VALUING PUT OPTIONS ON SINGLE STOCK FUTURES: DOES THE PUT-CALL PARITY RELATIONSHIP HOLD IN THE SOUTH AFRICAN DERIVATIVES MARKET?

*A Biebuyck, JH Van Rooyen**

Abstract

This study attempts to determine whether mispricing of options on single stock futures is present in the South African derivatives market. The valuation of options on single stock futures is considered through the put-call parity relationship. The theoretical fair values obtained, are compared to the actual market values over a period of three years, that is, from 2009 to 2011. Only put options are considered in this research.The results show that arbitrage put option opportunities do present themselves for the chosen shares. The actual put options were found to be underpriced in 5 out of 6 (83%) of the cases considered over the evaluation periods chosen. The mispricing was significant for both the BHP Billiton options with 100% and in the case of Sasol options (66%) of the time. Whether profitable arbitrage opportunities is possible, will depend on the magnitude of the mispricing and the transaction fees payable. Further, more extensive research may help identify tendencies which may be of use for the formulation of arbitrage strategies.

Keywords: Put-Call Forward Parity, Arbitrage Trading, Mispricing, Violations

*Stellenbosch University, Department of Business Management, Private Bag X1, Matieland, 7602, South Africa
Email: jvrooyen@sun.ac.za

1 Introduction

A SSF is a futures contract in which a single equity, listed on an exchange (Johannesburg Stock Exchange (JSE)), is the underlying instrument. SSFs are therefore derivatives instruments whose depends on movements in the underlying asset, in this case equity. An investor may acquire a long futures position, hereafter called "long", if he/she believes that the share price will increase in the future and will enter a short position, hereafter called "short", if the investor believes the share price will decrease. The difference between the spot price and the futures price, at expiration, will determine the gain or loss of the position. An SSF contract can be reverse-traded at any time before expiration, with the return determined by the difference between the initial value of the contract and the current spot price of the SSF contract.

SSF contracts were first introduced in South Africa in 1999 and have grown in popularity ever since. The increase in demand may be attributed to the wide range of investors that SSFs appeal to. A major benefit being the gearing due to the low outlay to gain substantial exposure.

Apart from the SSF's, options on SSF's may also be traded on the JSE. An investor may choose to long or short a call or a put on a SSF. Longing a call option on an SSF means the investor is protected against an increase in the price of the underlying. Longing a put option means that the investor expects a decrease in the price of the underlying. If an option is out-of-the-money on expiry date, the option is allowed to expire worthless. Only the premium is lost.

The put-call theorem describes the relationships between put and call option prices. This theorem works on the principle that the payoff or difference between a long call and a short put (together describing a leveraged underlying) is equal to a long underlying and the present value of the exercise price which is equivalent to a bond. If this relationship is violated, an opportunity to realise riskless profits becomes possible. These violations mean that the market prices of puts and calls do not equal their theoretical or model prices. This mispricing allows for arbitrage opportunities and unique profit opportunities to increase investment returns.

The results of this research is of particular interest to individual and institutional investors who may realise returns in excess of the risk-free interest rate with little or no increase in risk. Arbitrage in this case means selling the overpriced instrument and buying the under-priced equivalent. This strategy results in a return that is above the risk-free rate of return. The investor may choose to realise the return either today or on expiry date of the arbitrage deal. Investors execute this strategy by buying and selling the underlying asset and the synthetic underlying asset in order to profit from the differential in the price. This has to take place on day 0. This strategy will alter investors' trading decisions as it may offer unique opportunities to earn a riskless profit without the use of own capital. Synthetics refer to when an

instrument that cannot be traded now, is replicated with derivative instruments.

The growing interest in SSFs is evidence of the need for research in this field. Research done by De Beer (2008: 133) found that trading in SSFs increases the spot market trading volumes and reduces the level of spot market volatility. This was achieved through the use of t-tests and GARCH[24] models to test the volumes and volatility respectively.

2 Objective of the study

The aim of this study is to value option contracts on SSF's, using the put-call forward parity theorem, and to determine whether the theorem holds in the South African derivatives market or whether mispricing exists in the market. Mispricing of options, generally, are not considered in this research. Only puts relative to calls through the put-call parity relationship is considered. The objective then is to determine whether setting up synthetic put (whether shorting or longing) will deliver arbitrage returns in excess of the risk-free interest rate. This may be achieved with no capital outlay by the arbitrager.

Although options and SSF's on a particular share have no claim to dividend streams, the share price is affected by dividend payouts. Higher cash dividends tend to imply lower call premiums and higher put premiums. Dividends streams were ignored in this research.

3 Scope of the research

This research does not address the possible effect of derivatives trading on general volatility in the market. Historic volatilities available on the JSE database is used for pricing and is assumed to be constant and not affected by trading/arbitrage activities for the sake of this research. This study also does not address the implied versus historic volatility issue.

Transaction fees were also ignored for the sake of this research. Only once the arbitrage profit moves out of these bounds will arbitrage trading be profitable.

SSF's contracts on two different shares were selected and were evaluated over a period of four years to determine whether mispricing exists. Only put options on the SSF for the same underlying were examined for mispricing in this research. The mispricing was done by calculating the put option prices with the Black model and then comparing the calculated put option prices (which will be termed the "actual" put prices) with the data obtained from the JSE through the application of the put-call parity formula. The calculated put price is therefore compared to the synthetic put (short underlying, long actual put and long bond).

The call mispricing is not covered in this research.

4 Methodology

In order to conduct this research, option contract prices were obtained from the Derivatives Division of the JSE (previously called the South African Futures Exchange (SAFEX)). The study period spanned four years: 2009 to 2012. The data was downloaded from the Johannesburg Stock Exchange's Electronic Derivatives Market statistics (EDMStats) downloadable files. This time period represents fairly recent data which may still be deemed applicable in the current SA derivatives market. Daily put and call closing prices as well as the closing prices of the underlying futures contract were used. The put and call prices were on options with the same expiration dates as well as the same exercise prices. The exercise price was the price of the underlying SSF. Only American-style option prices were calculated. American options can be exercised at any time before expiration date. The options used for valuation were options that had already expired.

The option contract data were acquired for a share that was deemed relatively illiquid and one that was deemed relatively liquid.. The shares were picked based on their trading volumes. This allowed for a comparison of the relationship between put and call prices and the effect trading volume may have on prices and any mispricing, should it exist. The liquid share of choice was BHP Billiton (BILQ) and the illiquid share of choice was Sasol (S0LQ). Once this data had been collected, the futures contracts were valued using the following formula obtained from the JSE website (JSE 2012b).

