling

The lack of long-term price history will lead to calibration issues during the modelling phase and the lack of 'organic' data may result in unstable VaR estimates. On the other hand a plausible industry scenario exists that many risk managers faced in the asset-backed securities market leading up to the GFC: how do financial sector enterprises (FSEs) manage the price risk of newly created assets / derivatives effectively where there is little or no history? With the help of the tools employed in this study, options exist to synthetically create additional history to aid in the risk analysis of illiquid securities but the effectiveness of this 'aid' depends on the ability to select a distribution more closely aligned to the true distribution.

The mean and variance parameters of the original PIE return data are examined to generate an additional 500 weekly samples from the Normal and Logistic distributions. Next a simulation based on 100,000 iterations is put in place to derive an expected outcome. Figure 6 compares the original return data with the samples drawn from the two distributions. As can be seen from the left hand panel, the Normal distribution simulates the 'average' returns quite well but fails to account for any extreme, or tail, moves that have occurred in the historical data. Indeed, the sample statistics, in Table 3, bear this out. The minimum and maximum samples generated by the Normal distribution lie *within* those from the historical data. Other statistics from the sample don't match the historical data either indicating its generally poor fit for risk management purposes.

**Figure 7.** PIE return data vs. hypothetical sampled returns
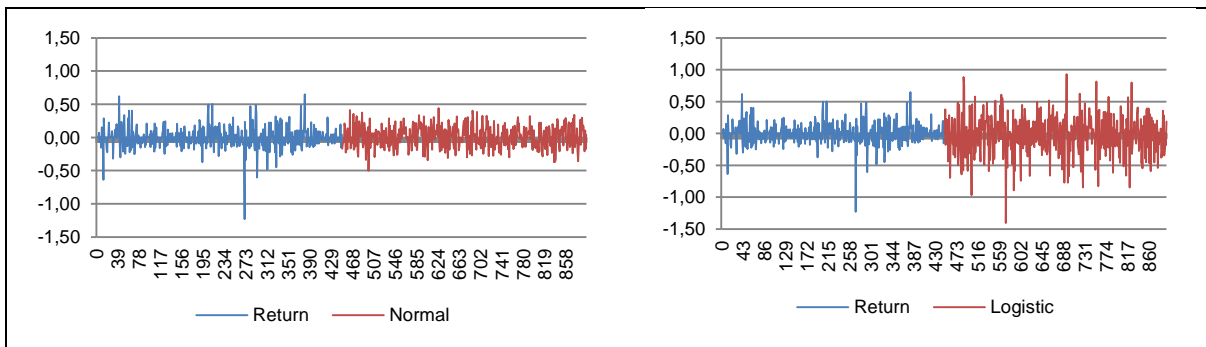


**Table 3.** Summary Statistics for Samples

| Statistic | Historical Returns | Normal samples | Logistic samples |
|---|---|---|---|
| Mean Return | 0.0012942 | 0.0107271 | -0.0324313 |
| Standard Deviation | 0.1615584 | 0.1629046 | 0.3080174 |
| Skewness | -0.6370697 | -0.0141445 | -0.299767 |
| Kurtosis | 9.281909 | -0.342546 | 1.0396698 |
| Minimum Value | -1.2299483 | -0.5015656 | -1.4060219 |
| Maximum Value | 0.6505876 | 0.4428324 | 0.9292129 |

The Logistic sample also has its issues: whilst it accounts for extreme observations present in the historical data, there are perhaps too many tail samples that occur at the expense of more average

returns. This would lead to excessive VaR violations than would be generally expected. Table 3 demonstrates that the Logistic distribution overstates the mean and variance. The extreme values, this time, lie *outside* those from the historical distribution.

This exercise is repeated with the REC data by generating 418 hypothetical future returns from the Normal and Logistic distributions. Figure 8 compares the original RECs data with samples drawn from the two theoretical distributions. As with the PIE example, the Normal distribution tends

to generate sample returns clustered about the mean and does not tend to produce any extreme moves as present in the historical returns. On the other hand, the Logistic distribution generates too many extreme and average returns. The summary statistics for the sample distributions are illustrated in Table 4 and indicate that whilst neither distribution appears to be an exact match to the underlying distribution, the Logistic is most appropriate representation from a risk management perspective due to the propensity for tail events to occur more frequently.

