ROBUSTIFYING THE MULTIVARIATE CHAIN-LADDER METHOD: A COMPARISON OF TWO METHODS

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Abstract

The expected result of a non-life insurance company is usually determined for its activity in different business lines as a whole. This implies that the claims reserving problem for a portfolio of several (perhaps correlated) subportfolios is to be solved. A popular technique for studying such a portfolio is the chain-ladder method. However, it is well known that the chain-ladder method is very sensitive to outlying data. For the bivariate situation, we have already developed robust solutions for the chain-ladder method by introducing two techniques for detecting and correcting outliers. In this article we focus on higher dimensions. Being subjected to multiple constraints (no graphical plots available), the goal of our research is to find solutions to detect and smooth the influence of outlying data on the outstanding claims reserve in higher dimensional data sets. The methodologies are illustrated and computed for real examples from the insurance practice.

Keywords: Multivariate Chain-ladder Reserving Method, Multivariate Outliers, Multiple Business Lines

1. INTRODUCTION

Because of the Solvency II regulations (f.i. for insurance companies operating in the European Union), the insurance companies have to put a Solvency Risk Capital aside. For every non-life insurance company, the starting point for the evaluation of this Solvency Risk Capital is the calculation of the outstanding claims reserve for the totality of its business. The outstanding claims reserve can be obtained as a result of applying the chain-ladder method for the run-off triangle of a univariate business line or a multivariate chain-ladder method for several run-off triangles of a company with multiple business lines.

In previous research, we discovered that the calculation of the outstanding claims reserve by means of the chain-ladder method is very sensitive towards outlying data. The statistical explanation for this sensitivity is straightforward because the development factors are averages and are not robust towards outliers. It was therefore interesting to be able to detect and correct the influence of outlying data on the outstanding claims reserve for one run-off triangle representing the univariate business and evaluate the difference between the outstanding claims reserve with outliers and without outliers. This means that we created the tool to compare the outstanding claims reserve in the two situations but without expressing a preference for one or another outcome. This remains an important task for risk management and the regulator.

However, a non-life insurance company will seldom limit its activity to one business line and will develop different business lines (it is not unusual that a non-life insurance company is doing business in the automobile sector as well as in the fire branch). As a consequence and because of the development of new guidelines, the insurance company is confronted with the quantification of the claims reserve for the whole of possibly correlated business lines. This implies that we have to study a portfolio of run-off triangles with possible correlation and that we have to consider multivariate claims reserving methods to calculate the ultimate claims reserve for the totality of the business of a non-life insurance company.

Our study of outlying data in the multivariate case is performed in two steps. A first extention treats the two-dimensional data sets. T. Verdonck and M. Van Wouwe (2011) studied the presence and the influence of outliers in a bivariate setting. This bivariate situation was chosen because the two-dimensional data set can still be visualized by a bagplot or an ellipsoid. In this way, the bivariate outlying data are easy to detect and they can be smoothed by graphical techniques. This detection and smoothing of the outliers is important. Merz and Wütrich (2008) defined a multivariate chain-ladder method model and derived the properties of the multivariate chain-ladder estimators and the predictors of the chain-ladder factors and the ultimate claims. The multivariate claims reserving methods and in particular the multivariate claims reserving method by Merz and Wütrich (2008) to calculate the age-to-age development factors and to determine the outstanding claims reserve for the different run-off triangles as a whole, are also very sensitive to outlying data. The two methodologies to correct the outliers by graphical techniques on bivariate data sets from practice, showed an excellent performance. It can be established that if no outliers are present in the data set, the results for the outstanding claims reserve are equal as to be expected.

If, however, we want to continue the process of detecting and smoothing outliers for multivariate data in higher dimensions, we have to act differently. A first reason is, that the tool of graphical representation of the data set, is not available anymore. A second problem occurs when exploiting the idea of depth functions to measure the distance of the data points to a center and to discover the outlying data. A robust estimator for the median is given by the MCD method (Rousseeuw-Ruts-Van Driessen) and is the Tukey median. The method is very time consuming and was therefore replaced by the FastMCD. The results for the value of the Tukey median by the application of the FastMCD method are very disappointing for three-dimensional (or higher dimensional) data sets. The reason for the poor results will be discussed and can be explained by graph theoretical foundations. As a consequence of these facts, we have to focus on the development of other methods to detect and correct the outliers in higher dimensions. In section 2,we indicate which multivariate chain-ladder method we will use to calculate the outstanding claims reserve. In section 3, we explain why different approaches should be used for bivariate and multidimensional (N>2) data sets. In section 4, we introduce two possible techniques to smooth the outliers in higher dimensions. The numerical results for both approaches are presented in section 5. The R-software will be used for this purpose.

