UPSIDE AND DOWNSIDE BETA PORTFOLIO CONSTRUCTION: A DIFFERENT APPROACH TO RISK MEASUREMENT AND PORTFOLIO CONSTRUCTION

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Abstract

Traditional financial measurements of risk are limited to variance-based methodologies. The most common measurement tool is beta. The beta calculation, however, is directionally agnostic and relies on the assumption of a normal distribution. This is a poor metric by which risk is measured, and is incomplete. The ability to break down beta into Upside and Downside beta allows investors the ability to more intelligently build risk into a portfolio. Using three-year trailing betas may also allow investors the ability to benefit from mean reversion and generate excess returns on a risk-adjusted basis.

Keywords: Beta Portfolio Construction, Risk Measurement, Portfolio Construction

1. Introduction

Traditional portfolio construction methods rely too heavily on illegitimate assumptions regarding risk. Mean-variance and other normal-distribution approaches underestimate tail risk and subject investors to higher probabilities of large drawdowns and increased portfolio volatility. This is in part due to the equal weighting of both left and right tail risks using a normal distribution. Historically, stock returns follow asymmetric distributions (Estrada, 2007) and the beta calculation used for risk measurement can be adjusted to split risk into two segments: upside and downside risk. From this, portfolios can be constructed more intelligently by allocating funds within a portfolio across securities based on their sensitivity to two separate market environments.

It can be assumed that investors are more sensitive to losses than it appears (Tversky & Kahneman, 1991; Ang, Chen, & Xing, 2006). The goal of this study is not to predict the future market returns, but to create portfolios that are more sensitive to upswings and less sensitive to downswings. If it is possible to isolate securities with these characteristics, portfolios that better capture an investor's risk tolerance and reduce volatility can be created.

2. Traditional Financial Theory of Risk

Traditionally, investment risk is associated with volatility. Mathematically, volatility is measured by variance, an assessment of dispersion around the mean. The more volatile a stock, the greater the probability of abnormal losses and difficulty of forecasting expected returns.

Variance has important implications for portfolio construction using Markowitz's Mean-Variance model (Markowitz, 1952). This approach, while intuitive and simple, breaks down investment options into two choices: the expected return of the stock or portfolio and the volatility, or variance of said stock or portfolio.

This parametric-decision making process leads to the assumptions that the expected return can be calculated using the CAPM approach (Markowitz, 1952), variance is a complete measure of volatility, and that stock returns follow a normal distribution. Additionally, it is assumed that investors are risk-averse and require a 'risk-premium' in the form of excess return for bearing additional risk. Furthermore, investors are assumed to display loss aversion, or higher sensitivity to losses than gains. Original research indicated that the pain of a loss is twice the pleasure of an investment gain (Kahneman, Knetsch & Thaler, 1990; Tversky & Kahneman, 1991). This leads to excessive portfolio turnover (Barber & Odean, 2000) and reduces the additional gains from additional risk that can amount to up approximately 3.5% reduction in annual performance (Barber, Odean & Zheng, 2000).

The CAPM approach (Markowitz, 1952) is a popular theoretical financial tool that hypothesizes the expected return of any stock can be calculated as the risk free rate plus a sensitivity factor known as beta (defined below) times the market risk premium, or the expected return of the market minus the risk-free rate of return. Simply put, stocks should earn at least the risk free rate, as measured by the three month T-bill, plus a multiple of market's gain over the risk free rate.

The multiple is known as the stock's beta, a measure of sensitivity to market movements, or more technically, the systematic risk or the covariance of returns between the stock and the market relative to the variance of the returns of the market. In sum, CAPM theorizes that the expected return on a stock is a function of beta and that higher beta stocks will outperform the market during upswings.

There are several issues with the Mean-Variance framework. First, Mean-Variance assumes investors and portfolio managers can accurately predict the expected return and probability distribution, despite the absence of theoretical and practical validity. In order to make this possible, a normal distribution of return probabilities is assumed.