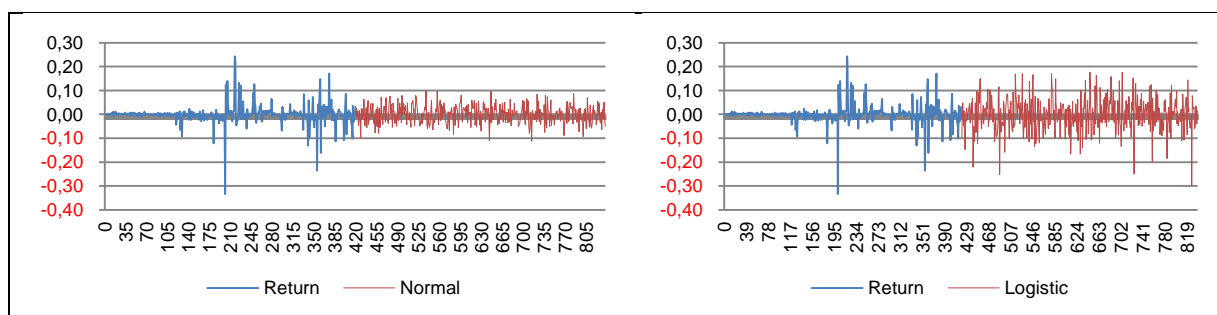**Figure 8.** REC return data vs. hypothetical sampled returns



**Table 4.** Summary Statistics for Samples

| Statistic | Historical Returns | Normal samples | Logistic samples |
|---|---|---|---|
| Mean Return | -0.000492 | 0.000235 | -0.002133 |
| Standard Deviation | 0.0401726 | 0.037572 | 0.0709011 |
| Skewness | -0.6404766 | -0.0298851 | -0.2496016 |
| Kurtosis | 21.325131 | -0.1428352 | 1.187132 |
| Minimum Value | -0.333154 | -0.112075 | -0.298536 |
| Maximum Value | 0.242703 | 0.097249 | 0.176529 |

Given that both PIE and REC return data appears to be best represented by a Logistic rather than a Normal distribution, according to the Anderson-Darling test statistic, the next step of the evaluation is to compare how each distribution performs within the VaR framework in terms of generating the expected number, and independence, of violations.

## 4. Modelling Approach

Risk managers are charged with understanding the empirical characteristics of financial asset prices, especially volatility clustering because if sustained high volatility is not anticipated and mitigated this increases the probability of an extreme tail event that could fatally impact on the availability of capital reserves to cushion losses (Kousky and Cooke 2010, 1).

Advanced models, such as a GARCH-type, have been shown empirically to be more successful in this regard (Engle, Focardi and Fabozzi 2007, 5), although non-parametric models are easier to implement. Realistically, there will never be a

perfect market risk model, which helps explain why stress testing has become a popular complement (Aragones, Blanco and Dowd 2001, 44). Taken together, this approach helps to mitigate the high level of model risk that was prevalent throughout the global financial sector during the lead up to the GFC (Avgouleas 2010, 392).

This research chose the GARCH framework of (Bollerslev, 1986) due to its widespread acceptance for VaR modelling. The illiquidity of both securities and absence of an active options market precludes the use of implied volatility in the modelling process so any VaR framework must rely on a volatility input derived from historical returns only. The addition of Monte Carlo techniques within the analyses ensures these models, rather than being deterministic, help to capture future price uncertainty.

The data from both series was subjected to a number of diagnostic tests (in EVIews) and, whilst not shown here due to brevity, the squared returns exhibits Auto Regressive Conditional Heteroscedacity (ARCH-LM test) making this class

of model an ideal medium in which to test our hypothesis.

### 4.1 GARCH Specification

As Jorion (2003, 363) explains, a GARCH model is a more sophisticated approach to estimating future volatility because it assumes that today's variance is dependent upon the latest 'innovation' in price and on the previous conditional (non-constant) variance. Given that a GARCH model is relatively simple to estimate and computationally straight forward for fixed weight portfolios the GARCH model is considered (Jorion 2000, 170) more precise compared to other models, principally in cases where there is volatility clustering (shown to exist in the actual return series).

Given that both securities display minimal skewness, this research chooses to present the basic GARCH incarnation as shown below.

