# THE SINGLE INDEX MODEL & THE CONSTRUCTION OF OPTIMAL PORTFOLIO: A CASE OF BANKS LISTED ON NSE INDIA

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**Abstract**

Risk and return plays an important role in making any investment decisions. Decisions that range from 'Should the investment be done?' and if yes, then 'which security should comprise portfolio?' In the present study 10 companies listed at National Stock Exchange (NSE) and CNX Bank Price Index was selected taking Jan 2009 to Dec 2013 as period of study. The monthly closing prices of the selected securities were used for the above mentioned period. Application of Single Index Model for the empirical analysis identified a portfolio of two companies based on the cut-off point.

**Keywords:** Risk & Return, Efficient Portfolio, Sharpe Index Model, CNX Bank, Cutoff Point

**JEL Classification:** G11, G21

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## 1. Introduction

Portfolio is a bundle of or a combination of individual assets or securities and the portfolio theory provides normative approach. It is based on the assumption that investors are *risk-averse*. This implies that investors hold well diversified portfolios instead of investing their entire wealth in a single or few assets. Investors who are risk-averse reject investment portfolios that are fair games or worse. Risk-averse investors are willing to consider only risk-free or speculative prospects with positive risk premiums. A rational investor is a person that desires to maximize their return with less risk on his investment in a portfolio. This means that the needs to construct an efficient portfolio minimum risk for a given expected return. This can be achieved with the help of single index (beta) model proposed by Sharpe.

### *Markowitz Portfolio Theory*

In the early 1950s, the investment community talked about risk, but there was no specific measure for the term. *Risk means uncertainty of future outcomes*. To measure risk or to avoid risk investor had to quantify their risk variable by building a basic portfolio model. The basic portfolio model was developed by Harry Markowitz (1952, 1959), who derived the expected rate of return. Markowitz showed that the variance of the rate of return was a meaningful measure of portfolio risk under reasonable set of assumptions, and he derived the formula for computing the variance of a portfolio as variance is a measure of risk. The Markowitz model is based on several assumptions regarding investor behavior. (1) Investor considers each investment alternative as being represented by a probability distribution of expected returns over some holding period. (2) Investors maximize one-period expected utility, and their utility curves demonstrate diminishing marginal utility of wealth. (3) Investors estimate the risk of the portfolio on the basis of the variability of expected return. (4) Investors base decisions solely on expected return and risk, so their utility curves are a function of expected return and the expected variance of returns only. (5) For a given level of risk, investor prefers higher returns to lower returns. Under these assumptions, a single asset or portfolio of assets is considered to be efficient if no other asset or port folio offers higher expected return with the same

### *The Sharpe Index Model*

Varian (1993) succinctly reviewed the history of modern portfolio theory as Markowitz's ground breaking research on portfolio optimization in his quest he finds out that implementation of Markowitz model is much more time-consuming and more complex by the number of estimates required. Markowitz model suffers from two drawbacks [see, Bodie, Kane, Marcus, Mohanty (2009)].

- First, the model requires huge number of estimates to fill the covariance matrix.
- Second, the model does not provide any guidelines to the forecasting of the security risk premiums that are essential to construct the efficient frontier of risky assets.

Identification of these discrepancies, with an attempt develop simple index model for portfolio. William Sharpe's effort towards to simplify the Markowitz model resulted in developing a single index model which substantially reduces its data and computational requirements [see, Sharpe (1963)]. The simplified model assumed that the fluctuations in the value of stock relative to other stocks do not depend on the characteristics of those two securities alone. Relationship between securities occurs only through their individual relationships with some indexes of business activity. The reduction in the number of covariance estimates made the job of security analysis and portfolio- analysis computation somewhat easy. Thus the covariance data requirement reduces from $(N^2 – N)/2$ under the Markowitz technique to only N measures of each security as it relates to the index. To find out proper covariance between securities Smith (1969) uses economic indexes such as gross national product and the consumer price index. The Sharpe technique requires 3N + 2 separate bits of information, as opposed to the Markowitz requirement of [N (N + 3)]/ 2.

The purpose of this paper is to construct the optimum portfolio by using single index model on Indian stock market (i.e. National Stock Exchange, Mumbai) and assuming a case where short selling is not allowed.