Market returns rarely follow a normal distribution, and more commonly exhibit abnormal gains and losses resulting in 'fat tails' when graphed. Problematically, the normal distribution creates an equal weighting amongst the tails for both downside and upside risk. Benoit Mandelbrot, the pioneer of fractal geometry, championed a power-law distribution to market returns that more accurately capture what he deemed as wild-randomness (Mandelbrot, 1961). Over the long term, stock prices typically display long periods of relatively minimal volatility and few, large swings, or shocks **(**Damodaran, 2008**)**. Given this, Mandelbrot's approach makes more sense and begs for a better, more complete understanding and use of risk in portfolio construction.

Given this dynamic, traditional risk measures based on normal distributions such as VaR and Mean-Variance portfolio construction clearly understate the probability of abnormal losses and lead to underestimations of risk. Given investor's risk aversion levels, differentiating between upside and downside risk may lead to an improved portfolio construction methodology. Upside risk is defined as a stock's return volatility in periods where the benchmark (for this study, the S&P 500) returns are positive, and downside risk is a stock's return volatility during periods of negative benchmark returns. This is similar to Semi-Variance (Markowitz, 1959) and the Sortino Ratio to measure downside exposure and relative performance. In order to capture these new risks, beta is used.

3. Breaking Down Beta: Upside and Downside Betas

Beta is a measure of sensitivity between two assets. For this study, the two assets will be an individual stock and a benchmark, either the S&P 500 or a custom benchmark, defined later in this study.

The numerator is the covariance between the stock and the market and the denominator is defined as the variance of the market. All beta calculations in this study use trailing 36 months returns. Beta is commonly used to predict expected returns and measure risk. Simply put, if a stock has a beta of 1.25, for every 1% move in the benchmark, the stock will theoretically move 1.25%. Therefore, high beta stocks are commonly labeled as those with historically more volatility, and therefore more perceived risk. The issue with the traditional beta calculation is the equal weighting to both upside and downside variance. A stock with more volatility during market downturns versus one with more volatility during market upswings can theoretically generate similar betas. However, this beta calculation captures both periods of market appreciation and depreciation, so it is directionally agnostic. It is unlikely, however, that a stock would have equivalent sized moves for both upside and downside periods. It would be ideal if a stock had a large positive sensitivity but low negative sensitivity. Rational investors would invest in stocks that are more sensitive to upswings than downswings. Investors are more concerned with downside volatility, and should therefore be able to build a portfolio that contains stocks with less sensitivity to down markets, and stocks with more sensitivity to up markets. This would allow an investor to maximize the capture of market upswings and minimize that of market downturns. In order to complete this, Upside and Downside betas are used.

3.1 Upside Beta

Upside beta is the stock beta measured for periods when the benchmark return is positive. This will allow an investor to understand which stocks have historically generated the highest returns during market upswings. Here, the variance is defined as the variance for periods such that the market return is greater than zero.

3.2 Downside Beta

Similar to Upside beta, except this beta is calculated using periods of negative benchmark return. In order to minimize portfolio risk, investors should allocate a percentage of portfolios to stocks with low downside betas to protect against market downturns. Here, the variance is defined as the variance for periods such that the market return is less than zero. Downside beta is a measure of stock sensitivity to market downswings.