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \qquad (1)$$

The term $\varepsilon_i = (z_i - \mu)$ where $z_i$ represents a random draw taken from the Normal distribution for the Normal GARCH and then from the Logistic distribution for the Logistic GARCH, such that:

$$\begin{aligned} \varepsilon_t^2 \,|\, I_{t-1} &\sim N(\mu, \sigma_t^2) \text{ and} \\ \varepsilon_t^2 \,|\, I_{t-1} &\sim LF(\alpha, \beta) \end{aligned} \qquad (2)$$

Where in (2), $\alpha = \mu$ and $\beta = \frac{\sqrt{3}\sigma}{\pi}$ represent the parameters of the Logistic distribution. Restrictions are placed on the parameter estimates to ensure that the conditional variance will always be positive.

$$\omega > 0, \; \alpha, \beta \geq 0, \; \alpha + \beta < 1 \qquad (3)$$

For the model, this paper sets the value of the parameters to near zero and constructs a time series for the conditional variance in (1) and calculates the likelihood of each observation. Summing these 500 values gives the log likelihood value which, in turn, is maximised using Palisade's RiskOptimizer as per the constraints in (3).

With the initial parameters estimated from the historical data, the mean and conditional variance for the first estimate of the series (simulation 0) are set to the long term mean and standard deviation derived from the historical data. Future estimates for conditional variance are then generated from (1) for simulations 1 to 500. The value-at-risk figure is simply the product of the conditional standard deviation and the level of significance chosen, in this case, at 90 per cent.

Figure 9 compares the week-ahead VaR estimate produced by each distribution for PIE and RECs respectively. Whilst at first glance there appears little difference between the GARCH estimates, the use of the Logistic distribution in the GARCH process appears better suited to capturing the stylised facts concerning volatility - mainly its reaction and persistence. It also better placed to account for the heavier tails present in both securities.

**Figure 9.** Comparison of GARCH estimates for PIE and RECs
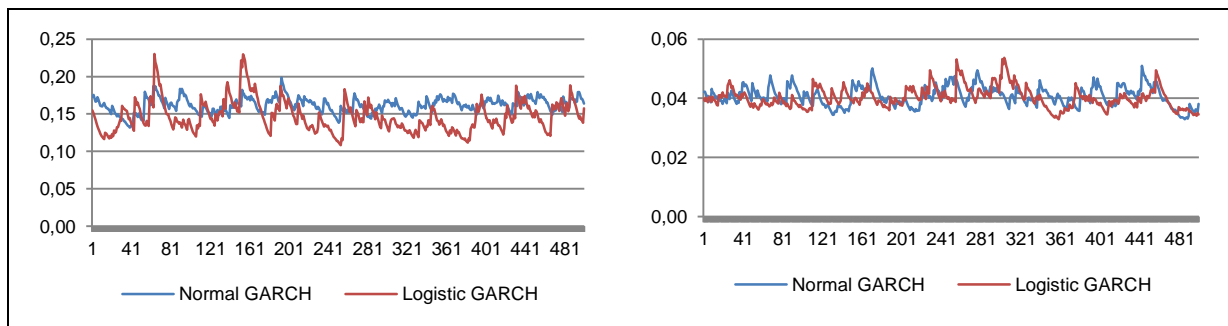


### 5. Model Validation

Back testing is the process of testing the accuracy of the VaR model using out-of-sample data. Failure of the back test indicates that the model may be mis-specified and that large estimation errors may exist (Alexander 2008b, 332).

With both models, if the next week's actual return exceeds the forecast VaR a violation is recorded. This process is repeated until the entire sample data is exhausted and the total number of VaR violations is recorded.

The *Conditional Coverage* test is employed to validate the models. The process of recording the VaR violations over 500 samples as representative of one trial which is repeated 100,000 times, using the Monte Carlo capabilities of @Risk, to arrive at a consistent number of VaR limit violations and test statistics for each model.

### 5.1 Coverage Tests

The conditional coverage test, introduced by Kupiec (1995) and extended by Christoffersen (1998), is a sophisticated and flexible backtesting

methodology. The test consists of an unconditional coverage test, based on the actual number of the violations of the previous day's VaR estimate in the out-of-sample test compared to the expected number of violations, and an independence test to see whether the VaR violations cluster.

The null hypothesis under an unconditional coverage test examines whether the observed violations follow an i.i.d. Bernouli process that are statistically similar to the significance level of the VaR, $\alpha$, that is, the expected number. The test is a likelihood ratio whose value of $-2\ln LR_{uc}$ is asymptotically distributed with one degree of freedom (Alexander 2008b, 337).

If the model passes the unconditional test, it could still be rejected because the VaR violations are not independent. This study follows Alexander (2008a, 359) by adopting expressions such as 'good' returns where a non-VaR violation was preceded by either a violation or a non-violation. A 'bad' return in contrast is where a VaR violation immediately follows the previous violation. Again, Christoffersen (1998) describes a test of the likelihood ratio whose value of $-2\ln LR_{ind}$ is chi-squared with one degree of freedom.