## 2. Literature Review

Markowitz (1952 and 1959) performed the pioneer work on portfolio analysis. The major assumption of the Markowitz's approach to portfolio analysis is that investors are basically risk-averse. This means that investors must be given higher returns in order to accept higher risk. Markowitz then developed a model of portfolio analysis. Markowitz (1952) and Tobin (1958) showed that it was possible to identify the composition of an optimal portfolio of risky securities, given forecasts of future returns and an appropriate covariance matrix of share returns. Sharpe (1963) attempted to simplify the process of data input, data tabulation, and reaching a solution. He also developed a simplified variant of the Markowitz model that reduces data and computational requirements. William Sharpe (1964) has given model known as Sharpe Single Index Model which laid down some steps that are required for construction of optimal portfolios. Elton and Gruber (1981), and Elton, Grube and Padberg [1976, 1977A, l978A, 1978B, 1979] have established simple criteria for optimal portfolio selection using a variety of models, such as single index, multi-index, and constant correlation models. These models are used to provide solution to portfolio problems by disallowing short sales of risky securities in portfolios and this can be done by using simple ranking procedures. Elton, Grube, Padberg (1977B) have also extended their analysis using a constant correlation model, as well as a single index model, to incorporate upper limits on investment in individual securities. Haugen (1993) stated that Index models can handle large population of stocks. They serve as simplified alternatives to the full-covariance approach to portfolio optimization. Although the Single Index Model offers a simple formula for portfolio risk, it also makes an assumption about the process generating security returns. According to Terol et al. (2006) Markowitz model is a conventional model proposed to solve the portfolio selection problems by assuming that the situation of stock markets in the future can be characterized by the past asset data. In addition, Briec & Kerstens (2009) stated that Markowitz model contributes in geometric mean optimization advocated for long term investments. On the other hand, the Simple Index Model is no longer good approximations to multi period. As seen by Frankfurter et al. (1976) according to this study, under conditions of certainty, the Markowitz and Simple Index Model approaches will arrive at the same decision set in the experiment. These results demonstrate that under conditions of uncertainty, Simple Index Model approach is advantageous over the Markowitz approach. It was found that variation in performance is explained in terms of the two essential differences in the models. First, fewer and different estimators are used in the Simple Index Model to summarize past history. Second, the linear assumption of the Simple Index Model does not necessarily hold. They finally found that in experiments, the Simple Index Model process performs worse than Markowitz process, and gives superior results when only short data histories are available. Omet (1995) argued that the two models are similar. Simple Index Model can be used, which is more practical than the Markowitz model in generating ASE efficient frontier. Dutt (1998) used Sharpe single index model in order to optimize a portfolio of 31 companies from BSE (Bombay Stock Exchange). Nanda, Mahanty, and Tiwari (2012) selected stocks from the clusters to build a portfolio, minimizing portfolio risk and compare the returns with that of the benchmark index i.e. Sensex. Saravanan and Natarajan (2012) used Sharpe single index model in order to construct an optimal portfolio of 4 companies from NSE (National Stock Exchange of India) and used NSE NIFTY as market index. Meenakshi and Sarita (2012) stated that Sharpe's single index model is of great importance and the framework of Sharpe's single index model for optimal portfolio construction is very simple and useful.

## 3. Research Methodology

### I. *THE SINGLE INDEX MODEL*

The risk return model suggested by Sharpe is:

$R_i = \alpha_i + \beta_i I + e_i$ (1)

Where:

$R_i$ = expected return on security *i*

$\alpha_i$ = intercept of a straight line or alpha coefficient

$\beta_i$ = slope of straight line or beta coefficient

I = expected return on index (market)

$e_i$ = error term with the mean of zero and a standard deviation which is a constant

**A.** Return: The daily return on each of the selected stocks is calculated with the following formula.

$$R_{it} = \frac{P_{it}}{P_{it-1}} - 1$$

Where Pt, Pit–1 are the share price at time t and t-1 for security i.

$R_{it}$= Return on security 'i' at time't'.

t= price of security at time "t"

t-1= price of security a year earlier or when portfolio was constructed if it's only a year old.