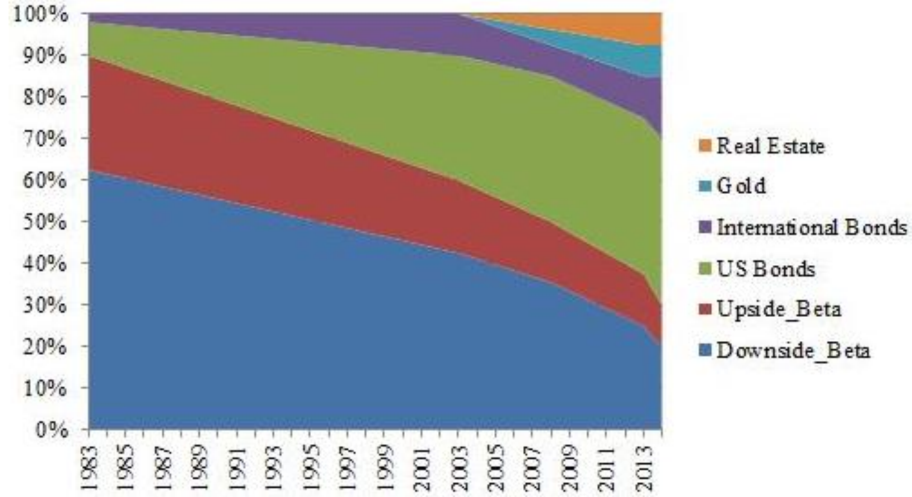
4. Portfolio Construction

To test this theory, an initial population of 2,923 stocks and their month-end prices was gathered using S&P500 and Russell 3000 composites and Morningstar monthly total returns. An initial filter was set to screen out any stocks with an inception date after 12/1/1994. This constraint was used to create eligible pools of stocks with a minimum of twenty years of trading. This reduced to population size to 1,172 potential stocks for portfolio construction.

Next, trailing 36 month Upside and Downside betas were calculated beginning in 1983. Upside periods were defined as months when the S&P500 return was positive and vice versa for downside periods. Portfolios were then created using the available stocks for each year.

Each annual portfolio begins with the eligible equities for the given year, subject to the constraints previously defined. Portfolio construction followed a Target-Date glide path approach (Vanguard's approach to Target Date Funds), as shown in Figure 1.

Figure 1. Target Date Glide-Path Model for Portfolio Construction



Traditionally, Target Date funds invest in US Equities, International Equities, US and International Bonds, and inflationary securities, here defined as Gold and Real Estate. The securities used for the non-equities are shown in Figure 2**.** This provides for a dynamic construction so that the study replicates a typical investor's thirty year portfolio allocation, as opposed to fixed allocations over the course of the portfolio, which is unrealistic. Essentially, modeling the portfolio in this fashion removes any advantages of tactical allocation. This weighting scheme is also unbiased regarding market timing and portfolio rebalancing.

Figure 2. Indices used for Portfolio Construction

Gold	Last London Gold Fix of the Year
Real Estate	FTSE Nareit All REITS total returns
US Bonds	Barclays US Aggregate
International Bonds	Barclays Global Aggregate Unhedged

In constructing these portfolios, substitutions were made for US and International Equities by swapping downside beta stocks for US Equities and upside beta stocks for International Equities. The reason is this: downside beta stocks are those that display minimal sensitivity to down markets, and are therefore less volatile equities. On the other hand, upside beta stocks represent equities with heightened sensitivity to market upswings, and will therefore be more volatile. This relationship, from a volatility standpoint, closely resembles the relationship between domestic and foreign equities. Additionally, research suggests that high-beta stocks generally underperform low-beta stocks during periods when the S&P 500 drops by more than 10% (Grundy & Malkiel, 1996) and it is therefore more favorable to have a higher allocation to low-beta stocks for portfolio protection purposes.

To pick the individual securities that comprise the Upside and Downside portfolios, the stocks were first sorted by smallest downside beta and assigned the required number of stocks to meet the allocation: for each year, the highest quintile was used, and the individual allocations were based on the allocation for the Upside or Downside portfolio in the given year provided in the glide path. Next, the stocks were then sorted by highest upside beta and assigned to the upside allocation. It should be noted that given an overlap, the stock was deferred to the downside allocation, in-line with investor preferences regarding upside and downside volatility.