The conditional coverage test combines the unconditional and independent tests, in which the asymptotic distribution of $-2\ln LR_{cc}$ is chi-squared with two degrees of freedom. In order for these models to be valid, they should pass the conditional coverage test.

## 6. Results

The tests are implemented in Excel, and results are displayed in Table 5. At the 10% level of significance both distributions produce approximately the same number of violations over 100,000 runs and neither model fails the conditional coverage test. The 10% critical value of the chi-squared distribution with two degrees of freedom is 4.61 and both conditional coverage statistics are less than this.

In terms of violations both models are very similar: an average of 54 for the Normal GARCH and 60 for the Logistic GARCH models for PIE, and 50 and 57 respectively for RECs. For both securities, the standard deviation for the Normal distribution is larger than for the Logistic. Indeed for PIE, the Normal distribution had a standard deviation of 11 versus 8 and approximately 90 per cent of simulations fell between 44 and 73 whereas for the Logistic the range of violations was 45 to 52. Similarly for RECs, the standard deviation of the Normal distribution was 13 but only 7 for the Logistic distribution. The respective range of violations determined that approximately 90% of simulations fall between 38 and 63 violations for the Normal model and 47 and 70 for the Logistic model.

**Table 5.** Backtesting Results & Coverage Tests for PIE and RECs

| *After 100,000 Iterations* | Normal | Logistic | *After 1,000,000 Iterations* | Normal | Logistic |
|---|---|---|---|---|---|
| Number of observations | 500 | 500 | Nnumber of observations | 500 | 500 |
| Number of VaR violations | 60 | 53 | Number of VaR violations | 52 | 54 |
| Number of non-VaR violations | 440 | 447 | Number of non-VaR violations | 448 | 446 |
| Percentage of VaR violations | 12.0000% | 10.6000% | Percentage of VaR violations | 10.4000% | 10.8000% |
| Percentage of expected violations | 10.0000% | 10.0000% | Percentage of expected violations | 10.0000% | 10.0000% |
| Unconditional coverage test | -262.06223 | -233.8327 | Unconditional coverage test | -229.568196 | -237.1459986 |
| Independence test | 1.444688 | -1518.1045 | Independence test | 1.689488304 | -2181.018436 |
| Percentage of stand-alone violations | 12.5000% | 10.2908% | Percentage of stand-alone violations | 10.0446% | 11.4350% |
| Percentage of consecutive violations | 8.3333% | 13.2075% | Percentage of consecutive violations | 13.4615% | 5.5556% |
| Conditional coverage test | -260.61754 | -1751.9372 | Conditional coverage test | -227.878708 | -2418.164434 |

**Chi-squared critical values**

| | 1% | 5% | 10% |
|---|---|---|---|
| 1 degree of freedom | 6.63 | 3.84 | 2.71 |
| 2 degrees of freedom | 9.21 | 5.99 | 4.61 |

For PIE, the unconditional coverage test will reject below 38 and above 73 violations for the Normal distribution, and will reject below 41 and above 66 for the Logistic. For RECs, the rejection zones for the Normal and Logistic models are below 38 and above 75, and below 33 and above 63 respectively. Any values within these ranges are statistically insignificant from 50 – the expected number of violations at the 10% level. Regardless of the distribution employed, for both securities, the resulting VaR model will tend to generate more

violations than the expectation due to the persistent level of high volatility resulting from illiquidity.

In addition, over 90% of all trials resulted in independent violations i.e. they tended not to cluster meaning that a sudden spike in volatility will not produce a string of VaR violations. Figure 8 compares the violation profiles for the Normal and Logistic GARCH models for both securities over the course of 500 samples drawn from 100,000[th] iteration. The thin vertical line represents

a standalone violation whereas thicker lines denote two or more consecutive violations.