The basic equation to price SSF contracts is the cost-of-carry pricing model:

- SSF price = Share price (spot) + Interest - Dividends
- Long position bid-offer equations (Standard Bank, 2006b: 21)

[24] GARCH or Generalised autoregressive conditional heteroskedasticity model.

$$SSF_{Bid} = S_{Bid} * (1 - c) * (1 + i)^{(t1/365)} - div * (1 + i)^{(t2/365)}$$
$$SSF_{Offer} = S_{Offer} * (1 - c) * (1 + i)^{(t1/365)} - div * (1 + i)^{(t2/365)} \quad (1)$$

Where:
SSF_{Bid} = SSF bid price
SSF_{Offer} = SSF offer price
S_{Bid} = bid price of underlying
S_{Offer} = offer price of underlying
r = risk-free interest rate
div = underlying asset's projected/expected dividend
t1 = number of days to expiry of particular SSF
t2 = number of days between the dividend date and SSF expiry date
c = commission

The call options and put options may be valued using the Black model.

$$c = e^{-rT}\left[f_0(T)N(d_1) - XN(d_2)\right]$$
$$p = e^{-rT}\left\{X\left[1 - N(d_2)\right] - f_0T\left[1 - N(d_1)\right]\right\}$$
$$d_1 = \frac{\ln(S_0/X) + \left[r + (\sigma^2/2)T\right]}{\sigma\sqrt{T}} \quad (2)$$
$$d_2 = d_1 - \sigma\sqrt{T}$$

Where:
c_0 = the call price on valuation date 0
p_0 = put price on valuation date 0
X = the strike price
F_0 = futures price on day 0
N = probability
e = Naperian constant, 2.71828
r = the risk-free interest rate
σ = volatility

The valuation of the options was done with Microsoft Excel. Once all the relevant values had been calculated, the put-call forward parity equation (Equation 3 below) was used to determine whether any mispricing existed for puts.

$$c_0 + \frac{(X - F(0,T))}{(1+R)^T} = p_0 \quad (3)$$

Where:
c_0 = call option price at time 0
p_0 = put option price at time 0
X = the exercise price of the option (futures price)
R = the risk-free interest rate
T = time to expiration
$F_{(0, T)}$ = futures price at time 0 spanning period T

The risk-free interest rate that was used in this study was the Johannesburg Interbank Agreed Rate (Jibar). The 3-month, 6-month, 9-month as well as the 12-month Jibar rates were used for valuation. The rates were acquired from the daily traded data, which were obtained from the EDMStats section of the JSE website. A year day count convention of 365 was used which is in accordance with the market practice in South Africa.

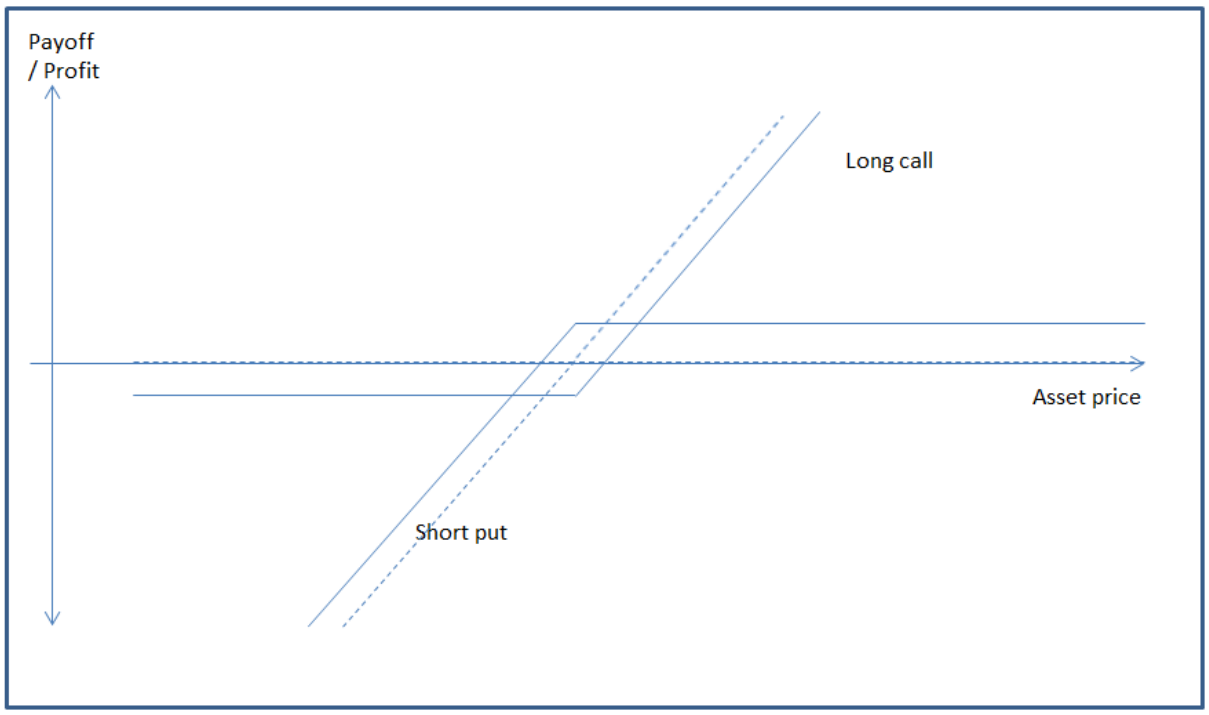
The fair values obtained from equation 3 were then compared to the actual values to determine whether mispricing was prevalent and whether this may mean that further research could unlock the potential to profit from mispricing. A study such as this is important as it may offer opportunities to realise returns above the risk-free rate.

5 Brief literature review

In order for arbitrage opportunities to exist, there needs to be violations of the put-call parity principle which opens up an arbitrage window. Arbitrage opportunities can lead to significant profit opportunities if acted upon in a timely manner with the correct arbitrage strategy. Research has been done over recent years on violations of put-call parity and its effect on the market.

It is important to point out that a special relationship exists between call options and put options. A call options gives the right to the holder (buyer) to buy an underlying (SSF) at a predetermined date at a predetermined price (the strike). One would normally buy a call if we expect that the price of the underlying will increase. Buying the call in this case therefore means that we expect the price of the SSF to increase in the future. The put is basically the opposite of the call as buying a put means that we expect the price of the SSF to fall. The payoff profiles of calls and puts are therefore opposite to each other. See Figure 1 below. Notice that the dotted line (payoff line of call and put) running at a 45 degree angle looks similar to the line of a long underlying. The fixed lines are the profit lines of the call and put. The payoff of a long call and a short put is the same as the payoff of a long leveraged underlying.

Figure 1. Payoff and profit profile of a long call and short put



Stoll (1969) found that the payoff of a long call and long bond is equivalent to a position of a long put option and long stock position. Stoll (1969) found that the main reason for divergences from put-call parity was the introduction of transaction costs into the theorem. He found that transaction costs can cause a divergence of between one or two percent on either side of the parity line (on the fiduciary call side or protective put side).