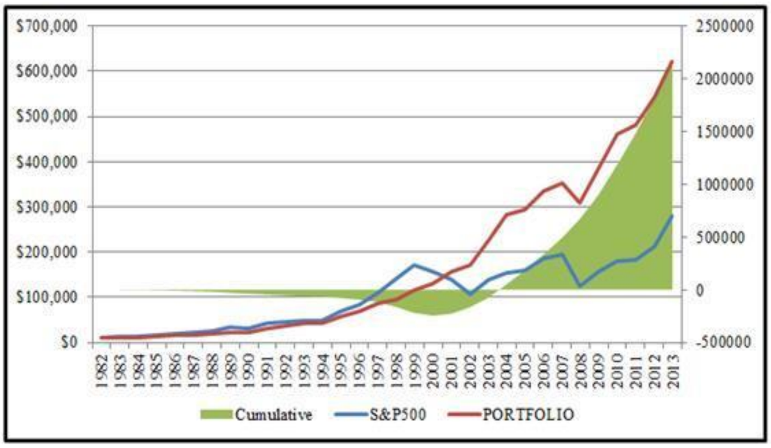
Obviously, as time continues more equities become listed. The upside and downside allocations continue to follow the glide path, but the individual weightings of the securities within the portfolio decrease over time as the number of eligible stocks increases. Additionally, each year, the three year trailing betas are recalculated so the portfolio construction is rebalanced every annum. This means that some stocks will remain in the portfolio, while others will be removed, but only those with at least three years of history will be eligible. For example, in 1983, there are 98 eligible equities, and the Upside allocation is 27.5% and downside allocation is 62.5%

according to the glide path. The top quintiles for each beta calculation are used for each sub portfolio, with the weights within the portfolios based on the allocations. In this example, given quintiles of twenty stocks, the twenty upside stocks would each account for 1.375% of the portfolio with the downside stocks 3.125% allocation to each. This correctly weights the returns in the total portfolio annual return.

5. Results

Overall, the new portfolio generated significant excess returns against both the S&P 500 and the Benchmark.

Figure 3. Portfolio vs. S&P 500

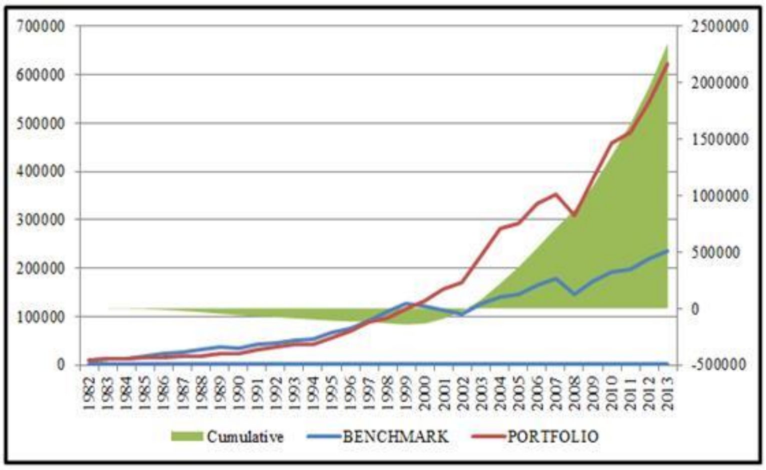


	S&P 500	Portfolio
Ave. Return	12.80%	14.81%
StDev	16.88%	11.27%
Risk Free	5%	5%
Sharpe	0.46	0.87
Best	37.20%	41.92%
Worst	-36.55%	-12.39%

Compared to the S&P 500, the Portfolio generated an average annual return of 14.81% versus 12.80% and standard deviation of 11.27% versus 16.88%, representing increased returns with lower risk.

The Benchmark used the securities listed in Figure 2 except the S&P 500 was used to US Equities and the MSCI EAFE Index was used to International Equities. The Portfolio outperformed the Benchmark in terms of average annual return with a lower standard deviation.

Figure 4. Portfolio vs. Custom Benchmark



	Benchmark	Portfolio
Ave. Return	11.40%	14.81%
StDev	11.94%	11.27%
Risk Free	5%	5%
Sharpe	0.54	0.87
Best	36.65%	41.92%
Worst	-18.34%	-12.39%

A simple regression analysis (Figure 5) was run to determine the level of dependence of the portfolio's return on the S&P 500. The adjusted R-square value of 0.64, combined with a statistically significant F-test value and t-statistic for the regression coefficient mean the S&P 500 explains approximately 64% of the portfolio's returns in any given year and is statistically significant. This means there is a significant portion of the returns that are unexplained, and could be attributed to alpha generated by the new risk methodology.