**Figure 10.** Week-ahead VaR Estimates and Violations
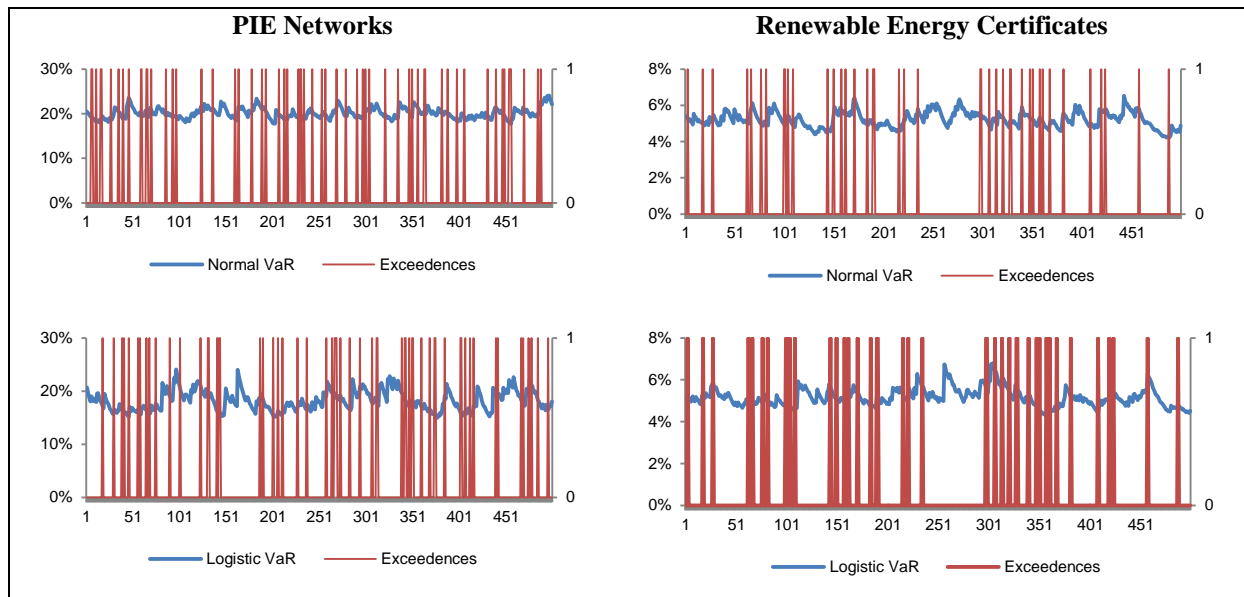


For PIE in this particular representation, the total number of consecutive violations for the Normal model was 13.11 per cent versus 15.00 per cent for the Logistic model, a difference of 1.89 per cent. For RECs, the Normal model had consecutive violations totalling 11.86 per cent of all observations against 9.76 per cent for the Logistic model, for a wider difference of 2.10 per cent which is a more significant difference for RECs as opposed to PIE. Overall employing the Normal distribution in the VaR the GARCH model does tend to result in a higher percentage of independence violations across a simulation composed of 100,000 iterations.

## 7. Conclusion

This research evaluates a more suitable probability distribution, or one better aligned to the underlying process rather than the standard assumption of normality, to model the price risk. This methodology is then applied to the illiquid securities PIE and RECS using a simple GARCH VaR framework. The hypothesis is evaluated by comparing the Normal and Logistic distributions (as chosen by @Risk as first and second best respectively) in forecasting the future volatility of each security. The resulting stochastic volatility forecast is used to determine the number and independence of VaR violations against what would be expected.

It is found that both distributions produce very similar results in terms of the number and independence of violations. What is significant is how and where the violations occur. Under the Normal distribution, the average number of

violations is closer to the expected amount; the average for the Logistic distribution is closer to upper boundary of expectation due to the more frequent occurrences of tail events than described by the historical data. The zone of acceptance, i.e. the upper and lower level of violations that will be accepted, is tighter when the Normal distribution is used. However, the Normal distribution tends to produce more clustered and dependent violations although not significant enough for the Normal GARCH model to fail the Conditional Coverage test. Whilst the results are similar, it is believed that implementing a Logistic GARCH model is more favourable from a theoretical perspective. This appears to better capture the extreme volatility dynamics present in illiquid securities.

The importance of constantly reviewing the underlying returns distribution cannot be understated. Securities markets are constantly evolving, reacting to new information and innovations which have the potential to deflect the path of a security in one of many competing directions. Asset price returns rarely, if ever, conform to neatly described and known PDFs. Therefore risk managers need to constantly review the historical data to pick up these subtle changes and alter the assumptions upon which their models are based. This can now comfortably be done through the use of easy-to-use software routines that integrate neatly into the Excel development environment. Such a modelling process described in this article would satisfy the Basel Advanced Measurement Approach. Finally the risk management function is carried out more accurately by not relying on common traditional assumptions applied by many corporates. Corporate owners and

managers secure greater confidence in the firms risk management function.