**B.** Standard Deviation:

The second phase in the context of testing of Sharpe's model for selection of appropriate securities in portfolio is used, the average returns of individual returns or portfolio are adjusted to that of risk free return (here 7.8 percent is considered as risk free rate based on the portfolio on 91-day Government of India treasury bills at the time of conducting a study). Therefore to estimates the coefficients with risk free adjusted average return on individual / portfolio and on market risk, the following model is used. The selection of any stock is directly related to its excess return - beta ratio:

$$Excess\ return = \frac{R_i - R_f}{\beta_i}$$

$R_i$ = the expected return on stock i; $R_f$ = the return on a riskless asset and $\beta_i$ = the expected change in the rate of return on stock i associated with one unit change in the market return.

The excess return is the difference between the expected return on the stock and the riskless rate of interest such as the rate offered on the government security or Treasury bill. The excess return to beta ratio measures the additional return on a security (excess of the riskless assets return) per unit of systemic risk or non-diversifiable risk. This ratio provides a relationship between potential risk and reward.

**C.** *Optimum Portfolio when Short Sales are Not Allowed*

Ranking of the stocks is done on the basis of their excess return to beta. Portfolio managers would like to include stocks with higher ratios. The selection of the stocks depends on a unique cut –off rate such that all stocks with higher ratios of $(R_i - R_f) / \beta_i$ are included and the stocks with lower ratios are left out. The cutoff point is denoted by C*.

$$C^* = \frac{\sigma_m^2 \sum_{i=1}^{i} \frac{(R_i - R_F)\beta_i}{\sigma_{ei}^2}}{1 + \sigma_m^2 \sum_{i=1}^{i} \frac{\beta_i^2}{\sigma_{ei}^2}} \quad (2)$$

Where:

$\sigma_m^2$= variance in the market index.

$\sigma_{ei}^2$ = variance of a stock's movement that is not associated with the movement of the market index; this is the stock's unsystemic risk.

Assuming that the short sales is not allowed and unlimited lending and borrowing can take place at risk free rate of return ($R_f$), the optimum amount of investment in security would be given by

$$X_i = \frac{Z_i}{\sum_{j \in K} Z_i} \times 100 \quad \text{For all } Z_i > 0 \quad (3)$$

Where:

K is the set of securities in the optimal portfolio

$$Z_i = \frac{\beta_i}{\sigma_{ei}^2}\left[\frac{\overline{Rj} - Rf}{\beta_i} - C\right] \quad (4)$$

### II. *DATA*

Here in this study daily data have not been used for analysis purpose reason for this are being mentioned below:

a. Non normality: The daily stock return for an individual security exhibits substantial departures from normality that are not observed with monthly data. The evidence generally suggests that distributions of daily returns are fat-tailed relative to a normal distribution [Fama (1976)].

b. Variance estimation: The first issue is the time-series properties of daily data. As a consequence of non-synchronous trading, daily excess returns can exhibit serial dependence. Attempts to incorporate such dependence into variance estimates have appeared in the event study literature [see, Ruback (1982)]. The second issue is stationarity of daily variances. There is evidence that the variance of stock returns increases for the days immediately around events such as earnings announcements [see, Beaver (1968), Patell and Wolfson (1979)].

The monthly closing price of stocks listed on National Stock Exchange (NSE) and monthly closing index value of CNX BANK INDEX have been used for construction of optimal portfolio applying Sharpe's Single Index Model. The closing prices were collected for a period of five year starting from January 2009 and ending at December 2013. This study takes 9 banks listed on NSE. Selection is done on the basis of market capitalization. This study has used secondary data and for risk free securities 91 days T-Bill has been used as a proxy for risk free rate and sourced from Reserve Bank of India. Company's names are mentioned below in table 1.

**Table 1.** List of Companies

| Company's Name |
| --- |
| 1. ICICI Bank Ltd |
| 2. HDFC Bank Ltd |
| 3. State Bank of India |
| 4. Axis Bank Ltd |
| 5. Kotak Mahindra Bank Ltd |
| 6. IndusInd Bank Ltd. |
| 7. Bank of Baroda |
| 8. Yes Bank Ltd |
| 9. Punjab National Bank |
| 10. Federal Bank Ltd |

*Source: National Stock Exchange*

**Overview of CNX BANK:**

The CNX Bank Index is an index comprised of the most liquid and large capitalized Indian Banking stocks. It provides investors and market intermediaries with a benchmark that captures the capital market performance of the Indian banks. The Index has 12 stocks from the banking sector, NSE.