Cremers and Weinbaum (2010) found in their research that a deviation from put-call parity contains information about future expected returns. They indicated that the degree to which deviations can be predicted is larger when the option liquidity is high and the stock volumes are low. Their research suggested that violations of put-call parity can simply be due to market imperfections, data-related issues and short selling constraints. The research used volatility spreads to measure the deviations from put-call parity. They found that relatively expensive call options outperform relatively expensive put options.

Research conducted by Goh and Allen (1984) indicated that the more in-the-money a call is, the more likely it is to be overvalued relative to the put and the more likely the long hedge will be profitable. This is therefore an example of a violation of the put-call parity principle.

In the discussion above it must be borne in mind that the behavioural aspects of investors decisions must also not be ignored when considering reasons for put-call parity violations. The past financial crisis underlined the importance of the actions of investors and also the effect of news and collective actions of investors.

Each JSE SSF contract is standardised in terms of its size, expiration date and tick movement. Each

contract is based on 100 shares of the underlying equity. The contract will specify the particular underlying share, the price of the contract and the expiration date (Standard Bank, 2006a: 68). SSF investors have three alternative actions that may be taken to close out the position. The contract may be physically settled, meaning that the commitment will be carried out. The actual number of shares will be traded between the two counterparties. Next the contract can be settled in cash, meaning cash will change hands at expiration and no physical delivery will take place. The alternative available to investors is that the contract may be rolled over to the next expiration date. All JSE SSF contracts are standardised. They expire on the third Thursday of March, June, September and December.

Table 1. Contract specifications of single-stock futures

Code	The three-letter stock code followed by a Q (e.g. SOLQ)
Underlying	In this case shares
Contract size	100 times the futures price
Contract months	March, June, September, December
Quotations	Price per underlying share to two decimals
Minimum price movement	R1 per contract (R0, 01 of the share price)
Initial margin	Approximately 10% of contract value
Settlement method	Physically and cash settled
Clearinghouse fees	R0, 30 per futures contract R0, 15 per option contract
Commissions	15-40 basis points to enter or exit a position
Brokerage	Fixed amount plus VAT per trade
Dividends	Reflected in the price of the futures contract
Corporate Events	SSF contracts will adjust to reflect changes in the underlying shares
Options on SSF contracts	Each option is on one futures contract
Strike Price intervals	R5, 00 in the futures price

Source: JSE (2012c); Nedbank (2012:9)

SSFs have hedgers and speculators as users. Hedgers seek to reduce risk by protecting an existing share portfolio against possible adverse price movements or locking in future anticipated purchase prices of shares. Speculators use SSFs in the hope of making a profit based on the short-term movements in the underlying share price, closing out the position before expiry date.

Futures allow the investor an opportunity to use gearing. In the South African market an initial investment of about 10% of the underlying value will give the investor the full exposure to price movements (Harris, S. 2005: 75). According to Harris (2005) this has proved very popular for retail investors as futures trading require a low initial capital outlay. The use of gearing makes possible substantially increased returns, and conversely, increased losses. Trading futures is a very popular strategy as it gives the investor a lot of exposure to price movements with a small commitment of capital with less impact on company cash flow. The use of leverage also enables investors to gain exposure to high-value shares otherwise not possible. By initiating a leveraged position in an SSF contract, it will mean that cash is free to invest in other parts of the market (De Beer, 2008:27). The transactions costs for SSFs are lower than the costs of trading in the underlying securities. There are also no uncertified securities taxes (UST) or STRATE[25] costs (Nedbank, 2012:3).

SSFs also allow investors to hedge against changes in index compositions, both when a share is added to an index and when a share is demoted from an index. Managers who follow index compositions normally include the correct weights of shares involved in the index they are trying to replicate. This may causes distortions in the prices of the securities. SSFs allow the managers to gradually increase (longing SSF's) or decrease (shorting SSF's) exposure to stocks of an index being replicated. SSFs allow managers to move out of the security in an orderly fashion even when there are liquidity problems (JSE, 2012b).

SSFs can also be used to reduce the risk of an existing portfolio without actually trading the shares.

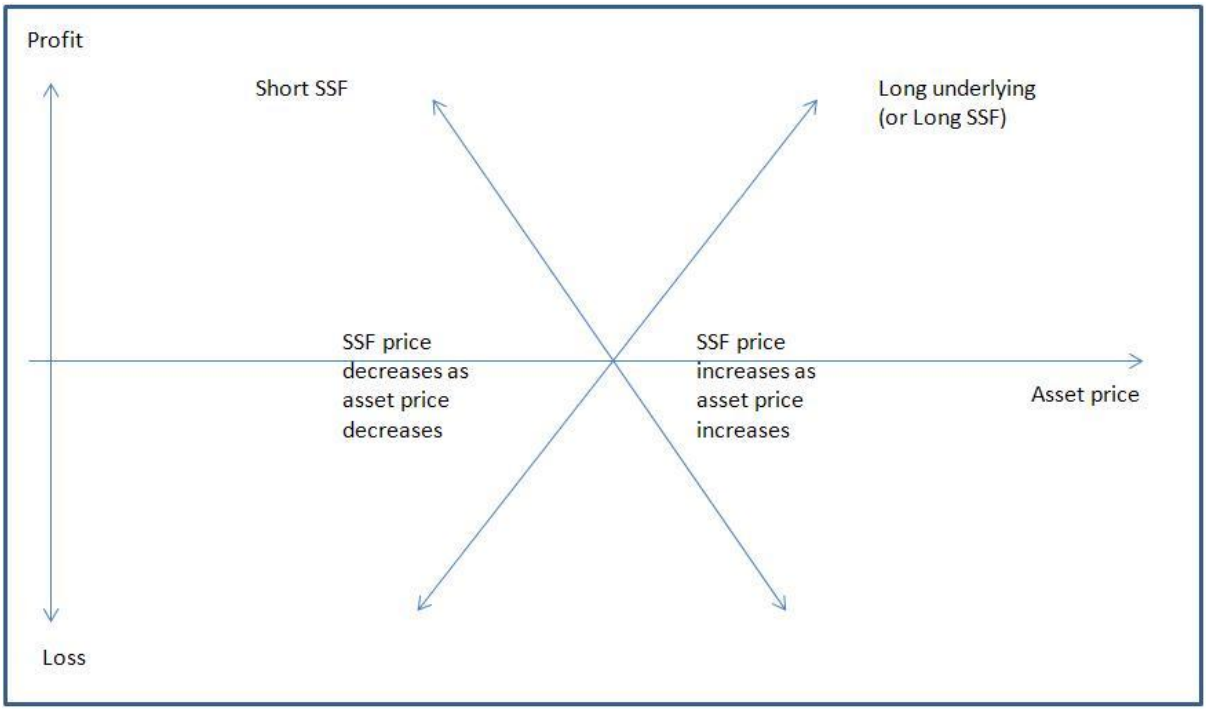
[25] Share Transaction Totally Electronic – the electronic settlement of share transactions and recording of ownership

By shorting a SSF, the investor can protect the value of a portfolio, without having to sell any shares (Standard Bank, 2006b). Because SSF contracts do not give the holder any shareholder rights (voting rights and dividends) this feature is very attractive if there is a temporary decline in the market. The use of SSFs in a portfolio setting can also lead to significant tax savings. If the objective is to reduce equity exposure but selling the stock will create significant tax liability, the use of SSFs will help achieve this. The shares do not actually have to be sold therefore avoiding the tax liability (Mitchell, 2003:72).