## References

1. Agrawal, G. 2009. 'Impact of Sample Size on the Distribution of Stock Returns: An Investigation of Nifty and Sensex', *IUP Journal of Applied Finance,* 15.1, pp. 5-13.
2. Alexander, C. 2008a. *Market Risk Analysis II: Practical Financial Econometrics*. London: Wiley.
3. Alexander, C. 2008b. *Market Risk Analysis IV: Value At Risk*. London: Wiley.
4. IDG Communications. 2011. *IT Sector might shrink further before it gets better.* http://www.arnet.com.au/article/386954/australia_it_sector_might_shrink_further_before_it_gets_better_report/ (accessed December 8, 2011).
5. Anderson, G. and Strate, B. (2009) Focus: Expanded renewable energy target scheme legislation passed. http://www.aar.com.au/pubs/ener/foeneraug09.htm (accessed December 3 2009).
6. Aragones, J. R., Blanco, C and K. Dowd. 2001. *Incorporating Stress Tests into Market Risk Modeling.* http://www.fea.com/resources/pdf/a_stresstest.pdf (accessed December 10, 2011).
7. ASX., http://www.asx.com.au/asx/research/company Info.do?by=asxCode&asxCode=PIE
8. Avgouleas, E. 2010. *Lessons from the Financial Crisis; Causes, Consequences, and Our Economic Future*, ed. R Kolb, 391-395. New York: J Wiley & Sons.
9. Bollerslev, T 1986, 'Generalized Autoregressive Conditional Heteroskedasticity', *Journal of Econometrica 31,* pp. 307 – 327.
10. Citibank. 2011. *Diversification – the strategy amid the market.* http://www.citibank.com.hk/english/investment/pdf/invpul.pdf (accessed December 8, 2011).
11. Engle, R. Focardi, S and F. Fabozzi. 2007. *ARCH/GARCH Models in Applied Financial Econometrics.* http://pages.stern.nyu.edu/~rengle/ARCHGARCH.pdf (accessed December 10, 2011).
12. Fusaro, P. 2008. *The Professional Risk Manager's Guide to the Energy Market.* New York: McGraw Hill.
13. Heiat, A. 2005. *Using an Excel Extension for Selecting the Probability Distribution of Empirical Data*. http://epublications.bond.edu.au/ejsie/vol2/iss1/5 (Accessed June 17, 2012).
14. Jorion, P. 2000. *Value-at-Risk: The benchmark for controlling market risk.* New York: McGrawHill.
15. Jorion, P. 2003. *Financial Risk Manager Handbook.* New York: Wiley.
16. Kousky, C and R. Cooke. 2010. *Adapting to Extreme Events: Managing Fat Tails*. http://www.rff.org/rff/documents/RFF-IB-10-12.pdf (accessed December 10, 2011).
17. Kupiec, P. 1995, 'Techniques for Verifying the Accuracy of Risk Management Models', *Journal of Derivatives,* 3, pp. 73 – 84.
18. Christoffersen, P. 1998, 'Evaluating interval forecasts', *International Economic Review* 39(4), pp. 817-840.
19. Intelligent Energy Systems. 2002. Modelling the Price of Renewable Energy Certificates. http://www.orer.gov.au/publications/modelling.html (accessed September 15, 2009)
20. Office of the Renewable Energy Regulator. http://www.orer.gov.au/About-the-Schemes/about-schemes (accessed December 9, 2011).
21. PIE Networks. *Becoming Australia's Premier Wi-Fi Company*. http://www.pienetworks.com.au/index.php/pienews/asx-announcements/cat_view/14-announcements (Accessed December 7, 2011).
22. Palisade Corporation, 2008. *Guide to Using @Risk: Risk Analysis and Simulation Add-In for Microsoft Excel.* Ithaca: Palisade.
23. Rachev, S.T. 2003. *Handbook of Heavy Tailed Distributions in Finance,* ed. Elsevier Science, ix. Amsterdam: Elsevier Science B.V.
24. Renewable Energy Scheme 2009 (Cth)
25. Super Review. 2011. *Asset Allocation: Smoothing out the bumps.* http://www.superreview.com.au/atricle/asset-allocation-smoothing-out-the-bumps/530261.aspx (accessed December 8, 2011).
26. Tan, K and Chu, M, 2012. 'Estimation of Portfolio Return and Value at Risk Using a Class of Gaussian Mixture Distributions', *The International Journal of Business and Finance Research, 6.1,* pp.97-107.
27. Trading Economics. 2011. *Market Capitalisation of Listed Companies.* http://www.tradingeconomics.com/australia/marketcapitalisation.html (accessed December 12, 201).