IMPROVEMENT OF OPERATIONAL RISK MEASUREMENT UNDER THE SOLVENCY II FRAMEWORK

D. Stepchenko*, G. Pettere**, I.Voronova*

Abstract

Operational risk is one of the core risks of every insurance company in accordance to the solvency capital requirement under the Solvency II regime. The target of the research is to investigate the improvement possibilities of the operational risk measurement under Solvency II regime. The authors have prepared the algorithm of the operational risk measurement under Solvency II framework that helps improve the understanding of the operational risk capital requirements. Moreover, the authors have prepared the case study about a practical usage of the suggested algorithm through the example of one non-life insurance company. The authors use, in order to perform the research, such corresponding methods as theoretical and methodological analysis of scientific literature, analytical, statistical and mathematical methods.

Keywords: Modelling, Operational Risk, Skew t-copula, the Solvency II Framework

**Faculty of Engineering Economics and Management, Riga Technical University, Riga, Latvia*
Tel: +371-2214-6683
***Department of Engineering Mathematics, Riga Technical University, Riga, Latvia*
****Faculty of Engineering Economics and Management, Riga Technical University, Riga, Latvia*
Tel: +371-2633-0355

1 Introduction

Insurance in one of the most important areas in every country’s economics therefore it requires more sophisticated and sensitive risk evaluation in order to ensure stability and solvency of an insurance company.

The Solvency II Framework has been under constant development for many ages due to the necessity of new approaches to ensure a more sensitive and sophisticated measurement, management and assessment of risk. In accordance with the Solvency II Directive’s requirements, the insurance companies of the European Union should establish an effective risk assessment system with the aim to ensure policyholders’ interests safety and the ability to prosper within the tough market environment.

The fact is that the Solvency II regime sets a lot of challenges to every insurance company since there is a need to seek for new approaches of risk measurement and their implementation in its processes and organizational structure.

The major problem lies in the fact that the Solvency II Directive’s requirements are still under discussions and for that reason it is difficult to understand how to assess the risk.

The target of this research is to study the improvement possibilities of the operational risk measurement under the Solvency II regime.

The object of this paper is a measurement of operational risk. Therefore, the subject of this paper is the improvement of operational risk measurement using the skew *t*-copula.

In order to achieve the set objective, the authors use a theoretical and methodological analysis of scientific literature**,** as well as statistical and mathematical methods.

The main issue within the process of conducting the research was to interconnect the risk management with the risk measurement in an insurance company with the aim to improve risk assessment.

The article encompasses three main sections. The overview of the suggested improvement of operational risk measurement is presented in Section 2. In Section 3, the authors of the paper introduce the case study of enhancement of operational risk measurement in an insurance company through modelling. The final section summarizes the findings and conclusions of the research and assesses the improvement of operational risk measurement.

2 Improvement of risk measurement in insurance company

The Solvency II Directive is based on the three-pillar approach where each pillar fulfills its own function: quantitative requirements, qualitative and supervision requirements, disclosure requirements that mean prudential re-porting and public disclosure (FAQs, 2007).

Operational risk (OR) is the risk of a loss resulting from inadequate or failed internal processes, people and systems, or from external events. This definition includes legal risk but excludes strategic and reputational risk (Embrechts and Hofert, 2011).

In the Solvency II framework and the Basel II regime, the basic principles and requirements for operational risk assessment in insurance and banking industries are described.

The usage, integration and implementation of the suggested principles as well as the requirements of operational risk assessment are under active discussions in the latest years. Many researchers (Embrechts and Hofert, 2014; Embrechts and Puccetti, 2008; Dutta and Perry, 2006; El-Gamal et al., 2007; Peters et al., 2013; Peters and Shevchenko, 2013; Frachot et al., 2001; Strelkov, 2008) are investigated those issues.

