SECURITIZATION, LOAN MODIFICATION AND THE SUPPLY OF SUBPRIME MORTGAGE CREDIT IN THE US

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Abstract

This paper develops a continuous time, contingent claims model of mortgage valuation with strategic behavior to show that mortgages that are securitized are characterized by significantly higher loan to value ratios than mortgages held on the balance sheet of the originator, if securitized mortgages cannot be renegotiated. Insofar as securitization inhibits loan modification, it serves as a credible threat to the borrower that default will provoke foreclosure. This enhances the value of the lender's claim on the loan collateral, the home, and she is willing to lend more per dollar of collateral value. An important implication of the analysis is that the higher loan to value ratio for the securitized mortgage does not imply that the securitized mortgage is characterized by looser underwriting standards than the mortgage held on balance sheet. Higher loan to value ratios for securitized mortgages do not necessarily constitute evidence that securitization encourages risky lending.

Keywords: Securitization, Loan Modification, Supply of Subprime Mortgage Credit, US

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1. Introduction

"...even though workouts may often be the best economic alternative, mortgage securitization and the constraints faced by servicers may make such workouts less likely." (Bernanke, 2008a).

"... subprime servicers claimed they were unable to help borrowers because trust agreements under Securities and Exchange Commission rules did not allow them to alter the terms of the loans ..." (Cutts and Merrill, 2008).

This paper contemplates the impact on the lending behavior of subprime originators of the perceived and possibly real limitation that securitization imposes on loan renegotiation. Would an originator offer a borrower a loan on the same terms regardless of whether the loan is going to be securitized or not? Does the prospect of possible loan renegotiation, versus foreclosure, in future `bad' states of the world make a difference?

The result obtained here is that securitization does make a difference to the lending behavior of originators, if the originators (and investors) believe, at the time of origination, that securitization would inhibit loan modifications in the future. In particular, securtized mortgages will have higher loan to collateral value (LTV) ratios than portfolio mortgages. This may make securitized mortgages appear riskier than portfolio mortgages, when in fact they are not. The result is obtained in the *absence* of informational asymmetries, often invoked to explain risky securitized subprime lending. In the model developed here, originators, borrowers and investors have complete information pertaining to the credit worthiness of the borrower and the characteristics of the home collateralizing the mortgage.

We proceed as follows. Section 2 establishes some stylized facts pertaining to subprime mortgages and securitization. Section 3 describes the simplified mortgage contract to be employed in the model. Section 4 describes the nature of the default risk to be studied. Sections 5 and 6 describe the strategic interaction between borrower and lender in the cases of securitized mortgages and portfolio mortgages. Section 7 reports results (loan-to-value ratios) and reviews sensitivity analysis for key contractual parameters. Section 8 concludes.

2. The lending environment

We describe some stylized facts pertaining to the subprime mortgage market that feature prominently in the model developed here. Some of these `facts' are contested in the literature. As we proceed, we point to sources for useful accounts of the various debates.

2.1 Subprime mortgages were intended to be short-term loans

Subprime mortgages are best viewed as `bridge' (temporary) loans to borrowers with impaired credit histories. Most subprime mortgages issued through the end of 2007 were hybrid adjustable rate loans. Rates were fixed for an initial period (usually 2 or 3 years) and then became indexed to a floating rate (typically 6-month Libor plus a credit spread) for the remaining 28 or 27 years of the 30-year term. The structure of these loans was intended to induce early loan termination (prepayment) at the end of the fixed rate period.[1] The objective was to limit the lender's exposure to a term much shorter than 30 years.[2] The contracts were indeed characterized by high rates of early termination.[3]

2.2 Subprime mortgages are effectively non-recourse loans

If the sale of the foreclosed property does not cover the outstanding loan balance, accrued interest and expenses incurred, the lender may be entitled to seek a `deficiency judgment' against other assets of the borrower. While no state forbids deficiency judgments in all cases, some states forbid deficiency judgments for residential properties (e.g. Arizona, North Dakota, Oregon), purchase mortgages (e.g. California, North Carolina) or properties abandoned for a period of time (e.g. Washington).[4] Furthermore deficiency judgments are usually only available to lenders following a judicial foreclosure action which is much more time consuming (and costly) than a power-of-sale (nonjudicial) action. The costs associated with pursuing deficiency judgments are generally prohibitive.

Ghent and Kudlyak (2011) provide a counterpoint to this position. They claim that recourse in the form of deficiency judgements is available in some states (albeit subject to restrictions) and that the threat of recourse mitigates the strategic default contemplated in this paper. Hatchondo et al (2013) offer a rebuttal arguing that the role of recourse is limited as households can discharge the obligation associated with a deficiency judgement in bankruptcy.

2.3 Mortgage foreclosures are costly

“Foreclosures are extremely costly. ... transaction costs ... typically run at one-third or more of a home's value ...” (Summers, 2008).

The magnitude of mortgage foreclosure costs borne by mortgage lenders and holders of mortgage backed securities are largely a function of the real estate foreclosure process. While foreclosure law varies from state to state, the foreclosure process is typically lengthy. Cutts and Merrill (2008) estimate the average time elapsed from the date of the last paid mortgage installment to the foreclosure sale to be 355 days.[5] During this period, the servicer (and ultimately the investor) is responsible for lost interest and principal payments, property taxes, costs associated with the preservation and maintenance of the property and hazard insurance. The foreclosure sale itself incurs legal and marketing costs (commissions to agents, etc.).

2.4 Securitized subprime mortgages are not easily modified

The rate at which securitized subprime mortgages have been modified to prevent foreclosure, has been surprisingly low given the unprecedented increase in delinquencies and the attempts by regulators to encourage loan modification. A brief review of reasons offered to explain the muted response of mortgage servicers follows.

2.4.1 Contractual constraints

The powers and obligations of the servicer to a securitized mortgage pool are defined in the pooling and servicing agreement (PSA). PSAs generally afford the servicer limited discretion to modify loans.[6]

2.4.2 Servicer incentives

Loan renegotiation is time consuming and expensive for servicers. Servicer fee arrangements currently in effect, did not anticipate the dramatic increase in delinquencies stemming from the subprime crisis. Consequently servicers have an incentive to minimize any additional cost associated with modifying loans. Foreclosure is often a lower cost option from the servicer's narrow perspective (Eggert, 2007). The problem is compounded when servicers have no economic exposure to the performance of the securitized mortgage pool.