Traders of SSFs contracts do not receive any shareholders rights. Therefore, investors do not have any voting rights that could prove important when voting on corporate action events. Dividends are taken into account in the pricing of the SSF but holders of SSFs do not receive any dividends. This has negative implications for investors that dependent on the cash flow that dividends provide.

The risk profile of a SSF contract will be the same as the underlying share, i.e. symmetric. If the price of the underlying share increases then the value of the SSF will increase. Figure 2 shows the relationship between a long underlying and short SSF position. The risk profile when the investor shorts a SSF contract is the opposite of the long share position or a long SSF. The net payoff of combining a long and a short position is 0 (ignoring transaction fees) and it termed the perfect or theoretical hedge.

Figure 2. Risk profile of a long and short SSF contract



Source: Standard Bank, 2006b: 12 (adjusted)

6 Pricing

The variables used to price a SSF' contracts is the underlying share price, interest rate, dividends and commission. The spot price of the underlying share is the main determinant of the futures price and is used in the calculation of the interest and commission components. The interest is calculated on the value of the underlying share for the remaining period of the contract. As mentioned before, the holder of a SSF contract does not receive ordinary dividends, thus the bid and offer prices are adjusted to reflect this[26]. Commission is charged as a percentage of the underlying share value (Standard Bank, 2006b: 17-18).

The basic equation used to price SSF contracts is the cost-of-carry model:

SSF price = Share price (spot) + Interest - Dividends

Long position bid-offer equations (Standard Bank, 2006b: 21)

[26] Special dividends are managed by the Johannesburg Stock Exchange (JSE) by means of an adjustment to the quantity of the holder's SSF position based on a factor of the special dividend.

$$SSF_{Bid} = S_{Bid} * (1 - c) * (1 + i)^{(t1/365)} - div * (1 + i)^{(t2/365)}$$
$$SSF_{Offer} = S_{Offer} * (1 - c) * (1 + i)^{(t1/365)} - div * (1 + i)^{(t2/365)} \quad (4)$$

Where:

SSF_{Bid} = SSF bid price
SSF_{Offer} = SSF offer price
S_{Bid} = bid price of underlying
S_{Offer} = offer price of underlying
r = risk-free interest rate
div = underlying asset's projected dividend
t1 = number of days to expiry of particular SSF
t2 = number of days between the dividend date and SSF expiry date
c = commission

If the share generates a very large dividend, the SSF will be priced at a discount to compensate the investor for not receiving the dividend. The estimation of dividends, when pricing SSFs, is done by looking at the share's dividend history to create an estimate of future dividends (Standard bank, 2006b: 18). If, at a later date the announced dividend is different to the estimated amount, there will be an adjustment to the fair value of the SSF.

Other issues such as market trends also affect the pricing of SSFs which may cause the futures price to diverge from fair value (JSE, 2012b). Wasendorf and Thompson (2004: 44-45) also indicated that the choice of the interest rate, the timing and uncertainty of dividends and the compounding method (continuous or discreet) all complicate the pricing of SSFs.

7 Analysis

The first step in the valuation process in this research was to value the future contracts over the study period. Table 2 below shows a short extract of the futures valuations. The same method was consistently applied for each month during the four year period.

Table 2. Calculation of the BHP Billiton futures contracts prices

Date traded	Contract code	Expiry date	Strike price	Spot price	Volatility (%)	JIBAR rate (%)	Day count	Carry cost	Futures price (FVF)
3/1/2012	BILQ	15/3/2012	N/a	248.75	37.25	5,60	72	2.747836	251.4978
3/1/2012	BILQ	21/6/2012	N/a	248.75	37.25	5,86	168	6.709299	255.4593
4/1/2012	BILQ	15/3/2012	N/a	252.00	37.25	5,60	71	2.745074	254.7451
4/1/2012	BILQ	21/6/2012	N/a	252.00	37.25	5,86	167	6.756500	258.7565
30/1/2012	BILQ	15/3/2012	N/a	263.90	37.25	5,60	45	1.82195	265.7220
30/1/2012	BILQ	21/6/2012	N/a	263.75	37.25	5,86	141	5.973973	269.8740
31/1/2012	BILQ	15/3/2012	N/a	248.75	37.25	5,60	45	1.822961	265.8630
31/1/2012	BILQ	21/6/2012	N/a	248.75	37.25	5,86	141	5.977142	270.0171

The Jibar rate and the year day count used were extracted from the daily trading data obtained from the JSE. Firstly, the finance costs were calculated using the above equation. The futures prices were then calculated by adding the pro rata calculated finance costs to the spot prices.

After the future contract prices were calculated, the put option prices were calculated using the Black model.

The next step involved determining the parity put prices using the put-call forward parity theorem. The calculated futures prices were used as inputs to determine the put-call parity prices. The valuation involved identifying put option contracts with the same exercise prices as well as the same time to expiration.

8 Put mispricing evidence

A number of different scenarios were considered in order to determine whether the mispricing was evident during different stages of the option contract term. This was done for both Sasol contracts as well as the BHP Billiton contracts. Specific option contracts were isolated and evaluated. Options that were in-the-money and out-of-the money were evaluated as well as option contracts that were very close to expiration. Table 3 illustrates a BHP Billiton put option that was traded during March 2009 and expired on June 18, 2009. As can be seen the put options were in-the-money as the spot price was below the strike price for the majority of the month.