In the latest researches, in order to model operational risk losses depending on covariates to use an extension of the Peaks-over-Threshold method and the block maxima approach to a non-stationary setup that allows the dependence (on covariates) to be parametric, non-parametric, or semi-parametric and can also include interactions (Chavez-Demoulin et al., 2014).

Moreover, most heated discussions are going on in relation to the possibility that the capital, to cover the possible losses of the operational risk, can be directly proportional to the volume of gross profit in banking industry.

Thus, traditionally it is assumed that the amount of the capital, to cover the possible losses of the operational risk, is equal to the sum of capital charges for each type of the incurred unexpected event in insurance. However, the described approach requires an ideal dependence among the occurred events, which is unreasonable and unrealistic in business conditions of insurance industry.

The authors of the article suggest using copulas to model the capital volume to cover the operational risk. In fact, copulas allow to model multivariate probability distribution using one-dimensional parametric dependences. The fact is that copulas are used to describe the dependence between random variables. Actually, the copula's function enables the task of specifying the marginal distribution to be decoupled from the dependence structure of variables.

Consequently, copula's function allows us to exploit univariate techniques at the first step, and secondly, is directly linked to non-parametric dependence measures. This avoids the flaws of linear correlation that have, by now, become well known. (Cherubini et al., 2004)

Many authors have applied the difference copulas approaches to model the capital to cover the risks and other financial processes (Nelsen, 1999; Angela et al., 2009; Cherubini et al., 2004; Kollo and Pettere, 2010; Srelkov, 2009).

To model a capital to cover the operational risk the authors use skew *t*-copula. Skew *t*-copula is constructed from a multivariate skewed distribution, which has the covariance matrix when the number of degrees of freedom is more than 4 (Kollo and Pettere, 2009). Actually, this enables to model distributions with heavier tail area.

Since the operational risk encompasses a number of sub-risks, the authors suggest establishing the risk catalogue to investigate more deeply the nature of risks. Basically, the scope of risks that should be included in the analysis will depend on the purpose and context of the assessment (QIS5, 2010).

The authors of the paper suggest for the statistics for modelling the capital to cover the operational risk to use historical data from loss database. The fact is that loss database introduces all incurred operational risk events with details about the losses within a concrete period.

Loss databases, both internal and external, are important aspects of an operational risk program. An understanding of interconnectivity of different risks is a prerequisite to controlling problems and assessing practices. Firms should strive to understand the causes and related factors relevant to operational risk losses. Comprehensive qualitative information can help managers identify the commonalties among loss events. Seeing these patterns or common threads may allow managers to recognize red flags in their own controls before incidents occur. Quantitative tools further enhance a database by allowing it to be used for benchmarking (IAFE, 2011).

The authors of the paper have prepared the algorithm of measurement of the capital to cover the operational risk (see Figure 1).

The fact is that the authors of the paper have prepared the case study based on the algorithm presented in Figure 1 (see Section 3).

3 Case study: Assessment of operation risk

Due to the nature of operational risk that is less depended on macroeconomical cycles it can be modelled by skew *t*-copula and estimated tail dependence in each situation for modelling distributions with heavier tail area. The main idea of the case study is to approve the possibility of identification of VaR (the acronym standing for Value at Risk) for the operational risk portfolio using simulation technique.

Because of the correlation between different operational risk sub-risks, VaR of them (portfolio) has to be smaller than simply added corresponding VaR of each sub-risk.

The fact is that VaR is a quintile of a distribution and used as a (non-coherent) risk measure (CEA.2007).