2.4.3 Tax laws and accounting standards

Tax and accounting laws governing the Special Purpose Entity (SPE) issuing the securities collateralized by the mortgage portfolio may also inhibit modification.[7] For example, if the SPE is structured as a Real Estate Mortgage Investment Conduit (REMIC) for tax purposes, the mortgage pool must remain static. With few exceptions, loans cannot be removed, replaced or modified. In addition, FAS 140 states that for an SPE to qualify for securitization accounting, the assets transferred must be ``passive in nature,'' precluding any action that is not ``inherent in servicing'' such as monitoring delinquencies and executing foreclosures. The statement is silent with respect to loan modification. Servicers are consequently reluctant to modify loans.

2.4.4 Investor conflicts

Servicers are required to act in the best interests of the investors. Holders of different classes of securities (tranches) may have conflicting interests pertaining to modifying delinquent loans such that they are reclassified to `current,' leaving the servicer hesitant to modify.

The extent to which these impediments reduce loan modifications that prevent foreclosure is contested. The latest word in this debate appears to be Kruger (2013). He provides compelling evidence to support the contention that securitization does indeed inhibit loan modification.[8]

What matters for the model developed here is that, at the time of origination and securitization, originators and investors believe that it will be difficult to modify securitized mortgages to prevent foreclosure at the time of default.

3. The subprime mortgage contract

We model the subprime mortgage contract as a short-term (2 or 3 years), non-recourse, fixed rate loan. The borrower has the option to default and lender has the option to foreclose, if, and only if, the borrower has defaulted. The lender advances funds in exchange for the borrower's promise to make a continuous flow of payments over some interval of time, $[0,T]$. The contract may call for a lump-sum payment at T. For an initial sum of 1 and a payment flow at a rate of p per year over $[0,T]$, and a lump-sum payment, P, at T, the contractual loan rate, c, satisfies:

$$1 = p\int_0^t e^{-c\tau}\mathrm{d}\tau + b(t)e^{-ct} \tag{1}$$

The outstanding loan balance at $t \in [0,T]$ is:

$$b(t) = e^{ct} - (e^{ct} - 1)p/c \tag{2}$$

The lump-sum payment is:

$$P = b(T) \tag{3}$$

If, at any time prior to maturity, the borrower offers a payment flow smaller than p, or the borrower offers a lump-sum payment smaller than P at maturity, the lender has the right, but not the obligation, to foreclose. If she chooses to foreclose, the lender seizes and sells the home, incurring foreclosure costs $l(h,t)$. Any net proceeds from the sale, $h(t) - l(h,t)$, that exceed the outstanding loan balance $b(t)$, are refunded to the borrower.

4. Default risk

Mortgage defaults occur when either the borrower experiences some sort of credit event (unrelated to the value of the home) and is unable to service the mortgage, or the value of the home declines such that it is rational for the borrower to exercise his default option.[9] We ignore borrower credit risk. The borrower is always able to service his mortgage, if it is rational to do so. The only source of default risk emanates from the stochastic nature of the market value of the home. We refer to this as `rational default.'[10] We assume that the value of the home, $h(t)$, follows a continuous Markov process over time:

$$\mathrm{d}h(t) = \alpha(h,t)\mathrm{d}t + \sigma h(t)\mathrm{d}z(t) \tag{4}$$

where $z(t)$ is a standard Brownian motion, σ is a constant volatility parameter and $\alpha(h,t)$ is the instantaneous expected drift in h. The home generates a continuous flow of housing services accruing at rate $d(h,t)$. This flow accrues to the borrower provided that foreclosure has not occurred.

We assume that $h(t)$ is costlessly and continuously observed by both borrower and lender and that both parties have access to a market in which they can construct a transaction cost free hedge against h-risk. Such a market is said to be dynamically complete with respect to h-risk. Hence the borrower and lender value their claims under the same martingale measure, Q. Under Q, the process for $h(t)$ is

$$\mathrm{d}h(t) = (rh(t) - d(h,t))\mathrm{d}t + \sigma h(t)\mathrm{d}\tilde{z}(t) \tag{5}$$

The instantaneous risk free rate, r is assumed to be constant and $\tilde{z}(t)$ is a standard Brownian motion under Q.

5. Behavior of the contracting parties

We model the behavior of the borrower and lender as a noncooperative game. Each party chooses strategies to maximize the value of his/her claim.[11] We restrict our attention to games where the borrower is always in effective control of the timing of loan termination.[12] At every point in time, the borrower exercises choice over the instantaneous debt service flow offered to the lender, p^*.[13] The borrower makes this offer with full knowledge of the lender's rational response. The borrower knows, at every point in time, the (minimum) debt service flow, $\hat{p} \le p$, that prevents foreclosure. At this debt service level, the lender is indifferent between foreclosure and allowing the loan to continue. Foreclosure (only) occurs when the borrower chooses to offer $p^* < \hat{p}$.[14] The borrower can also terminate the loan by prepaying the outstanding balance at any point prior to maturity.[15]

A debt service offer of $\hat{p} < p^* < p$, constitutes default, but will not provoke foreclosure. We refer to this as strategic default or strategic debt service. We assume that when the lender accepts such a debt service offer, she surrenders any claim on the unpaid interest and principal. In other words the outstanding balance, $b(t)$, is adjusted as if the full contractual payment has been made. The contract is effectively modified (renegotiated), in favor of the borrower.

The borrower's optimization problem can be represented by the Bellman equation

$$rB(h,t) = \max_p\{d(h,t) - p + \frac{1}{\mathrm{d}t}\,\mathrm{E}^Q[\mathrm{d}B(h,t)]\} \tag{6}$$

E^Q is the expectation operator under the equivalent martingale measure. Under Q, the risk free rate, r, is the appropriate discount rate. The optimal choice of the debt service flow is the function $p^*(h,t)$.

The lender maximizes the value of her claim, given the borrower's offer and the indentures of the loan contract. The lender's optimization (stopping) problem is[16]

$$L(h,t) = \max\{\Omega_L(h,t), \frac{1}{r}[p^*(h,t) + \frac{1}{\mathrm{d}t}\,\mathrm{E}^Q[\mathrm{d}L(h,t)]]\} \tag{7}$$

The current values of h and t embody all relevant information upon which the current actions of the lender and borrower are based. Past actions do not influence current payoffs. At any point in time the state of the game is determined by the current realization of $h(t)$ and the current actions of the players.[17]

For a finite term loan, the state space $\mathrm{H} \times \mathrm{T}$, where $\mathrm{H} \equiv [0,\infty)$ denotes the range of values for h, and $\mathrm{T} \equiv [0,T]$ denotes the range of values for t, contains all possible states for the players' strategies.[18] A strategy constitutes the specification of a number of regions or closed subsets in $\mathrm{H} \times \mathrm{T}$ in which specific actions are taken by the player and, for the borrower, the function $p^*(h,t)$.[19] For example, the borrower defaults whenever $(h,t) \in \mathrm{D}$, where D is a closed subset of $\mathrm{H} \times \mathrm{T}$. His prepayment policy, P, is another closed subset of $\mathrm{H} \times \mathrm{T}$. The lender forecloses whenever $(h,t) \in \mathrm{F}$, where $\mathrm{F} \subset \mathrm{H} \times \mathrm{T}$. Similarly, any other actions may be represented by closed subsets of $\mathrm{H} \times \mathrm{T}$.