Table 3. Extract of BHP Billiton put option valuations using put-call forward parity (strike price R178, 20)

Date	Put Expiry	Futures Price	Fraction of year	Jibar Rate	Strike price	Spot price	Put price	Parity put price	Mis-pricing
2009/03/02	2009/06/18	178.89	0.2904	9.33%	178.82	155.75	34.53	34.79	-0.2585
2009/03/03	2009/06/18	191.58	0.2877	9.33%	178.82	155.40	35.30	35.56	-0.2519
2009/03/04	2009/06/18	200.25	0.2849	9.33%	178.82	165.01	30.30	30.55	-0.2481
2009/03/05	2009/06/18		0.2822	9.33%	178.82	166.40	29.54	29.79	-0.2456
2009/03/06	2009/06/18		0.2795	9.33%	178.82	174.50	26.18	26.39	-0.2093
2009/03/13	2009/06/18		0.2603	9.33%	178.82	168.49	19.81	18.23	1.5868
2009/03/16	2009/06/18		0.2521	9.33%	178.82	179.00	21.90	20.00	1.8958
2009/03/17	2009/06/18		0.2493	9.33%	178.82	184.05	22.75	22.08	0.6674
2009/03/18	2009/06/18		0.2466	10.22%	178.82	155.75	10.83	23.31	-12.4836
2009/03/23	2009/06/18		0.2329	10.22%	178.82	177.90	9.44	13.20	-3.7543
2009/03/24	2009/06/18		0.2301	10.22%	178.82	197.33	10.36	15.85	-5.4868

If it is assumed that the parity put price is fair, then the actual put was undervalued the first five days. As the spot price approached the strike price, the degree of mispricing stayed fairly constant being either over- or undervalued. The last two days were substantially different as the parity put was much higher than the actual put. The spot increased to above the strike causing material under valuation of the actual put compared to the parity put. This may be due to positive sentiment in the market pushing the actual put to below its parity value. Figure 3 graphically illustrates the mispricing for the above mentioned put option for different expiry dates. Table 4 and Figure 4 illustrates the same for a put expiring 15 December 2010.

Figure 3. BHP Billiton put option with a strike price of R178,82 expiring 18 June 2009

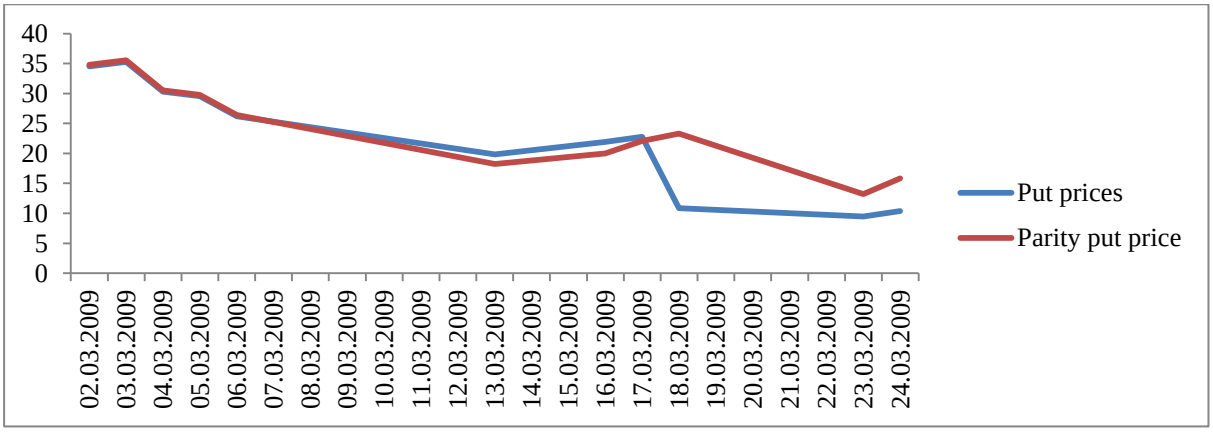


Table 4. Extract of BHP Billiton put option valuations using put-call forward parity (strike price R180)

Date	Put expiry	Futures Price	Fraction of year	Jibar rate	Strike price	Spot price	Put price	Parity put price	Mis-pricing
2010/10/01	2010/12/15	228.66	0.2027	6.33%	180.00	225.90	0.02	0.67	-0.64
2010/10/04	2010/12/15	242.27	0.1945	6.33%	180.00	225.61	0.02	0.54	-0.52
2010/10/05	2010/12/15	247.85	0.1918	6.33%	180.00	228.00	0.01	0.50	-0.49
2010/10/06	2010/12/15	249.56	0.1890	6.33%	180.00	232.26	0.01	0.30	-0.29
2010/10/07	2010/12/15		0.1863	6.33%	180.00	232.90	0.01	0.35	-0.35
2010/10/08	2010/12/15		0.1836	6.33%	180.00	236.10	0.00	0.16	-0.16
2010/10/11	2010/12/15		0.1753	6.33%	180.00	235.60	0.00	0.24	-0.24
2010/10/12	2010/12/15		0.1726	6.33%	180.00	232.28	0.00	0.21	-0.21
2010/10/19	2010/12/15		0.1534	6.33%	180.00	234.35	0.00	0.19	-0.19
2010/10/20	2010/12/15		0.1507	6.33%	180.00	239.10	0.00	0.12	-0.12
2010/10/25	2010/12/15		0.1370	6.33%	180.00	243.90	0.00	4.90	-4.90

Figure 4. BHP Billiton put option with a strike price of R180 expiring 15 December 2010

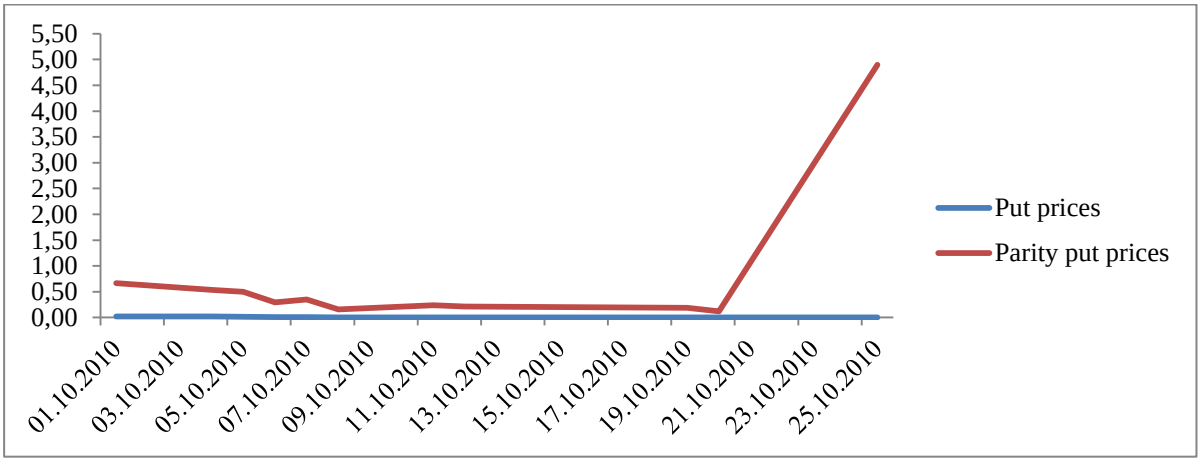


Table 5 illustrates the mispricing present for a Sasol put option that was also in-the money during the month of January 2009. This specific option contract was due to expire on March 19, 2009. The actual put was undervalued during the whole evaluation period. It is important to note that the spot price in Table 5 is more subject to change which may explain the relative material underpricing of the actual put. News events over that period may have had a material influence on the spot price.