2. MULTIVARIATE CHAIN-LADDER RESERVING METHOD

2.1. Notation and Method

We will consider multivariate data and hence assume that the portfolios consist of N run-off triangles of observations of the same size. We assume that $X_{ij}^{(n)}$ and $C_{ij}^{(n)}$ (for $1 \le i,\ j \le I$) are respectively the incremental and the cumulative claims amount of accident year i and development year j belonging to subportfolio n (with n=1,…,N). Furthermore let R_i and R respectively denote the outstanding claims reserve of accident year i and the overall reserve. The values of X_{ij} for $i + j \le n + 1$ represent the past claims data and will be used to make predictions about the claims that need to be paid in future calender years, namely X_{ij} where $i + j > n + 1$. For representation of the data it is common to use a run-off triangle as in Table 1.

Assume that the subportfolios consist of N run-off triangles of observations of the same size.

Table 1. Claims Development Triangle Number n

accident year i	development year j: 0 … j … J
0	realizations of r.v. $C_{ij}^{(n)}$
$\vdots$	i.e. observations $\mathcal{D}_I^{(n)}$
$I-j$	
$\vdots$	$\hat{C}_{i,j}^{(n)}$ predicted
I	quantities for $C_{i,j}^{(n)}$

Note: n, 1 ≤ n ≤ N, refer to subportfolios (triangle); i, 0 ≤ i ≤ I, refer to accident years (rows); j, 0 ≤ j ≤ J = I, refer to development years (columns).

Cumulative claims of run-off triangle n for accident year i and development year j are denoted by $C_{ij}^{(n)}$.

For $n \in \{1,...,N\}$, $i \in \{1,...,I\}$, and $j \in \{1,...,J\}$, the individual development factors for accident year i and development year j are defined by

$$F_{i,j}^{(n)} = \frac{C_{i,j}^{(n)}}{C_{i,j-1}^{(n)}}$$

$$\boldsymbol{F}_{i,j} = \left(F_{i,j}^{(1)}, \ldots, F_{i,j}^{(N)}\right)$$

$$\boldsymbol{D}(a) = \begin{pmatrix} a_1 & & 0 \\ & \ddots & \\ 0 & & a_N \end{pmatrix} \text{ and } \boldsymbol{D}(a)^b = \begin{pmatrix} a_1^b & & 0 \\ & \ddots & \\ 0 & & a_N^b \end{pmatrix}$$

are the $N \times N$ diagonal matrices of the N-dimensional vectors $\boldsymbol{a} = (a_1, \ldots, a_N)' \in \mathrm{R}^N$ and $\left(a_1^b, \ldots, a_N^b\right) \in \mathrm{R}^N$ for an exponent }, $b \in R$, respectively so that

$$\mathbf{C}_{i,j} = \mathbf{D}(\mathbf{C}_{i,j-1}) \cdot \mathbf{F}_{i,j} = \mathbf{D}(\mathbf{F}_{i,j}) \cdot \mathbf{C}_{i,j-1}$$

$$\forall j = 1, \ldots, J \text{ and } \forall i = 0, \ldots, I.$$

We use the multivariate chain-ladder time series model of Merz and Wütrich (2008) with $N = 3$. The parameter estimations are obtained by an iterative algorithm as proposed by Merz and Wütrich (2008).

3. DETECTION OF THE ATYPICAL OBSERVATIONS

Detecting outlying data in one dimension is easy. In passing from univariate data sets to multivariate data, the basic idea to start the process of detecting outliers is to introduce some measure of distance to evaluate how far away an observation lies from the center of the data set. A very common choice for this measure is the Mahalanobis distance.

The definition of the Mahalanobis distance is: let C be a positive definite $p \times p$-matrix and let t be a p-vector, then the Mahalanobis distance $md(x; C; t)$ of vector x towards C and t is defined by $md(x;C;t) = (x-t)^t C^{-1}(x-t)$. For our application, t and C are respectively the arithmetic mean and the classical covariance matrix.