The loan contract is terminated whenever foreclosure or prepayment occurs, or when the maturity date is reached. The boundaries of F and P are referred to as the termination boundaries of the game, and the regions themselves are the termination regions. The open subset of $\mathrm{H} \times \mathrm{T}$, in which the loan contract is not terminated or the contractual payment flow, p is not changed, is referred to as the continuation region, C.

6. Value to borrower and lender

Let $\Omega_L(h,t)$ and $\Omega_B(h,t)$ represent the termination values of the lender's claim and borrower's claim, respectively. The continuation value of the lender's claim, $L(h,t)$, is simply the value to the lender of the remaining cash flows from the loan if the collateral value at time t is h and the loan has not been terminated at an earlier date. $L(h,0)$ represents the maximum amount of credit that the lender would extend to the borrower in exchange for the promised sequence of contractual payments. Similarly, $B(h,t)$ represents the continuation value of the borrower's position, taking into account his options under the contract, assuming the contract has not yet been terminated.

The arbitrage or replication arguments of contingent claims pricing imply that over a small time interval, Δt, $L(h,t)$ and $B(h,t)$ satisfy (approximately) the following equations in C

$$\begin{aligned} L(h,t) &= p^*(h,t)\Delta t + E_t^Q[L(h+\Delta h, t+\Delta t)]e^{-r\Delta t} \\ &= p^*(h,t)\Delta t + L^+(h,t) \end{aligned} \tag{8}$$

$$\begin{aligned} B(h,t) &= [d(h,t) - p^*(h,t)]\Delta t + E_t^Q[B(h+\Delta h, t+\Delta t)]e^{-r\Delta t} \\ &= [d(h,t) - p^*(h,t)]\Delta t + B^+(h,t) \end{aligned} \tag{9}$$

$L^+(h,t)$ and $B^+(h,t)$ are respectively the `ex debt service' and `ex dividend' values of the claims. Allowing $\Delta t \to 0$

$$\frac{1}{2}\sigma^2 h^2 L_{hh} + [rh - d]L_h + L_t + p^* = rL \tag{10}$$

$$\frac{1}{2}\sigma^2 h^2 B_{hh} + [rh - d]B_h + B_t + d - p^* = rB \tag{11}$$

Subscripts denote partial derivatives and arguments of the functions are suppressed.

Boundary conditions at loan maturity and the free-boundary conditions that hold on the termination boundaries of $\mathrm{H} \times \mathrm{T}$, allow us to solve for the players' optimization problems. The boundary conditions will be determined by the specific indentures of the contract, and the restrictions imposed on the strategy space of the borrower and lender.

The free boundary conditions that characterize optimal policies, and determine the sets D, F and P are termed `value matching' and `high contact' or `smooth pasting' conditions (Dixit, 1993). The value matching condition requires that the continuation value and the termination value of a particular claim be equal on the boundaries of the termination regions. The smooth pasting condition requires that the first derivative in the h direction of the value function of the option exerciser be continuous on the boundary of these sets. For example, suppose that the borrower is in control of termination of the game along a particular boundary, $\underline{h}(t)$. The value matching condition implies $B(\underline{h},t) = \Omega_B(\underline{h},t)$, and the smooth pasting implies $B_h(\underline{h},t) = \partial\,\Omega_B(\underline{h},t)/\,\partial h$. This calculation assumes that the strategies followed by the players are fixed. Consequently, it determines a subgame perfect Nash equilibrium in the Markov strategies which is characteristic of a Markov perfect equilibrium.

Equations (10) and (11) constitute the continuous time representation of the solutions to the claim values, $L(h,t)$ and $B(h,t)$, for a general class of one state variable games. Problems with linked partial differential equations subject to free boundary conditions can be solved analytically only for some special cases. In the context of the problem presented here, if the loan is perpetual, the drift and volatility parameters for the market value of the home are time invariant, and there is a single free boundary condition, then analytic solutions for $L(h,t)$, $B(h,t)$ and the optimal boundary value of h are obtainable. Analytical solutions are also obtainable in the case of finite term loans if the options available to borrower and lender could only be, or would only be, rationally exercised at the maturity date. An example of such a case would be a zero coupon loan with no prepayment option.

For the problems we wish to consider here we are compelled to resort to numerical techniques for solutions. We consider two variations of the game, portfolio mortgages and securitized mortgages. In the case of portfolio mortgages we allow for the borrower to engage in strategic default. In the case of securtized mortgages, we rule out strategic default. We assume that the lender always forecloses if the borrower defaults. In each case we solve the same partial differential equations. All that changes from one case to the other will be the specification of $p^*(h,t)$ and the boundary conditions.

6.1 Securitized Mortgages

We assume that the servicer is contractually obligated to foreclose whenever the borrower offers $p^* < p$ for any $t < T$, or if he offers $P^* < P$ at T. There is no scope here for the borrower to explore the possibility of offering the lender payments that fall short of the contractual amounts, but do not induce foreclosure. Default is always characterized by the borrower offering the lender a debt service flow of zero. The debt service flow equals the contractual payment flow in the continuation region.

$$p^*(h,t) = \begin{cases} 0 & \text{for } (h,t) \in \mathrm{D} \\ p & \text{for } (h,t) \notin \mathrm{D} \end{cases}$$

Since his control variable, $p^*(h,t)$, is binary, the borrower's control problem is reduced to an `optimal stopping' problem. At every point in time he can either terminate the game (default or prepay) or continue (make the contractual debt service payment). Under these circumstances, $\mathrm{D} \equiv \mathrm{F}$. The lender's actions are completely determined by the borrower's behavior for every $(h,t) \in \mathrm{H} \times \mathrm{T}$.