Table 5. Extract of Sasol put option valuation using put-call forward parity (Strike price R332, 20)

Date	Put expiry	Futures price	Fraction of year	Jibar rate	Strike price	Spot price	Put price	Parity put price	Mis-pricing
2009/01/02	2009/03/19	299.14	0.2110	12.10%	332.20	292.10	53.9259	57.1989	-3.2730
2009/01/05	2009/03/19	370.58	0.2027	12.10%	332.20	315.00	41.8189	44.7199	-2.9009
2009/01/06	2009/03/19	355.62	0.2000	12.10%	332.20	325.00	37.2204	39.9575	-2.7371
2009/01/07	2009/03/19	335.39	0.1973	12.10%	332.20	312.50	42.5790	45.5069	-2.9279
2009/01/16	2009/03/19		0.1726	12.10%	332.20	290.00	57.7105	60.9257	-3.2152
2009/01/27	2009/03/19		0.1425	12.10%	332.20	290.00	55.6060	56.3586	-0.7526
2009/01/28	2009/03/19		0.1397	12.10%	332.20	278.00	62.9401	64.0731	-1.1330
2009/01/29	2009/03/19		0.1370	12.10%	332.20	275.60	65.8260	89.1358	-23.3098
2009/01/30	2009/03/19		0.1342	12.10%	332.20	272.01	8.7366	57.5160	-48.7795

Figure 5. Sasol put option with a strike price of R332,20 expiring 19 March 2009

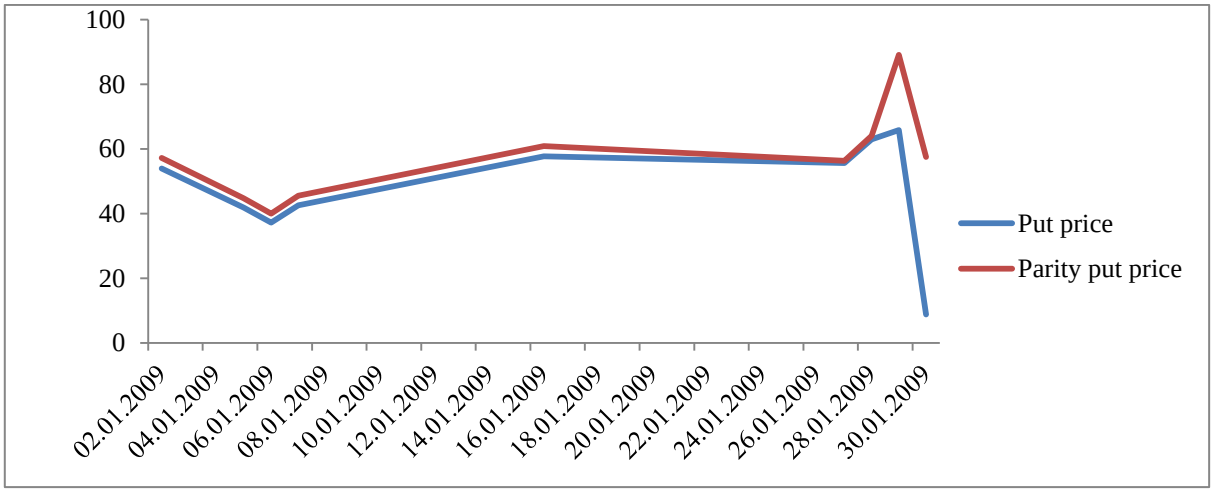
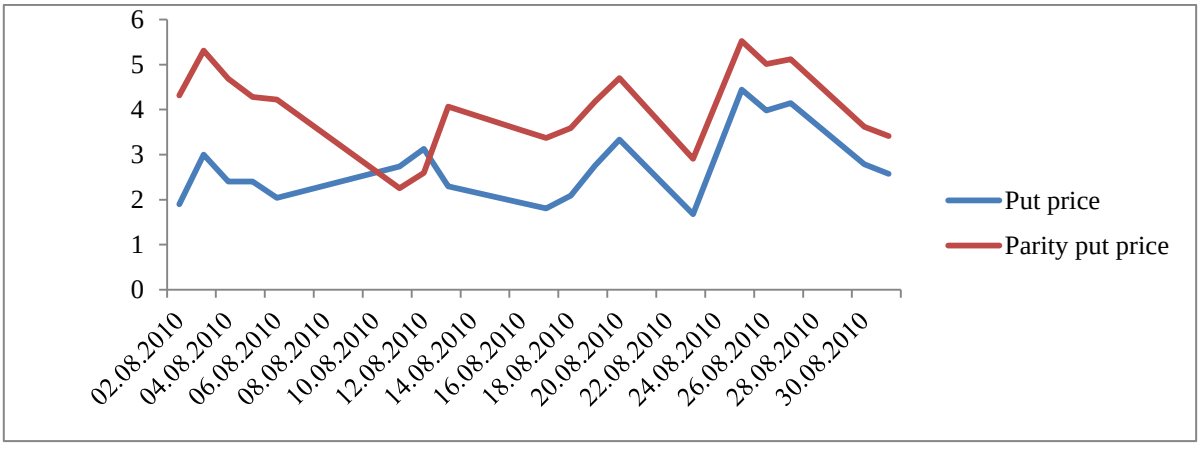


Table 6 below shows an extract of the Sasol put option prices. The put option was fairly close to expiry and was close to being in-the-money. This indicates that the further away the option contract was from being in-the-money the smaller the mispricing that occurred which is logical due to the relatively low option price. Figure 6 gives a graphical representation of Table 6. Put options are said to be out-of-the-money when the spot price of the underlying is above the strike price and therefore will not be exercised. Table 6 illustrates a Sasol put option that was out-of-the-money during August of 2010. Note that the overpricing decreased as the spot price moved towards the strike price. As shown below, the actual put prices were lower overall relative to the parity put prices obtained using the put-call forward parity theorem. Figure 6 graphically demonstrates the mispricing.