Figure 5. Regression Analysis

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.821658344
R Square	0.675122434
Adjusted R Square	0.640639675
Standard Error	0.108598625
Observations	30

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	0.710737884	0.710737884	60.26439689	1.86235E-08
Residual	29	0.342016177	0.011793661		
Total	30	1.052754061			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A
0.2234	0.728067128	0.093786648	7.763014678	1.46463E-08	0.536251895	0.919882361	0.536251895	0.919882361

There is a couple interesting takeaways from the study. The first is the laggard returns of the Portfolio relative to the S&P 500 in the years preceding a market top as shown in Figure 13**.**

From 1994-1999 the downside and upside allocations are generally consistent in ranges from 51.5-46.5% and 22-19.5%, respectively. During this time, the market gained on average 24% per year. In the three year trailing periods (i.e. 1991-1997) the market returned an average of approximately 18%. The S&P 500 increased 30.23% in 1991, an outlier for this period. Removing this year decreases the average annual return to approximately 15%.

Given this information, the Portfolio has a natural lag that will underweight stocks with high betas *in the current year*, which often times means overvalued stocks. Given this, the Portfolio naturally excludes stocks that have significantly appreciated in the current year. Additionally, the higher weighting of downside volatility stocks insulates the Portfolio against market crashes, as it will ideally be constructed with stocks featuring low downside betas. Essentially, the Portfolio construction allows an investor to shy away from rapidly increasing shares that lead to rapidly falling prices and outperformance during market corrections.

This is an important observation. Many investors would scoff at the notion of underperforming bull markets, yet in the same breath applaud the outperformance of bear markets. This is in-line, however with risk aversion. Furthermore, given the true distribution of stock returns, investors should be more concerned with higher probabilities of market bubbles and crashes than typically forecasted. Over the long term, the Portfolio will exhibit lower volatility, and therefore lower risk. Traditionally, the lack of risk premium would necessitate lower returns. However, looking across the Portfolio returns suggests the opposite over the long term.

During both the dot-com and credit crash and rebound, the limited losses during recessionary corrections and impressive rebound repudiate the risk/reward relationship claim. In fact, the lagged nature of the portfolio gives rise to another interesting takeaway. The highest Upside beta stocks during a market upswing are those that have gained the most, and typically decline the furthest. Following a market crash, the prevailing 36 month beta should capture those stocks that have appreciated the most. Adding these stocks into the portfolio 1-3 years following the crash allows an investor to take advantage of mean-reversion. In the dot-com bubble, tech stocks in the highest quintile of the upside beta group from 1997-1999 would begin being eligible for the portfolio from 2000-2002, and the investor is able to buy these shares after the prices have fallen.

Given that market's typically overreact, it is presumed that the sell-off is over exaggerated, allowing investors to buy shares of companies that previously traded well above intrinsic value (bubble-inflated prices) at well below intrinsic value (recessionary fear sell-off).

This lagged boom-and–bust buying and selling cycle is a more rational investment methodology for investors. Given the market dynamics that are evolving, the probability of market shocks continue to grow, as evidenced by the fat-tail distribution of stock returns. This portfolio construction methodology naturally allows investors to navigate these market cycles more intelligently.

In order to ensure the outperformance was not purely attributed to strategic asset allocations given the glide path approach, the portfolios were reconstructed with fixed weightings as shown in Table 7.