6.1.1 Strategies at maturity

At maturity, the borrower offers a lump sum payment, P^*, to offset the outstanding balance, $P = b(T)$. Since an offer of $P^* < P$ forces foreclosure, his rational offer is

$$P^* = \begin{cases} 0 & \text{for } h(T) \le P \\ P & \text{for } h(T) > P \end{cases}$$

At maturity there is a single, termination (default and foreclosure) region, $(h,T) \in \mathrm{F}$ if $h(T) \le P$. The values of the claims at T are

$$L(h,T) = \begin{cases} \max\{0, h(T) - l(h,T)\} & \text{if } h(T) \le P \\ P & \text{if } h(T) > P \end{cases} \quad (12)$$
$$B(h,T) = \max\{0, h(T) - P\}$$

6.1.2 Strategies prior to maturity

There is a lower termination (default and foreclosure) region that extends back from T to the loan origination date, $t = 0$. Default occurs in this region when the continuation value of the borrower's claim is driven to zero. Since default forces foreclosure, the termination values of the claims in this region are

$$\begin{aligned} \Omega_L(h,t) &= \max\{0, h(t) - l(h,t)\} \\ \Omega_B(h,t) &= 0 \end{aligned} \quad (13)$$

The value of the lender's claim is never less than zero since she can abandon the collateral if the foreclosure costs exceed its market value. Due to the non-recourse nature of the loan, the borrower's claim is also never less than zero.

The boundary of this region, $\underline{h}(t)$, is the lower termination boundary for the game. In discretized form, the value matching condition for the borrower's problem, $B(\underline{h},t) = \Omega_B(\underline{h},t)$, implies

$$\left[d(\underline{h},t) - p\right]\Delta t + B^+(\underline{h},t) = 0 \quad (14)$$

$B(h,t)$ is strictly positive whenever the flow of housing services from the home exceeds the contractual debt service flow, $d(h,t) > p$. Thus any (h,t), such that $d(h,t) > p$, is not an element of the lower default region.

It is rational for the borrower to continue servicing the debt when the service flow from the home falls short of the contractual payment flow, $d(h,t) < p$, if the ex dividend value of his claim is sufficiently large, $B^+(h,t) > [p - d(h,t)]\Delta t$.

Prior to maturity, $\underline{h}(t) \le b(t)$, since there is some finite probability that the value of the home will recover sufficiently such that it exceeds P. Thus, $B(h,t) > 0$ for $\underline{h}(t) < h(t) < b(t)$, even though the borrower would receive nothing in the event of default and foreclosure. The borrower's default decision problem is similar to the stopping problem faced by the holder of an American option. In the interval, $\underline{h}(t) < h(t) < b(t)$, the `intrinsic value' of the borrower's claim is zero, but the `time value' is positive.

From the lender's perspective, default does not occur `soon enough' along the lower termination boundary. The lender would always prefer the borrower to default at the last moment the loan could be fully repaid by the liquidated home, $h(t) = b(t) + l(\underline{h},t)$. The borrower's rational behavior of timing default so as to maximize the value of his claim, is detrimental to the value of the lender's claim.

A second termination (default and foreclosure, or prepayment) region exists for sufficiently `high' values of the home if the credit spread, $c - r$, is sufficiently large. This upper region has a lower bound, $\overline{h}(t)$, the upper termination boundary for the game. If foreclosure costs or refinancing (prepayment) costs are positive, this region does not extend to the maturity date.[20] As the home value rises, the probability of default diminishes. The credit spread, originally set when the home value was lower, now seems unwarranted. Faced with the prospect of making the high contractual payments for the remaining term of the loan contract, the borrower will choose to default or prepay the loan if his proceeds from doing so exceed the continuation value of his claim.[21] If the cost incurred by the borrower in negotiating a new loan to refinance the outstanding balance, $f[b(t)]$, is less than the foreclosure costs, $l(h,t)$, the borrower will prepay the loan instead of defaulting.[22] The termination values of the claims in this region are

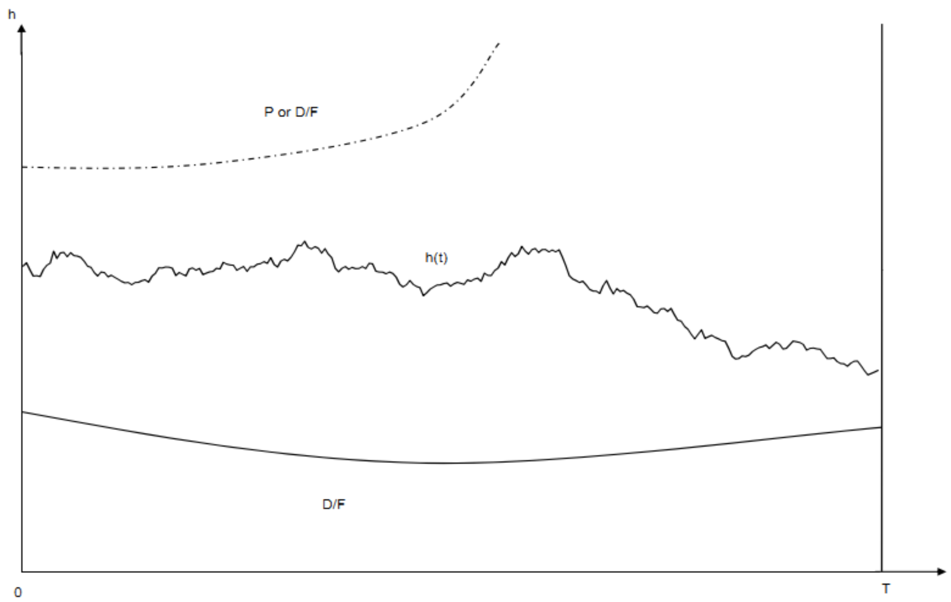
$$\begin{aligned} \Omega_L(h,t) &= b(t) \\ \Omega_B(h,t) &= h(t) - \min\{l(h,t), f[b(t)]\} - b(t) \end{aligned} \quad (15)$$

On the boundary of the upper termination region, the value matching condition, $B(\overline{h},t) = \Omega_B(\overline{h},t)$, implies that the foreclosure- or refinancing costs incurred by the borrower in default are exactly equal to the present value of the extra cost associated with servicing the loan over the remaining term to maturity at the contractual rate c which is now greater than the fair market rate for the lower default risk.[23] In discretized form, the value matching condition may be expressed as

$$[d(\overline{h},t) - p]\Delta t + B^{+}(\overline{h},t) = \overline{h}(t) - \min\{l(\overline{h},t), f[b(t)]\} - b(t) \quad (16)$$

The borrower's optimal strategy is characterized by a termination set which consists of two disjoint regions or subsets in $\mathrm{H} \times \mathrm{T}$. The continuation region C is then defined by $\underline{h}(t) < h(t) < \overline{h}(t)$. Figure 1 illustrates the typical termination regions in the state space.[24]

Figure 1. Strategy space: Securitized mortgages



6.2 Portfolio mortgages

Since the mortgage is held on the balance sheet of the lender, there are no contractual constraints associated with securitization, compelling the lender to foreclose when the borrower defaults. The lender will not foreclose if doing so does not increase the value of her claim.

We describe the strategy space of the game, starting at the maturity date of the loan.