Table 6. Extract of Sasol put option valuation using put-call forward parity (Strike price R270)

Date	Put expiry	Futures price	Fraction of year	Jibar rate	Strike price	Spot Price	Put price	Parity put price	Mis-pricing
2010/08/02	16/09/2010	300.08	0.1205	6.81%	270.00	297.50	1.8949	4.3187	-2.4238
2010/08/03	16/09/2010	292.46	0.1178	6.81%	270.00	290.00	2.9998	5.3107	-2.3110
2010/08/04	16/09/2010	295.43	0.1151	6.81%	270.00	293.00	2.4054	4.6855	-2.2801
2010/08/05	16/09/2010	297.39	0.1123	6.81%	270.00	295.00	2.4054	4.2791	-1.8737
2010/08/06	16/09/2010	296.84	0.1096	6.81%	270.00	294.50	2.0384	4.2244	-2.1861
2010/08/11	16/09/2010	292.47	0.0959	6.81%	270.00	288.00	2.7354	2.2502	0.4853
2010/08/12	16/09/2010	290.03	0.0932	6.81%	270.00	285.65	3.1265	2.5931	0.5334
2010/08/13	16/09/2010	287.61	0.0904	6.81%	270.00	289.50	2.2941	4.0663	-1.7722
2010/08/16	16/09/2010	291.43	0.0822	6.81%	270.00	290.25	1.9233	3.5389	-1.6156
2010/08/17	16/09/2010	292.02	0.0795	6.81%	270.00	290.50	1.8030	3.3662	-1.5632
2010/08/18	16/09/2010	292.22	0.0767	6.81%	270.00	288.15	2.0904	3.5880	-1.4976
2010/08/19	16/09/2010	289.80	0.0740	6.81%	270.00	284.45	2.7567	4.1829	-1.4263
2010/08/20	16/09/2010	286.03	0.0712	6.81%	270.00	281.50	3.3374	4.6972	-1.3598
2010/08/23	16/09/2010	283.01	0.0630	6.81%	270.00	288.10	1.6788	2.9093	-1.2305
2010/08/25	16/09/2010	289.49	0.0575	6.81%	270.00	277.05	4.4401	5.5221	-1.0820
2010/08/26	16/09/2010	285.96	0.0548	6.81%	270.00	278.02	3.9786	5.0126	-1.0340
2010/08/27	16/09/2010	278.29	0.0521	6.81%	270.00	277.00	4.1414	5.1203	-0.9789
2010/08/30	16/09/2010	279.21	0.0438	6.81%	270.00	280.00	2.7846	3.6178	-0.8332
2010/08/31	16/09/2010	278.12	0.0438	6.81%	270.00	280.85	2.5741	3.4098	-0.8357

Figure 6. Sasol put option with a strike price of R270 expiring 16 September 2010

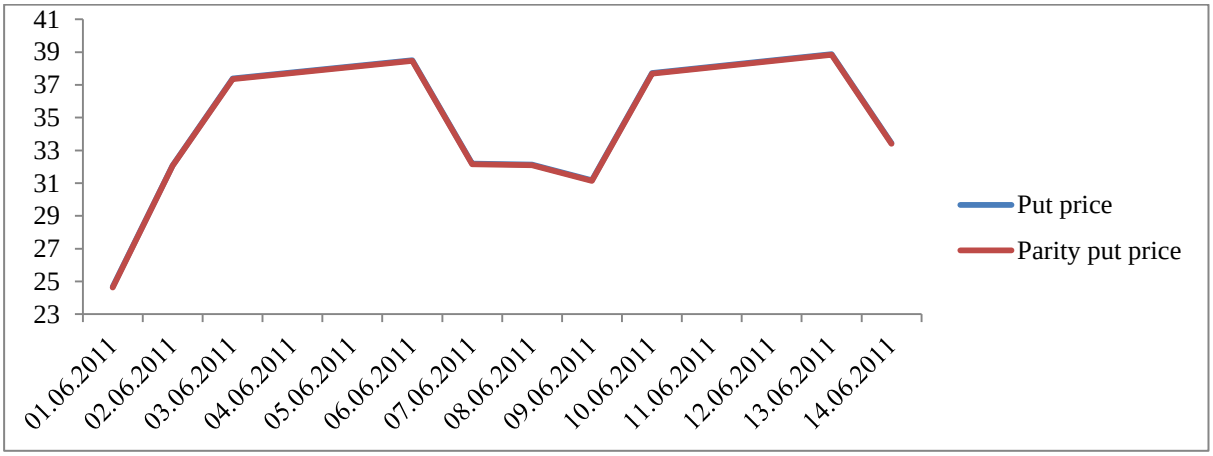


The last aspect looked at in identifying mispricing was to consider option contracts that were traded during the months that they were due to expire. A Sasol put option and a BHP Billiton put option was considered to accomplish this. The Sasol contract evaluated was trading in-the money during the evaluation period and the BHP Billiton put option was trading out-of-the-money during the evaluation period. Table 9 illustrates the pricing of the Sasol contract during June 2011. The contract's expiration date was June 15 2011. Figure 7 graphically illustrates the results as depicted in Table 7.

Table 7. Extract of Sasol put option valuation using put-call forward parity (strike price R385)

Date	Put expiry	Futures price	Fraction of year	Jibar rate	Strike price	Spot price	Put price	Parity put price	Mis-pricing
01.06.2011	15.06.2011	366,78	0,0384	5,73%	385,00	360,60	24,6645	24,6337	0,0308
02.06.2011	15.06.2011	352,88	0,0356	5,73%	385,00	352,50	32,0801	32,0487	0,0314
03.06.2011	15.06.2011	363,92	0,0329	5,73%	385,00	347,03	37,3834	37,3510	0,0323
06.06.2011	15.06.2011	361,89	0,0247	5,73%	385,00	346,00	38,4940	38,4566	0,0374
07.06.2011	15.06.2011	354,42	0,0219	5,73%	385,00	352,40	32,1965	32,1563	0,0402
08.06.2011	15.06.2011	362,53	0,0192	5,73%	385,00	352,50	32,1260	32,0841	0,0419
09.06.2011	15.06.2011		0,0164	5,73%	385,00	353,50	31,1706	31,1268	0,0438
10.06.2011	15.06.2011		0,0137	5,73%	385,00	346,98	37,7197	37,6751	0,0445
13.06.2011	15.06.2011		0,0055	5,73%	385,00	346,00	38,8791	38,8296	0,0495