**Statistics of CNX BANK:**

**Table 2.** Statistics of CNX Bank

| | Quarter To Date | Year To Date | 1 Year | 5 Year | Since Inception |
| --- | --- | --- | --- | --- | --- |
| Returns | 11.92 | 11.91 | 12.15 | 25.24 | 19.55 |

*Source: National Stock Exchange*

| | 1 Year | 5 Year | Since Inception |
| --- | --- | --- | --- |
| Standard Deviation | 30.52 | 28.63 | 32.55 |
| Beta (Nifty) | 1.47 | 1.22 | 1.05 |
| Correlation | 0.87 | 0.89 | 0.82 |

*Source: National Stock Exchange*

Fundamentals

| Profit Earning Ratio | Price to Book Ratio | Dividends |
| --- | --- | --- |
| 14.3 | 2.24 | 1.4 |

*Source: National Stock Exchange*

## 4. Result and Discussion:

Firstly the securities are ranked according to their excess return to beta ratio from highest to lowest. Among 9 companies 5 companies offer less return than risk free rate. The cut-off value has been calculated in order to find out optimum C*. The highest value thus achieved is considered as the optimum *C**. From table 3 it can be seen that out of 10 banks 2 banks are having excess return and their values are more than cut-off value. Here, the cut-off value is 0.438 see table 4.

**Table 3.** Optimal Portfolios

| Optimal Portfolio ($R_f$ = 7.8 per cent) | | | | | | |
| --- | --- | --- | --- | --- | --- | --- |
| Bank | Rank | Mean Return ($\bar{R}_i$) | ($R_i$- $R_f$) | Beta | Unsystematic Risk | Excess Return |
| Federation Bank | 1 | 30.156 | 22.356 | 0.933 | 0.5785 | 23.96 |
| Yes Bank | 2 | 8.62 | 0.82 | 1.332 | 0.2368 | 0.61 |
| ICICI Bank | 3 | 7.907 | 0.107 | 1.221 | 0.1144 | 0.087 |
| Axis Bank | 4 | 7.88 | 0.08 | 1.217 | 0.3168 | 0.065 |
| PNB | 5 | 7.141 | -0.659 | 1.101 | 0.371 | -0.598 |
| SBI | 6 | 6.405 | -1.395 | 0.988 | 0.2132 | -1.411 |
| BOB | 7 | 6.04 | -1.76 | 0.934 | 0.363 | -1.884 |
| Kotak Mahindra Bank | 8 | 5.668 | -2.132 | 0.877 | 0.44 | -2.431 |
| HDFC Bank | 9 | 4.309 | -3.491 | 0.664 | 0.134 | -5.257 |
| IndusInd Bank | 10 | -8.49 | -16.29 | 0.728 | 0.544 | -22.38 |

*Sources: Author own calculation*

From the above table 3 it can be seen that on the basis of excess return ranks are given to different companies. There are 10 securities in the table 3. They are already ranked. Ranking is done on the basis of excess return. Selecting a security in an optimal portfolio it is necessary to compare excess return. It can be seen from the table that out of 10 securities returns on 4 securities are positive & other 6 securities have negative excess return. Here the excess return is largest for Federation bank and least for HDFC bank Ltd. further from among mentioned securities only four securities shows beta value are above 1 and rest are less than it.

It can be seen from table 4 that cutoff point is 0.438, two companies i.e. Federal Bank Ltd. & Yes Bank Ltd. shows C* more than that of cutoff point. Other companies i.e. 8 companies cutoff point is lower than that of required. All the companies whose C* is greater than that of cutoff point can be included in the portfolio.