6.2.1 Strategies at maturity

The borrower offers a lump-sum payment in lieu of the outstanding balance, $P = b(T)$. In the presence of positive foreclosure costs, $l(h,T)$, the borrower offers the smallest payment, P^*, that does not provoke foreclosure

$$P^* = \min\{P, \max\{0, h(T) - l(h,T)\}\} \quad (17)$$

This implies a single loan modification (strategic debt service) region at maturity, $(s,T) \in \mathrm{M}$ if $h(T) \leq b(T) + l(h,T)$.[25]

Consequently the values of the claims at maturity are

$$\begin{aligned} L(h,T) &= P^* \\ &= \min\{b(T), \max\{0, h(T) - l(h,T)\}\} \end{aligned} \quad (18)$$

$$B(h,T) = h(T) - P^* = \max\{\min\{l(h,T), h(T)\}, h(T) - b(T)\} \quad (19)$$

For $b(T) > h(T) - l(h,T) > 0$, the borrower avoids foreclosure by offering the lender an amount equal to what she would receive if she liquidated the collateral, $h(T) - l(h,T)$. This allows the borrower to retain $l(h,T)$, the amount that would be dissipated if foreclosure occurred. If $h(T) - l(h,T) \leq 0$, the borrower retains $h(T)$, while the lender receives nothing. It is never rational for the borrower to provoke foreclosure at maturity.

6.2.2 Strategies prior to maturity

The debt service flow, $\hat{p}$, that leaves the lender indifferent between foreclosing and allowing the loan to continue satisfies (in discretized form)

$$\hat{p}(h,t)\,\Delta t = \max\{0, \Omega_L(h,t) - L^+(h,t)\} \quad (20)$$

where $\Omega_L(h,t)$ represents the value of the lender's claim if she forecloses

$$\Omega_L = \min\{b(t), \max\{0, h(t) - l(h,t)\}\} \quad (21)$$

To determine $\hat{p}(h,t)$, the borrower must take into account the value to the lender of the subgames along which the contract is not terminated.[26] Since the lender cannot default if the borrower offers the contractual payment flow, p, and the continuation value of the borrower's claim is strictly decreasing in p^*, his debt service offer will never exceed p for $t < T$.[27] This and the fact that the lender will accept any debt service flow greater than or equal to $\hat{p}(h,t)$, yields the borrower's optimal debt service flow offer for every $(h,t) \in \mathrm{H} \times \mathrm{T}$

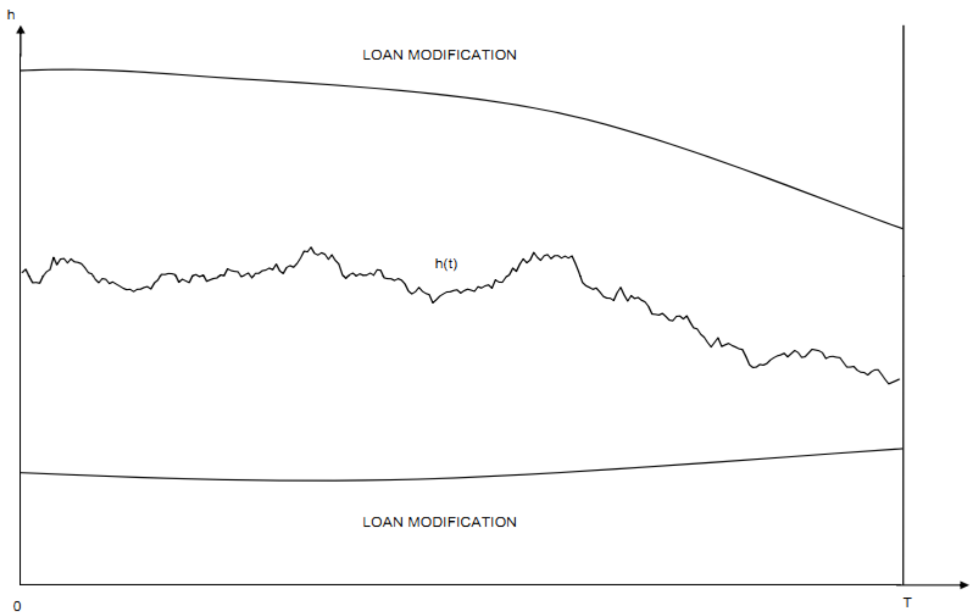
$$p^*(h,t) = \min\{p, \hat{p}(h,t)\} \quad (22)$$

Due to the finite term of the loan, $\hat{p}(h,t)$ is a function of the home value *and* time remaining to maturity.

There are two disjoint loan modification (strategic default) sets in the state space for $t < T$. For sufficiently low home values, we find $p^*(h,t) < p$ due to the low value of the lender's claim in foreclosure.[28] The presence of foreclosure costs would further reduce the value of the lender's claim. This adds to the borrower's ability to `extract' value from the lender.[29] As the term to maturity diminishes, the upper boundary of the lower default set increases. With less time remaining to maturity the probability that the collateral value will recover diminishes, lowering the ex debt service value of the lender's claim. Smaller debt service flows are required to keep the lender from foreclosing.

For sufficiently high home values we again find $p^*(h,t) < p$.[30] The lender is willing to accept debt service flows below the contractual flow since the probability of default diminishes as $h(t)$ increases. This region extends to T. As the term to maturity declines, the lower boundary of the upper modification region, $\overline{h}(t)$, declines as well. With less time remaining to maturity, the risk of the home value deteriorating before maturity, becomes smaller and the lender is willing to accept progressively smaller debt service flows without foreclosing. Figure 2 illustrates the typical loan modification regions in the state space.[31]

Figure 2. Strategy space: Portfolio mortgages



Since foreclosure imposes a `dead-weight' loss on the borrower, he never induces foreclosure along the equilibrium path of the game.

7. Loan to value ratios

Equilibrium values for $L(s,0)$ and $B(s,0)$ are computed using a finite difference procedure.[32] We assume that foreclosure costs and housing service flows are linear functions of the home value

$$l(h,t) = l_0 + l_1 h(t) \tag{23}$$

$$d(h,t) = d_1 h(t) \tag{24}$$

and that prepayment/refinancing costs are a linear function of the outstanding loan balance

$$f(b,t) = f_0 + f_1 b(t) \tag{25}$$

The following base case parameter values are used in the analysis presented here

Home:	r	d_1	σ	l_0	l_1	
	0.05	0.05	0.20	0.10	0.15	
Loan:	T	c	p	P	f_0	f_1
	2.00	0.08	0.09	0.98	0.00	0.05

The fair value of the securitized mortgage at origination, for a particular value of the home, $L_s(h,0)$ is shown in the first column of table 2 (Appendix A).[33] Table 3 illustrates the borrower's strategy sets.[34] The fair value for the portfolio mortgage, $L_p(h,0)$, is displayed in table 4 (Appendix B). The borrower's strategy sets are illustrated in table 5.[35]

The smallest initial value of the home, $h_s^*(0)$ for which $L_s(h,0) = 1$, is the minimum home value required for the securitized mortgage loan to be made (at par). At this home value, the credit spread, $c - r$ is the fair (`correct') spread. For the base case, $h_s^*(0) = 1.35$. The LTV for the loan is $1/h_s^*(0) = 0.74$. For the portfolio mortgage, $h_p^*(0) = 1.72$, and the LTV is $1/h_p^*(0) = 0.58$. The difference is material.