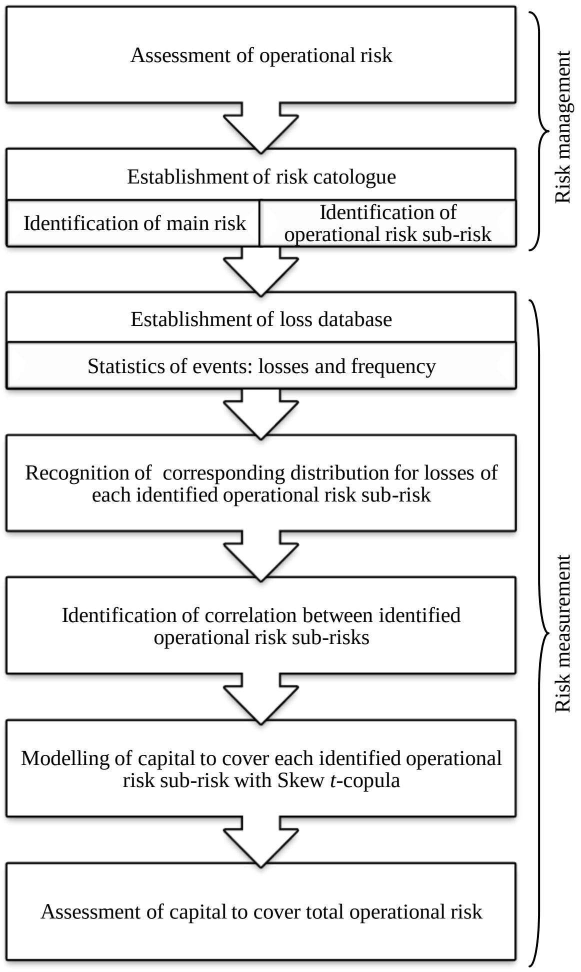
The model created by the authors includes the following operational risk sub-risk:

Legal risk (LR) means the possibility that lawsuits, adverse judgments from courts, or contracts that turn out to be unenforceable, disrupt or adversely affect the operations or condition of an insurer. The result may lead to unplanned additional payments to policyholders or that contracts are settled on an unfavorable basis, e.g. unrecoverable reinsurance (CEA. 2007).

Organizational risk (OR) means possible losses due to unclear organizational structure (unclear processes, unclear responsibilities split between units etc.).

Informational risk (IR) means possible losses due to failures in the IT system.

Figure 1. The algorithm of measurement of the capital to cover the operational

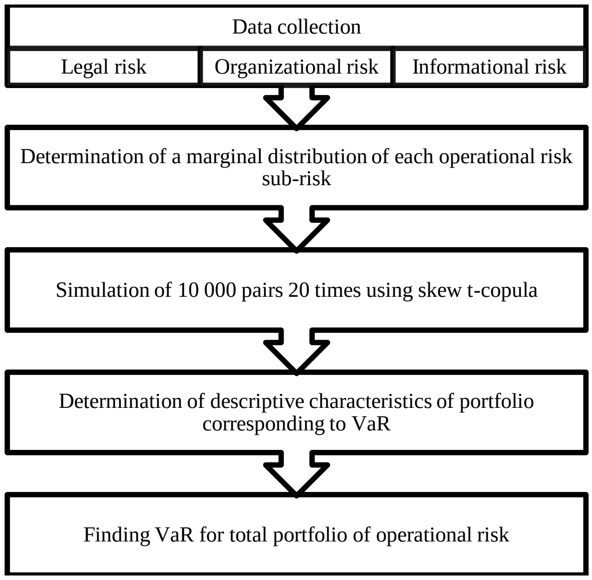


The simulation model performed during case study is based on three risks due to the reason to show the model advantages. In reality the proposed model is possible to use for any number of risks. The historical data was based on recorded data in relation to three operational risk sub-risk from the annual loss database.

Basically, the model is based on several main steps that are in details described in Figure 2.

However, the simulation of 10 000 pairs 20 times in the model is performed using the skew t-copula described in (Kollo and Pettere, 2010, 2011).

Figure 2. The description of model for determination of VaR for an operational risk portfolio



The correlation matrix is following:

$$R = \begin{pmatrix} 1 & -0.143 & 0.357 \\ -0.143 & 1 & -0.118 \\ 0.357 & -0.118 & 1 \end{pmatrix}$$

The legal risk and informational risk (first and third) are positively correlated but the other are negative. Descriptive statistics of the marginal distributions of the mentioned risks are presented in Table 1.