Figure 7. Sasol put option with a strike price of R385 expiring 15 June 2011



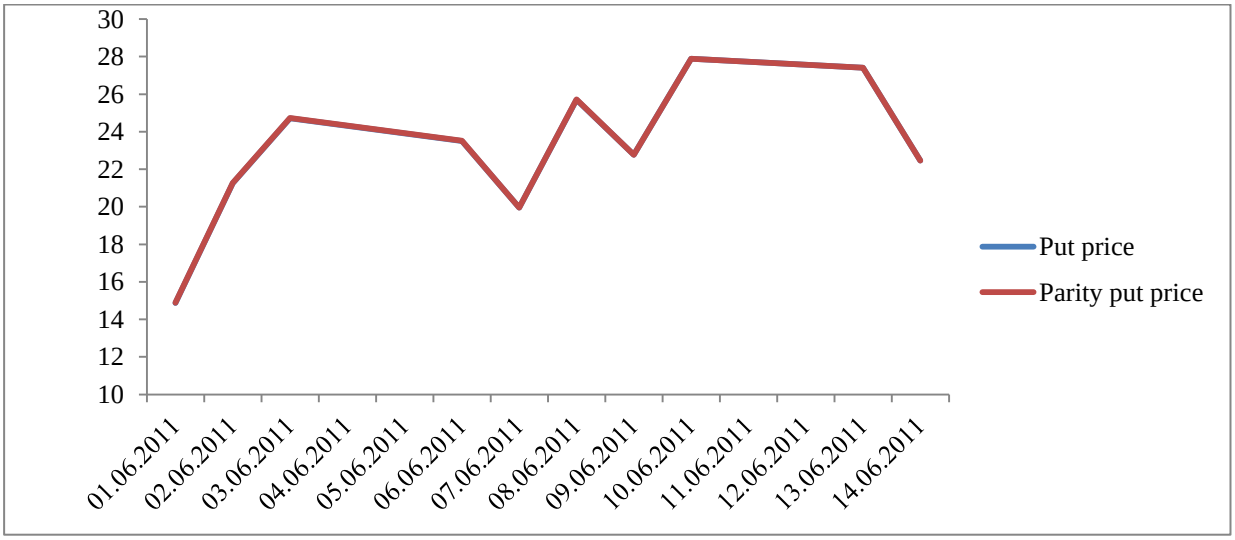
The Sasol parity put options were lower compared to the actual option prices. Although mispricing was present, the level of mispricing was very small indicating that arbitrage trading strategies will not produce net profit (if transaction fees are taken into account). The mispricing found was small indicating that as the option contracts neared their expiration dates the degree of mispricing decreased as can be seen in Figure 10.

The same analysis with the BHP Billiton contracts displayed results opposite to that of the Sasol contracts. Table 8 illustrates the BHP Billiton results. The BHP Billiton parity put option was more expensive, even though the time to expiration of the two contracts was the same. Although mispricing was present, the relative degree of mispricing was very low, making arbitrage trading strategies very difficult to implement and as the contract neared expiration the relative degree of mispricing decreased. Figure 8 illustrates this graphically.

Table 8. Extract of BHP Billiton put option valuation using put-call forward parity (Strike price R280)

Date	Put expiry	Futures price	Fraction of year	Jibar rate	Strike price	Spot price	Put price	Parity put price	Mis-pricing
01.06.2011	15.06.2011	266,7291	0,0384	5,73%	280,00	360,60	14,8660	14,8836	-0,0176
02.06.2011	15.06.2011	259,2137	0,0356	5,73%	280,00	352,50	21,2450	21,2612	-0,0163
03.06.2011	15.06.2011	255,4674	0,0329	5,73%	280,00	347,03	24,7097	24,7247	-0,0150
06.06.2011	15.06.2011	256,5822	0,0247	5,73%	280,00	346,00	23,5087	23,5199	-0,0112
07.06.2011	15.06.2011	260,2176	0,0219	5,73%	280,00	352,40	19,9522	19,9621	-0,0099
08.06.2011	15.06.2011	254,2816	0,0192	5,73%	280,00	352,50	25,7103	25,7189	-0,0086
09.06.2011	15.06.2011	257,2355	0,0164	5,73%	280,00	353,50	22,7699	22,7773	-0,0074
10.06.2011	15.06.2011	252,0924	0,0137	5,73%	280,00	346,98	27,8822	27,8883	-0,0061
13.06.2011	15.06.2011	252,5771	0,0055	5,73%	280,00	346,00	27,4121	27,4145	-0,0024
14.06.2011	15.06.2011	257,5293	0,0027	5,73%	280,00	351,49	22,4660	22,4672	-0,0012

Figure 8. BHP Billiton put option with a strike price of R280 expiring 15 June 2011



The mispricing of the put options therefore may offer investors an opportunity for arbitrage trading. Because the actual put options were found to be predominantly underpriced, investors could benefit from arbitrage trading by longing the cheaper actual put option and shorting the synthetic put option. The put-call forward parity equation shows that a synthetic put option can be created by being short a call option, long a bond, with the face value of the bond being equal to the present value of the exercise price minus the futures price (Chance, 2003:230) and short the underlying. An investor may execute the following transactions in order to take advantage of the situation where the parity put is more expensive:

- Long the actual put option and short the parity put.

Shorting the following overall position on the same underlying (the synthetic put):

- Short the call option and pay the option premium at the beginning of the trade
- Long the bond and invest the cash from the transaction at the beginning of the trade
- Short the underlying futures contract at the beginning of the trade.

The above mentioned transactions will lead to a risk-free profit (depending on transaction fees) due to the relative mispricing of the put option. In order for the arbitrage profits to be realised, the options need to be on the same underlying, have the same exercise prices as well as the same time to expiration. The existence of transaction costs places an upper and lower bound on the arbitrage profit. The arbitrage profit must exceed either of the these bounds to be profitable.

9 Summary of findings

The results show that arbitrage put option opportunities do present themselves for the chosen shares. The actual put options were found to be underpriced in 5 out of 6 (83%) of the cases considered over the evaluation periods chosen. The mispricing was significant for both the BHP Billiton options with 100% and in the case of Sasol options (66%) of the time. Whether profitable arbitrage opportunities is possible, will depend on the magnitude of the mispricing and the transaction fees payable. Although the frequency of mispricing may seem very significant, it is the degree of mispricing that is important. Large discrepancies may arise on any one date due to market inefficiencies and may disappear the next. Behavioral issues and mean reversion may certainly play a role.

The findings are summarised in Table 9 below.

Table 9. Summary of findings

	Sasol		BHP Billiton	
	Actual put overpriced	Actual put underpriced	Actual put overpriced	Actual put underpriced
Far from expiry and in-the-money	-	Table 3	-	Table 5
Far from expiry and out-of--the-money	-	Table4	-	Table 6
Close to expiry and in-the-money	Table 7	-	-	-
Close to expiry and out-of-the-money	-	-	-	Table 8

The low number of contracts included in this research cannot be used to determine any patterns. However, it does indicate that mispricing may possibly be used by traders to realise additional profits, which was what this research set out to do. A more complete analysis of most SSF's and option contracts over an extended period may help to identify tendencies.

10 Conclusions and recommendations

The research as discussed above reported evidence of put mispricing. Whether a trader will be able to profit from it, depends on the degree of mispricing, the level of transaction fees, the liquidity of the market and general market conditions. More comprehensive research may identify patterns on which a trading strategy may be based.

The impact of the volatility in the market should be assessed in future studies. This is an important aspect that may underline just how different companies may be. What affects the volatility of the one, may not affect the volatility of the other. This study was conducted during a period of relatively high volatility in the South African derivatives market. This led to mispricing as the put-call parity inequality does not always hold during highly volatile periods. It is recommended that during the valuation process, volatility over different periods is tested. This will allow for a comparison of the extent of mispricing during highly volatile periods and less volatile periods.

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