For home values less than $h_s^*(0)$ we have $L(h,0) < 1$. The lender is not willing to lend (`buy' the loan) for the contractual amount of 1. For example, when $h(0) = 1$, the lender is only willing to pay 0.816 for the contractual flow of debt service payments. This discount to par implies that the effective credit spread is greater than $c - r$.

Table 1 shows fair loan to value ratios for securitized and portfolio mortgages for different levels of l_1. Naive comparison of LTVs would suggest that the securitized mortgage is riskier than the portfolio mortgage, but both loans are issued (at par) at the *same* credit spread.

Table 6 (Appendix C) shows the effects of varying some loan contract terms and home value process parameters on the LTVs. Note that for some parameter values (e.g. $c = 0.09$), there is no LTV for the securitized mortgage. At this credit spread, $c - r = 0.04$, the borrower will prepay the loan (at origination) at home values high enough to induce the lender to lend the contractual amount of 1. The loan will never be made. The credit spread is too large. Similar reasoning applies in the cases where the refinancing costs, f_1, are (too) low.

Table 1. Fair loan to value ratios

l_1	$h_s^*(0)$	LTV_s	$h_p^*(0)$	LTV_p
0.00	1.25	0.80	1.46	0.68
0.05	1.29	0.77	1.54	0.65
0.10	1.32	0.76	1.62	0.62
0.15	1.35	0.74	1.72	0.58
0.20	1.38	0.73	1.83	0.55
0.25	1.40	0.71	1.95	0.51
0.30	1.42	0.70	2.09	0.48
0.35	1.45	0.69	2.25	0.44
0.40	1.47	0.68	2.43	0.41
0.45	1.49	0.67	2.65	0.38

8. Conclusion

There is broad consensus that moral hazard and informational asymmetries between originators and investors in securitized subprime mortgages were instrumental in the decline in lending standards in the years preceding the subprime crisis[36].

The implication is that originators who securitized their loans exploited the information asymmetry by extending credit to high risk borrowers who would not have received loans from lenders who kept the mortgages on their balance sheets, or they issued loans with higher loan to collateral value ratios and/or lower credit spreads when compared to portfolio loans with similar terms. In short, securitized mortgages were mispriced. Calls for the (increased) regulation of the securitization process to align incentives and mitigate moral hazard seem well founded.

This paper demonstrates that a naive comparison of LTV's across mortgages to provide evidence for this view is of no value. The environment in which borrower and lender negotiate the terms of a loan at origination (and subsequently, if terms are renegotiated) matters for determining the fair value of the claims of the borrower and lender on the underlying collateral (the home). The value of the lender's claim is the fair amount of credit that should be offered. Securitization implies a contracting environment quite different from the environment where the lender holds the loan. Hence there is no reason to expect securitized loans and portfolio loans to have similar LTVs for a given credit spread, or similar credit spreads for a given LTV, ceteris paribus, even in the *absence* of asymmetric information. In the model we develop, securitized loans have higher LTVs than portfolio loans, for a given credit spread. This does not constitute evidence of aggressive lending on the part of securitizing originators.

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Appendix A. Securitized Mortgage

Table 2. $L_s(h, 0)$

2.50	1.000	0.998	1.041	1.034	1.026	1.018	1.011	1.003	0.995	0.986	0.978
2.40	1.000	0.998	1.041	1.034	1.026	1.018	1.011	1.003	0.995	0.986	0.978
2.30	1.000	0.998	1.041	1.034	1.026	1.018	1.011	1.003	0.995	0.986	0.978
2.20	1.000	0.998	1.041	1.034	1.026	1.018	1.011	1.003	0.995	0.986	0.978
2.10	1.000	0.998	1.041	1.034	1.026	1.018	1.011	1.003	0.995	0.986	0.978
2.00	1.000	0.998	1.041	1.033	1.026	1.018	1.011	1.003	0.995	0.986	0.978
1.90	1.000	0.998	1.040	1.033	1.026	1.018	1.011	1.003	0.995	0.986	0.978
1.80	1.000	1.043	1.038	1.032	1.025	1.018	1.010	1.003	0.995	0.986	0.978
1.70	1.000	1.041	1.036	1.030	1.024	1.017	1.010	1.003	0.995	0.986	0.978
1.60	1.010	1.035	1.031	1.026	1.021	1.016	1.009	1.002	0.995	0.986	0.978
1.50	1.023	1.025	1.023	1.020	1.016	1.012	1.008	1.002	0.994	0.986	0.978
1.40	1.011	1.010	1.009	1.008	1.006	1.005	1.003	0.999	0.994	0.986	0.978
1.30	0.986	0.986	0.986	0.987	0.988	0.989	0.991	0.991	0.991	0.986	0.978
1.20	0.948	0.949	0.951	0.953	0.956	0.960	0.965	0.971	0.978	0.983	0.978
1.10	0.893	0.895	0.897	0.900	0.903	0.908	0.915	0.924	0.937	0.959	0.978
1.00	0.816	0.818	0.819	0.822	0.824	0.827	0.831	0.837	0.846	0.862	0.978
0.90	0.710	0.711	0.712	0.713	0.714	0.713	0.713	0.711	0.707	0.695	0.665
0.80	0.580	0.580	0.580	0.580	0.580	0.580	0.580	0.580	0.580	0.580	0.580
0.70	0.495	0.495	0.495	0.495	0.495	0.495	0.495	0.495	0.495	0.495	0.495
0.60	0.410	0.410	0.410	0.410	0.410	0.410	0.410	0.410	0.410	0.410	0.410
0.50	0.325	0.325	0.325	0.325	0.325	0.325	0.325	0.325	0.325	0.325	0.325
	2.00	**1.80**	**1.60**	**1.40**	**1.20**	**1.00**	**0.80**	**0.60**	**0.40**	**0.20**	**0.00**