The fact is that, before fitting to marginal distributions, the data was standardized and then the marginal distributions were approximated by Exponential and Gamma distributions.

Table 1. Descriptive Statistics of Data

Risks	LR	OR	IR
Size	12	12	12
Mean	7 564	45 618	5 425
Median	3 700	1 610	960
Standard deviation	11 151	143 207	9 342
Skewness	3	3	2
Kurtosis	9	12	5

The authors of the paper have identified that the legal risk should be obtained by the Exponential distribution, but for the organizational and informational risk, the Gamma distribution should be suitable.

The appropriateness of distributions to each sub-risk was measured by the Kolmogorov test (the 5% critical value equals 0,391).

The testing results are shown in Table 2.

Table 2. Results of Marginal Distributions Tests

Risks	Used distribution	Parameters	
LR	Exponential	λ	1.474
		Test value	0.164
OR	Gamma	α	0.101
		β	3.139
		Test value	0.169
IR	Gamma	α	0.227
		β	2.098
		Test value	0.0957

Based on Table 2, the authors can draw conclusion that all univariate marginal distributions are appropriate to the obtained model distributions.

The obtained marginal distributions were joined into a three-dimensional distribution by the skew *t*-copula.

The parameters $\mathbf{\Sigma}$ and $\boldsymbol{\alpha}$ are estimated from the first two moments (Kollo and Pettere, 2010).

Let $\bar{\mathbf{X}}$ and $\mathbf{S}_{\mathbf{X}}$ denote the sample mean and the sample covariance matrix, respectively. Then the estimates are

$$\hat{\mathbf{\Sigma}} = \frac{\nu-2}{\nu}(\mathbf{S}_{\mathbf{X}} + \bar{\mathbf{X}}\bar{\mathbf{X}}^T)$$

$$\hat{\boldsymbol{\alpha}} = \frac{b(\nu)\cdot\boldsymbol{\beta}}{\sqrt{b^2(\nu) - \bar{\mathbf{X}}^T\hat{\mathbf{\Sigma}}^{-1}\bar{\mathbf{X}}}}$$

where

$$\boldsymbol{\beta} = \frac{1}{b(\nu)}\hat{\mathbf{W}}\hat{\mathbf{\Sigma}}^{-1}\bar{\mathbf{X}}$$

With $\hat{\mathbf{W}} = (\delta_{ij}\sqrt{\hat{\sigma}_{ij}})$, $i, j = 1, ..., p$, where δ_{ij} is the Kronecker delta and

$$b(\nu) = \left[\frac{\nu}{\pi}\right]^{\frac{1}{2}} \cdot \frac{\Gamma(\frac{\nu-1}{2})}{\Gamma(\frac{\nu}{2})}$$

The number of degrees of freedom ν was taken as 4 (four) in order to use the multivariate *t*-distribution with maximally heavy tail area. The $\hat{\mathbf{\Sigma}}$ matrix is following:

$$\hat{\mathbf{\Sigma}} = \begin{pmatrix} 0.730 & 0.037 & 0.340 \\ 0.037 & 0.551 & 0.017 \\ 0.340 & 0.017 & 0.614 \end{pmatrix}$$

However, the calculated values of alfa are following:

$$\boldsymbol{\alpha}^T = (1.551\ \ 0.946\ \ 0.681)$$

In the experiment of simulation triples from the joint 3-variate skew *t*-copula were modelled. The number of replications was 20. The results of simulation are collected in Table 3. On the first line `Real values´ we have the 99.5% *VaR* for each sub-risk using inverse marginal distributions and sum of *VaR* (portfolio) in the current year.