Table 6. Annual Returns

YEAR	S&P500	PORTFOLIO	+/-
1983	22.34%	14.39%	-7.95%
1984	6.15%	3.58%	-2.57%
1985	31.24%	24.01%	-7.23%
1986	18.49%	12.24%	-6.25%
1987	5.81%	2.07%	-3.74%
1988	16.54%	12.96%	-3.58%
1989	31.48%	22.35%	-9.13%
1990	-3.06%	-1.92%	1.14%
1991	30.23%	41.92%	11.69%
1992	7.49%	15.00%	7.51%
1993	9.97%	13.50%	3.53%
1994	1.33%	0.33%	-1.00%
1995	37.20%	32.51%	-4.69%
1996	22.68%	22.59%	-0.09%
1997	33.10%	27.61%	-5.49%
1998	28.34%	8.69%	-19.65%
1999	20.89%	20.56%	-0.33%
2000	-9.03%	13.70%	22.73%
2001	-11.85%	19.12%	30.97%
2002	-21.97%	8.96%	30.93%
2003	28.36%	33.98%	5.62%
2004	10.74%	23.46%	12.72%
2005	4.83%	4.48%	-0.35%
2006	15.61%	13.85%	-1.76%
2007	5.48%	5.45%	-0.03%
2008	-36.55%	-12.39%	24.16%
2009	25.94%	24.83%	-1.11%
2010	14.82%	19.11%	4.29%
2011	2.10%	4.45%	2.35%
2012	15.89%	13.24%	-2.65%
2013	32.15%	14.48%	-17.67%

Table 7. Benchmark Weightings

US Stocks	International Stocks	US Bonds	International Bonds	Short-Term TIPS	
DownsideBeta	*UpsideBeta*	*US Bonds*	*International Bonds*	*Gold*	*Real Estate*
35.0000%	25.0000%	20.0000%	10.0000%	5.0000%	5.0000%

The Portfolio returns resulted in a similar pattern and risk characteristics:

Table 8. Risk and Return Characteristics with Fixed Weightings

YEAR	S&P500	PORTFOLIO	+/-
1983	22.34%	18.69%	-3.65%
1984	6.15%	-0.62%	-6.77%
1985	31.24%	22.29%	-8.95%
1986	18.49%	10.77%	-7.72%
1987	5.81%	2.04%	-3.77%
1988	16.54%	12.96%	-3.58%
1989	31.48%	23.36%	-8.12%
1990	-3.06%	-6.86%	-3.80%
1991	30.23%	58.91%	28.68%
1992	7.49%	17.55%	10.06%
1993	9.97%	14.66%	4.69%
1994	1.33%	2.00%	0.67%
1995	37.20%	39.56%	2.36%
1996	22.68%	31.26%	8.58%
1997	33.10%	38.63%	5.53%
1998	28.34%	10.33%	-18.01%
1999	20.89%	43.28%	22.39%
2000	-9.03%	15.06%	24.09%
2001	-11.85%	24.92%	36.77%
2002	-21.97%	1.01%	22.98%
2003	28.36%	58.16%	29.80%
2004	10.74%	31.31%	20.57%
2005	4.83%	6.05%	1.22%
2006	15.61%	19.04%	3.43%
2007	5.48%	4.19%	-1.29%
2008	-36.55%	-31.99%	4.56%
2009	25.94%	56.51%	30.57%
2010	14.82%	33.86%	19.04%
2011	2.10%	-2.86%	-4.96%
2012	15.89%	25.11%	9.22%
2013	32.15%	46.02%	13.87%

	S&P 500	Portfolio
Are. Return	12.80%	20.17%
StDev	16.88%	20.38%
Risk Free	5%	5%
Sharpe	0.46	0.74
Best	37.20%	58.91%
Worst	-36.55%	-31.99%

Changing the allocations such that the Portfolio consists of 50% Upside beta stocks and 50% Downside beta stocks ("The 50/50 Model") yields similar results:

Table 9. Risk and Return Characteristics with 50/50 Weightings

YEAR	S&P500	PORTFOLIO	*H-*
19S3	22.34%	14.64%	-7.70%
1984	6.15%	4.66%	-1.49%
19S5	31.24%	21.11%	-10.13%
1986	18.49%	13.23%	-5.26%
19S7	5.81%	2.71%	-3.10%
1988	16.54%	10.20%	-6.34%
1989	31.48%	18.38%	-13.10%
1990	-3.06%	-1.63%	1.43%
1991	30.23%	39.01%	8.78%
1992	7.49%	13.01%	5.52%
1993	9.97%	13.64%	3.67%
1994	1.33%	0.38%	-0.95%
1995	37.20%	29.94%	-7.26%
1996	22.68%	21.32%	-1.36%
1997	33.10%	24.77%	-8.33%
1998	28.34%	*7.79%*	-20.55%
1999	20.S9%	22.30%	1.41%
2000	-9.03%	12.S9%	21 92%
2001	-11.85%	IS.00%	29.85%
2002	-21.97%	7.26%	29.23%
2003	28.36%	38.62%	10.26%
2004	10.74%	23.51%	12.77%
2005	4.S3%	5.0S%	0.25%
2006	15.61%	15.9S%	0.37%
2007	5.48%	5.42%	-0.06%
200S	-36.55%	-18.08%	IS.4 7%
2009	2594%	34.83%	S$9%
2010	14.82%	23.S9%	*9.07%*
2011	2.10%	*2.09%*	-0.01%
2012	15.89%	17.27%	1 38%
2013	32.15%	25.73%	-6.42%

	S&P 500	Portfolio
Ave. Return	12.S0%	15.10%
StDev	16.SS%	12.21%
Risk Free	5%	5%
Sharpe	0.46	0.83
Best	37.20%	39.01%
Worst	-36.55%	-18.08%

The 50/50 model, while substantially increased risk, still offers an improved Sharpe Ratio, and would therefore be favored in portfolio construction methodologies that consider risk-adjusted returns.

6. Potential Issues and Biases

There are a couple potential flaws with the methodology. First, the model could be subjected to using this methodology constructed over different time periods time-period bias. Portfolios using this methodology constructed over different time periods would have different results.

Secondly, the initial population of stocks was created using S&P 500 and Russell 3000 index constituents. There exists the potential for survivorship bias, as the composition of the indices clearly changes as market caps change. For example, index constituents that were part of the Russell 3000 from 1980 to 2012 would be excluded, and this could have material effects on the portfolio returns. However, given the large sample size (>1000), the omission of securities represents minimal risk. The portfolios are reconstructed each year and do not represent fixed baskets of securities.

Third, beta calculations are subject to Standard Error. For this study, the coefficient that was generated was the beta that was used, however, incorporating the Standard Error and 95% confidence level range could change the composition of the portfolio. For example, compare a stock in the top quintile with a beta of 4.0 and a stock with beta of 3.75 in the next lowest quintile. If the first beta has a Standard Error of 0.5, a 95% confidence range would indicate the beta could be anywhere from 3.0 to 5.0. If, however, the second beta had a Standard Error of 0.05, the same 95% confidence range would be 3.65-3.85. It is possible the first stock, which is included in the Portfolio, actually has a lower beta (3.0 versus 3.65 at the lowest end of the confidence range) and the two stocks should be swapped. A further analysis could be conducted to consider the Standard Errors when selecting stocks for the Portfolio.

Lastly, as with all beta calculations, they are backward-looking, so any reliability on beta as a measure of future returns should be measured with a large grain of salt.

7. Conclusions

There are two takeaways from this study. The first is the obvious shortfall modern financial theory uses to assess risk. Beta is directionally agnostic and no additional value is given to stocks that capture more upside than downside. By dividing risk into Upside and Downside segments, investors can better understand risk and choose stocks more likely to most benefit from upswings and provide minimized volatility during downswings.

In using this methodology, there is a natural lag effect that is created that allows investors to potentially gain from mean-reversion. By using trailing three year betas, the Portfolio will invest in stocks with high betas over the previous three years. While this may initially underweight outperforming stocks, it will overweight these stocks following a market correction.

There are a few additional statements that need to be made. One potential update to the model would be to exclude securities that reside in the top quintiles for both upside and downside betas. This would further expose the portfolio to those securities with the purest exposure and enhance the risk/reward profile. Lastly, the model in general can be expanded in limitless ways by incorporating fundamental factors such as market cap (and change in market cap), relative value, dividend payout history, etc. Given the initial framework of an updated risk model, tactical allocation and security selection could be further refined to identify undervalued opportunities or tilt the portfolio based on other fundamental factors.

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