Table 3. Securitized mortgage: Borrower strategies

2.50	!	!									
2.40	!	!									
2.30	!	!									
2.20	!	!									
2.10	!	!									
2.00	!	!									
1.90	!	!									
1.80	!										
1.70	!										
1.60											
1.50											
1.40											
1.30											
1.20											
1.10											
1.00											
0.90											*
0.80	*	*	*	*	*	*	*	*	*	*	*
0.70	*	*	*	*	*	*	*	*	*	*	*
0.60	*	*	*	*	*	*	*	*	*	*	*
0.50	*	*	*	*	*	*	*	*	*	*	*
	2.00	**1.80**	**1.60**	**1.40**	**1.20**	**1.00**	**0.80**	**0.60**	**0.40**	**0.20**	**0.00**

Appendix B. Portfolio mortgage

Table 4. Ll(h, 0)

2.50	1.000	0.998	0.996	0.994	0.992	0.990	0.987	0.985	0.983	0.981	0.978
2.40	1.000	0.998	0.996	0.994	0.992	0.990	0.987	0.985	0.983	0.981	0.978
2.30	1.000	0.998	0.996	0.994	0.992	0.990	0.987	0.985	0.983	0.981	0.978
2.20	1.000	0.998	0.996	0.994	0.992	0.990	0.987	0.985	0.983	0.981	0.978
2.10	1.000	0.998	0.996	0.994	0.992	0.990	0.987	0.985	0.983	0.981	0.978
2.00	1.000	0.998	0.996	0.994	0.992	0.990	0.987	0.985	0.983	0.981	0.978
1.90	1.000	0.998	0.996	0.994	0.992	0.990	0.987	0.985	0.983	0.981	0.978
1.80	1.000	0.998	0.996	0.994	0.992	0.990	0.987	0.985	0.983	0.981	0.978
1.70	1.000	0.998	0.996	0.994	0.992	0.990	0.987	0.985	0.983	0.981	0.978
1.60	0.995	0.994	0.993	0.992	0.990	0.989	0.987	0.985	0.983	0.981	0.978
1.50	0.984	0.983	0.983	0.983	0.983	0.982	0.982	0.983	0.982	0.981	0.978
1.40	0.964	0.964	0.964	0.965	0.966	0.967	0.968	0.971	0.974	0.978	0.978
1.30	0.934	0.934	0.935	0.936	0.937	0.939	0.941	0.945	0.950	0.959	0.978
1.20	0.890	0.890	0.891	0.892	0.893	0.895	0.897	0.900	0.905	0.911	0.920
1.10	0.830	0.830	0.830	0.831	0.831	0.832	0.833	0.834	0.835	0.835	0.835
1.00	0.750	0.750	0.750	0.750	0.750	0.750	0.750	0.750	0.750	0.750	0.750
0.90	0.665	0.665	0.665	0.665	0.665	0.665	0.665	0.665	0.665	0.665	0.665
0.80	0.580	0.580	0.580	0.580	0.580	0.580	0.580	0.580	0.580	0.580	0.580
0.70	0.495	0.495	0.495	0.495	0.495	0.495	0.495	0.495	0.495	0.495	0.495
0.60	0.410	0.410	0.410	0.410	0.410	0.410	0.410	0.410	0.410	0.410	0.410
0.50	0.325	0.325	0.325	0.325	0.325	0.325	0.325	0.325	0.325	0.325	0.325
	2.00	**1.80**	**1.60**	**1.40**	**1.20**	**1.00**	**0.80**	**0.60**	**0.40**	**0.20**	**0.00**

Table 5. Lender serviced mortgage: Borrower strategies

2.50	+	+	+	+	+	+	+	+	+	+	
2.40	+	+	+	+	+	+	+	+	+	+	
2.30	+	+	+	+	+	+	+	+	+	+	
2.20	+	+	+	+	+	+	+	+	+	+	
2.10	+	+	+	+	+	+	+	+	+	+	
2.00	+	+	+	+	+	+	+	+	+	+	
1.90	+	+	+	+	+	+	+	+	+	+	
1.80	+	+	+	+	+	+	+	+	+	+	
1.70			+	+	+	+	+	+	+	+	
1.60								+	+	+	
1.50										+	
1.40											
1.30											
1.20											+
1.10										+	+
1.00	+	+	+	+	+	+	+	+	+	+	+
0.90	+	+	+	+	+	+	+	+	+	+	+
0.80	+	+	+	+	+	+	+	+	+	+	+
0.70	+	+	+	+	+	+	+	+	+	+	+
0.60	+	+	+	+	+	+	+	+	+	+	+
0.50	+	+	+	+	+	+	+	+	+	+	+
	2.00	**1.80**	**1.60**	**1.40**	**1.20**	**1.00**	**0.80**	**0.60**	**0.40**	**0.20**	**0.00**

Appendix C. Sensitivity Analysis

Table 6. Effect of parameters on LTV

p	LTV_s	LTV_p	c	LTV_s	LTV_p	f_1	LTV_s	LTV_p
0.00	0.62	0.57	0.05	0.38	0.29	0.00		0.58
0.02	0.64	0.55	0.06	0.66	0.50	0.02		0.58
0.04	0.67	0.53	0.07	0.71	0.55	0.04		0.58
0.06	0.70	0.56	0.08	0.74	0.58	0.06	0.74	0.58
0.08	0.73	0.57	0.09		0.60	0.08	0.74	0.58
0.10	0.75	0.59	0.10		0.62	0.10	0.74	0.58
0.12	0.77	0.61	0.11		0.64	0.12	0.74	0.58
0.14	0.80	0.62	0.12		0.65	0.14	0.74	0.58
0.16	0.81	0.63	0.13		0.66	0.16	0.74	0.58
0.18	0.83	0.64	0.14		0.67	0.18	0.74	0.58
0.20	0.84	0.65	0.15		0.68	0.20	0.74	0.58

l_0	LTV_s	LTV_p	σ	LTV_s	LTV_p	d_1	LTV_s	LTV_p
0.00	0.78	0.65	0.05	0.93	0.76	0.00	0.79	0.62
0.02	0.77	0.64	0.10	0.87	0.71	0.05	0.74	0.58
0.04	0.76	0.62	0.15	0.81	0.65	0.10	0.68	0.54
0.06	0.75	0.61	0.20	0.74	0.58	0.15	0.62	0.49
0.08	0.75	0.60	0.25	0.67	0.51	0.20	0.56	0.44
0.10	0.74	0.58	0.30	0.60	0.45	0.25	0.51	0.40
0.12	0.73	0.57	0.35	0.53	0.38	0.30	0.46	0.36
0.14	0.73	0.56	0.40	0.47	0.33	0.35	0.41	0.33
0.16	0.72	0.55	0.45	0.41	0.28	0.40	0.37	0.30
0.18	0.72	0.53	0.50	0.35	0.23	0.45	0.34	0.27
0.20	0.71	0.52	0.55	0.29	0.20	0.50	0.31	0.24