On the next lines, characteristics of 99.5% *VaR* for each sub-risk and portfolio obtained from modelled simulations.

Table 3. 99.5% *VaR* Obtained Using Simulation and its Characteristics

Risks	LR	OR	IR	Sum of *VaR*	Portfolio
99.5% VaR from distributions	40 078	947 292	55 567	1 042 937	
Mean of *99.5% VaR*	40 091	909 123	56 556	1 005 769	935 922
Median	40 034	91 1132	56 821	1 008 493	935 630
Standard deviation	1 005	41 170	2 888	42 721	4 4248
Skewness	0.232	-0.008	-0.399	-0.035	0.178
Coefficient of variation (%)	2.5	4.5	5.1	4.2	4.7

Based on Table 3, it is possible to conclude that the portfolio *VaR* obtained in simulation is smaller, and it means that the capital to cover these risks is less by 10.3%.

In order to evaluate the dependence between risks the authors have used tail dependence coefficient (Bortot, 2012).

Let assume that (X_1, X_2) is two dimensional vector with one dimensional marginal distributions $F_1(x)$ and $F_2(x)$. Then the upper tail coefficient is

$$\lambda_U = \lim_{u\to 1}\lambda_U(u)$$

where $\lambda_U(u) = P(F_1(x) > u / F_2(x) > u)$.

Similarly is defined lower tail coefficient

$$\lambda_L = \lim_{u \to 1} \lambda_L(u)$$

where $\lambda_L(u) = P(F_1(x) < u / F_2(x) < u)$.

$\lambda_U = \lambda_L = \lambda$ for symmetrical elliptical distribution, but for normal distributions λ equals zero. For two dimensional *t*-distribution with ν degrees of freedom

$$\lambda = 2T_{1,\nu}\left(-\sqrt{\frac{(\nu+1)\cdot(\rho-1)}{(\rho+1)}}\right)$$

where

$T_{1,\nu}(.)$ - the distribution function of standard *t* - distribution with ν degrees of freedom;

ρ – coefficient of correlation.

It is approved in (Bortot, 2012) that it is sufficient to study the upper tail dependence due to the lower tail dependence coefficient that is determined by the upper one. Let us denote by

$$\alpha_1^* = \frac{\alpha_1 + \alpha_2 \cdot \rho}{\sqrt{1+\alpha_2^2 \cdot (1-\rho^2)}}.$$

and

$$\alpha_2^* = \frac{\alpha_2 + \alpha_1 \cdot \rho}{\sqrt{1+\alpha_1^2 \cdot (1-\rho^2)}}.$$

Assume that $\alpha_1^* \le \alpha_2^*$. Then

$$\lambda_U = \lim_{u \to 1} \frac{P(F_1(x) > u, F_2(x) > u)}{P(F_2(x) > u)} = \lim_{x \to \infty} \frac{P(F_1(x) > F_2(x), X_2 > x)}{P(X_2 > x)}$$
$$\ge \lim_{x \to \infty} \frac{P(X_1 > x, X_2 > x)}{P(X_2 > x)}$$
$$= \lim_{x \to \infty} \frac{2 \cdot P(Y_1 > x, Y_2 > x) \cdot T_{1,\nu+2}\left((\alpha_1 + \alpha_2)\cdot\sqrt{\frac{(\nu+2)\cdot(\rho+1)}{2}}\right)}{2 \cdot T_{1,\nu+1}(\alpha_2^* \cdot \sqrt{\nu+1}) \cdot (1 - T_{1,\nu}(x))}$$
$$= \lambda \cdot \frac{T_{1,\nu+2}\left((\alpha_1 + \alpha_2)\cdot\sqrt{\frac{(\nu+2)\cdot(\rho+1)}{2}}\right)}{T_{1,\nu+1}(\alpha_2^* \cdot \sqrt{\nu+1})}$$