**Table 4.** Determining the Optimal Portfolio (From January, 2009 to December, 2013)

| Banks | Rank | $\frac{R_i - R_f}{\beta_i}$ | $(R_i - R_F)$ | $\beta_i$ | $\frac{(R_i - R_F)\beta_i}{\sigma_{ei}^2}$ | $\sigma_{ei}^2$ | $\frac{\beta_i^2}{\sigma_{ei}^2}$ | $\sum_{i=1}^{i} \frac{(R_i - R_F)\beta_i}{\sigma_{ei}^2}$ | $\sigma_m^2$ | $C_i$ |
|---|---|---|---|---|---|---|---|---|---|---|
| Federal Bank | 1 | 23.96 | 22.35 | 0.933 | 36.05 | 0.57 | 1.50 | 36.05557 | 0.008 | 0.300 |
| Yes Bank | 2 | 0.61 | 0.82 | 1.332 | 4.61 | 0.23 | 7.49 | 40.66807 | 0.008 | 0.385 |
| ICICI Bank | 3 | 0.087 | 0.10 | 1.221 | 1.14 | 0.11 | 13.03 | 41.81009 | 0.008 | 0.438* |
| Axis Bank | 4 | 0.06 | 0.08 | 1.217 | 0.30 | 0.31 | 4.67 | 42.11741 | 0.008 | 0.374 |
| PNB | 5 | -0.59 | -0.65 | 1.101 | -1.95 | 0.37 | 3.26 | 40.16173 | 0.008 | 0.347 |
| SBI | 6 | -1.41 | -1.39 | 0.988 | -6.46 | 0.21 | 4.57 | 33.69709 | 0.008 | 0.306 |
| BOB | 7 | -1.88 | -1.76 | 0.934 | -4.52 | 0.36 | 2.40 | 29.16861 | 0.008 | 0.252 |
| Kotak Mahindra Bank | 8 | -2.43 | -2.13 | 0.877 | -4.24 | 0.44 | 1.74 | 24.91915 | 0.008 | 0.213 |
| HDFC Bank | 9 | -5.25 | -3.49 | 0.664 | -17.2987 | 0.134 | 3.29 | 7.62046 | 0.008 | 0.087 |
| IndusInd Bank | 10 | -22.38 | -16.29 | 0.728 | -21.80 | 0.544 | 0.974 | -14.179 | 0.008 | -0.105 |

*Source: author own calculations*

**Table 5.** Determining the Weights for an Optimal Portfolio

| Bank | Rank | $\beta_i$ | $\sigma_{ei}^2$ | $\frac{\beta_i}{\sigma_{ei}^2}$ | $\frac{R_i - R_f}{\beta_i}$ | $\frac{\beta_i}{\sigma_{ei}^2}\left[\frac{\overline{Rj} - Rf}{\beta_i} - C*\right]$ | $\frac{Z_i}{\sum_{j \in k} Z_j} \times 100$ |
|---|---|---|---|---|---|---|---|
| Federal Bank | 1 | 0.933 | 0.57 | 1.61 | 23.96 | 37.917 | 97.51 |
| Yes Bank | 2 | 1.332 | 0.23 | 5.62 | 0.61 | 0.967 | 2.49 |

*Source: author own calculations*

Table 5 explains the results of analysis. The optimal portfolio, under assumption that short sales is not allowed, consists of two securities, namely Federal Bank Ltd, and Yes bank Ltd. The total investment needs to be shall in proportion of 97.51 percent and 02.49 percent in Federal Ban ltd. and Yes Bank Ltd. respectively. All the securities which have excess return to beta ratio greater than that of cut-off point are included in the portfolio. Such portfolio is the optimum portfolio and the securities included in the portfolio are the efficient securities. The study that follows 10 stocks needs 32 numbers of inputs as against 65 numbers of inputs for Markowitz Model. So, it can be stated that implementation of Markowitz model is much more time-consuming and more complex. And the framework of Sharpe's single index model for optimal portfolio construction is very simple and useful.

## Conclusion

This study aims at analyzing the opportunity that are available for investors as per as returns are concerned and the investment of risk thereof while investing in equity of firms listed in the national stock exchange. Sharpe's single-index model was applied by using the monthly closing prices of 10 companies listed in NSE and CNX BANK price index for the period from January 2009 to December 2013. From the empirical analysis it can be concluded that out of 10 companies 2 companies are selected for investment purpose.

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