[1]Prepayment penalties, usually in effect for the duration of the fixed rate period, mitigated prepayment before the end of the fixed rate period, see Gorton (2008) and Cutts and Van Order (2005).
[2]See Gorton (2008) and Bhardwaj and Sengupta (2008b).
[3]Approximately 80% of subprime loans originated from 2001 to 2006, had been terminated within 3 years of origination. The early terminations were largely due to prepayments for vintages up to and including 2004, with defaults accounting for a larger share of the early terminations for 2005 and 2006 vintages (Demyanyk, 2009).
[4]See Ghent and Kudlyak (2011).
[5]Choudhary (2008) indicates that for judicial foreclosures, the period may be as long as 2 years.
[6]Some PSAs essentially bar loan modifications, while others limit the number of loans that may be modified over the life of the securitization, see Eggert(2007).
[7]See Chaudhary (2007) and Chaudhary (2008).
[8]See Kruger (2013) and the work cited therein for an account of the debate.
[9]The non-recourse nature of the loan provides the borrower with an option to put the home back to the lender for the outstanding mortgage balance.
[10]Rational defaults are sometimes referred to as `strategic,' `walk-away' or `ruthless' defaults in the literature.
[11]The game is a continuous time, stochastic game of perfect information. The essential assumptions of these games is that the history of the game at each point in time can be summarized by a `state'. Current payoffs depend on this state and on current actions. Perfect information implies that players `move' sequentially and their actions are observed before the next move occurs (Fudenberg and Tirole, 1991, pp.72-73, p.503). The game theoretic approach to contract (re)negotiation follows Anderson and Sundaresan (1996) and Mella-Barral and Perraudin (1997). The valuation framework follows Jones and Nickerson (2002).
[12]The borrower always has a `first-mover' advantage.
[13]In the case of a loan that requires a lump-sum payment at maturity, the borrower exercises similar choice over this amount, P^*.
[14]Loan indentures such as `technical default' that allow the lender to foreclose whenever the value of the home is less than the outstanding loan balance, regardless of whether the borrower has defaulted or not, are not considered here. This guarantees that $\hat{p} \leq p$. Indentures of this nature are not typical of current US mortgage contracts.
[15]Prepayment penalties and refinancing costs may be incurred.
[16]$\Omega_L(h,t)$ represents the termination value of the lender's claim.
[17]We restrict our analysis to the Markov perfect equilibria of the game. We find these equilibria by restricting the strategy space of the players to the set of strategies in which the past influences current play only through its effect on a finite number of state

variables that summarize the direct effect of the past on the current environment. The past matters only to the extent that it directly affects the current payoffs of the players. A Markov perfect equilibrium is a profile of Markov strategies for the players that yields a Nash equilibrium in every proper subgame (Fudenberg and Tirole, 1991, p.501).

[18]For infinite horizon cases, $T = \infty$, players' strategies depend only on h. In this case the game is said to be `stationary' (Fudenberg and Tirole, 1991, p.521).

[19]For the valuation problems to be solved by the borrower and lender over the course of the game it is necessary that these sets be closed subsets of $\mathrm{H} \times \mathrm{T}$.

[20]For valuation purposes we only consider foreclosure costs of the linear form, $l(h,t) = l_0 + l_1 h(t)$. In this case, the upper default region is a compact set for finite term loans, i.e. there is an upper bound to the region. Since foreclosure costs are monotonically increasing in h, at sufficiently high levels of h, the foreclosure costs will exceed the benefits associated with termination in order to avoid the high credit spread.

[21]Since the foreclosure costs are a `deadweight loss' from the point of view of the borrower and lender, a clear incentive exists for the parties to renegotiate the terms of the contract (the credit spread, in particular) as $h(t)$ approaches $\overline{h}(t)$. Such renegotiation is ruled out here. Prepayment or default are (costly) substitutes for renegotiation.

[22]We assume that refinancing costs are of the form $f[b(t)] = f_0 + f_1 b(t)$, for valuation purposes. Since these costs are not increasing in h, the prepayment region will not have an upper bound.

[23]With sufficiently little time remaining to maturity, the cost incurred in servicing the loan at a rate greater than the fair market rate for the reduced default risk will be less than the foreclosure costs incurred by defaulting. Thus, the upper stopping region and the upper stopping boundary do not extend to T for any h.

[24]Appendix A shows the termination regions determined by numerical computation.

[25]This region is *not* a termination region for the game.

[26]The borrower evaluates future payoffs to the lender using the martingale equivalent probability measure, Q.

[27]The monotonicity of the value of the borrower's claim in p^* is a consequence of the constraints the loan contract considered here imposes on the lender. Under different constraints, the monotonicity would not exist. For example, if the lender could foreclose whenever $h(t) < b(t)$ (technical default), the monotonicity would not be preserved.

[28]Typically, $\underline{h}(t)$ will be significantly less than $b(t) + l(\underline{h}, t)$.

[29]While positive foreclosure costs enhance the scope for strategic default when the home value is low, they are not necessary for strategic default. In the absence of foreclosure costs, the borrower can avoid foreclosure at low collateral values by offering the lender the entire dividend flow from the collateral if the dividend flow is less than the contractual debt service flow. From the lender's point of view, receiving the dividend flow is just as good as owning the collateral. Of course, the borrower prefers this strategy to inducing foreclosure since there is some positive probability that the collateral value will recover.

[30]The upper modification region only exists if the credit spread, $c - r$, is positive. Prepayment may still occur if the credit spread is sufficiently large, refinancing costs are low and the loan balance is accreting.

[31]Appendix B shows the termination regions determined by numerical computation.

[32]We use the Crank-Nicolson scheme. Analytical solutions are obtainable in cases where the termination boundaries of the game are invariant with respect to time. This is the case for perpetual loans and finite maturity loans where loan termination is restricted to the maturity date.

[33]Note that time on the horizontal axis is time remaining to maturity.

[34]The symbol ! denotes prepayment and $*$ denotes default.

[35]The symbol $+$ denotes loan modification, $p^* < p$.

[36] Regulators concurring with this view include Bank for International Settlements (2008), Bernanke (2008b), Knight (2008) and President's Working Group on Financial Markets (2008). Researchers concurring with this view include Ashcraft and Schuermann (2008), Crouhy, Jarrow and Turnbull (2008), Demyanyk and Van Hemert (2011), Hull (2008), Keys et al (2009) and Mian and Sufi (2009). Dissenting views are offered by Bhardwaj and Sengupta (2008a), Gorton (2008) and Jaffee (2008).