In the case of $\alpha_1 = \alpha_2 = \alpha$ tail dependence coefficient can be calculated using formula:

$$\lambda_U = \lambda \cdot \frac{T_{1,\nu+2}\left(2\cdot\alpha\cdot\sqrt{\frac{(\nu+2)\cdot(\rho+1)}{2}}\right)}{T_{1,\nu+1}(\alpha^* \cdot \sqrt{\nu+1})},$$

where

$$\alpha^* = \frac{\alpha \cdot (1+\rho)}{\sqrt{1+\alpha^2 \cdot (1-\rho^2)}}.$$

The fact is that the difference of tail dependencies between *t*-distribution and skew *t*-distribution is determined by the ratio of univariate distribution functions of the *t*-distribution. It is shown in (Kollo, Pettere, Valge, 2015).that for the equal values of α the difference in tail dependence is not large.

The tail dependence coefficient calculations for given risks is presented in Table 4.

Table 4. Results of Tail Dependence Coefficient for the Risk

Risks	Legal risk – Organizational risk	Organizational risk – Informational risk	Legal risk – Informational risk
λ	0.030612	0.032737	0.099198
α_1^*	1.033629	0.716906	1.513446
α_2^*	0.395529	0.414779	0.700886
$T_{1,\nu+2}$	0.996458	0.980886	0.997954
$T_{1,\nu+1}$	0.791520	0.801877	0.911078
λ_U	**0,038537**	**0.040046**	**0.108657**

The measurement of the operational risk based on copulas allow modelling multivariate probability distribution using one-dimensional parametric dependencies.

The measurement of the operational risk is based on the skew *t*-copula since it allows modelling distributions with heavier tail area and correlation between marginal distributions. However, there are discovered several valuable advantages of skew *t*-copula usage in operational risk measurement:

skew *t*-copula has a very simple and clear simulation rules;

using copula is possible to simulate portfolio of risks keeping correlation between them;

calculated necessary capital for portfolio is less than sum of capitals needed for each risk;

by choosing degrees of freedom is possible to find appropriate skewness of copula for simulation;

another advantage of simulation is the possibility to calculate average measure of necessary characteristic;

further tail dependence can be evaluated between risks.

During the case study, it has been proved that because of the correlation among different sub-risks of the operational risk, their *VaR* (portfolio) is smaller than a simply added corresponding *VaR* of each sub-risk that allows keeping optimal volume of capital to cover the possible losses due to occurrence of the operational risk. Because *VaR* is not coherent risk measure the *VaR* for simulated portfolio will always be less than sum of *VaR* of different risks. Thus, the proposed method would not allow over-reserving and putting gap capital to other needs of an insurance company.

4 Conclusion

The dynamic nature of risk under changing insurance market conditions sets a lot of challenges to every insurance company.

Moreover, the new Solvency II Directive's requirements, which will soon come in force, set a lot of challenges to every insurance company in the countries of the European Union in relation to the establishment of more sensitive and sophisticated risk coverage in order to ensure solvency to ensure the safety of the policyholders.

The fact is that the new regime requirements might create additional problems for an insurer.

The authors of the paper have interconnected the risk management with the risk measurement in an insurance company with the target to improve the operational risk assessment.

Basically, the authors suggest the algorithm of the operational risk evaluation to measure the capital to cover it. The measurement of the operational risk is based on copulas since they allow to model multivariate probability distribution using one-dimensional parametric dependencies.

Furthermore, the authors have prepared the case study in accordance with the suggested algorithm. The main idea of the case study is to approve that because of the correlation between different operational risk sub-risks, *VaR* of them (portfolio) has to be smaller than a simply added corresponding *VaR* of each sub-risk.

The suggested approach of the capital measurement to cover the operational risk will enable every insurance company to control and properly assess the capital required for the operational risk in line with the Solvency II Directive requirements and establish a more sophisticated and sensitive risk assessment in future.

In future, the authors plan to continue the present research on an insurance company's risk assessment.

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