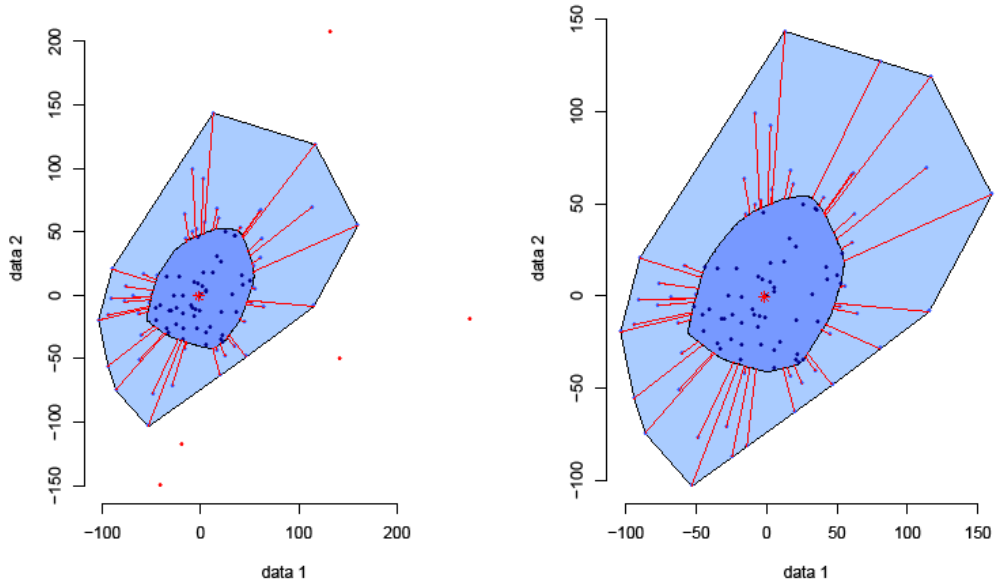
V. Hodge and J. Austin (2004) discuss several outlier detection methodologies for multivariate data. Because there are no unambiguous total orderings for multivariate data sets, the reduced sub-ordening based on the generalised distance metric using the Mahalanobis distance measure is recommended. Laurikkala et al. (2000) noted that the Mahalanobis measure is the most accurate for multivariate data. Their findings are supported by a panel of experts. There are a number of drawbacks for the application of the Mahalanobis distance measure. It turns out that the Mahalanobis distance is computationally expensive to calculate for high dimensional data sets. Like many other statistical methods, the Mahalanobis distance measure suffers from the problem of increasing dimensionality, because it requires a pass through the entire data set to identify the attribute correlations. As the dimensionality of the data increases, the data are spread through a larger space and therefore become less dense. This is known as 'the Curse of Dimensionality' and makes it harder to discern the convex hull separating the outliers from the good data points.

It is also proven that this Mahalanobis distance suffers from the masking effect, which causes outliers to have a small Mahalanobis distance and not being flagged as abnormalities. To solve this problem, Rousseeuw (1984) introduced robust estimates for the center and the covariance matrix. The Minimum Covariance Determinant method is a robust estimator of the location and the scatter of the multivariate data in R^p that looks for those h observations in the data set, whose classical covariance matrix has the lowest determinant. The MCD estimate of location and scatter is respectively the average and the covariance matrix of these h data. In practice, h=0.75n is a very common choice but the robustness and efficiency of this MCD method is highly depending on the choice of the h points.

Another method is the Minimum Volume Ellipsoid (MVE) where the idea is to find a subset of size h, for that the enclosing ellipsoid has the minimal volume. The MVE is a robust classifier that fits the boundaries around specific percentages (f.i. 50%) of the data irrespective of the sparseness of the outlying region and the outlying data do not skew the boundary ellipsoid. However, this result relies on a good spread of the data. Barnett and Lewis (1994) show that MVE is only applicable for lower dimensional data because also this method suffers from 'the Curse of Dimensionality'.

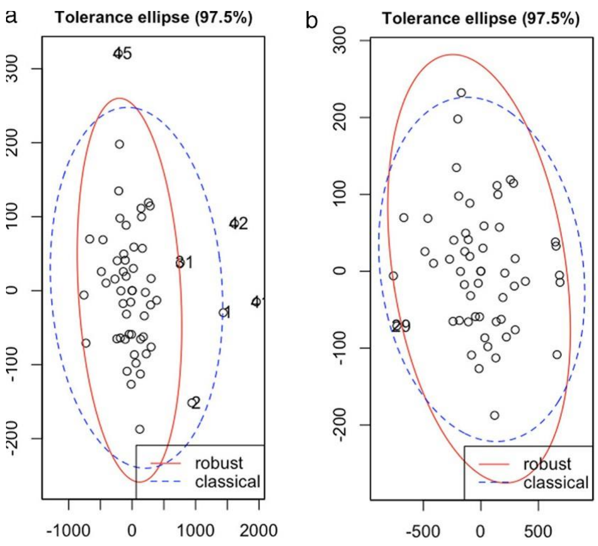
In looking at multivariate data sets, we have to make a distinction between bivariate data and higher dimensional data. The obvious reason is that for a bivariate data set, we can still analyse the data by a graphical tool. This tool, the bagplot (Rousseeuw et al. (1999)), is a natural extention of the boxplot and relies on a ranking system for the data set based on the concept of halfspace depth. The halfspace depth of a point Z can be seen as the minimal number of observations in a closed halfspace of which the boundary plane passes through Z. The application of this specific choice of a depth function for a bivariate point x results in a halfspace depth of a bivariate point x as the smallest number of data points lying in a closed halfplane bounded by a line through x. It also allows a graphical representation of outliers by means of a bagplot. The bagplot is the total image of the data set and consists of the Tukey median (the bivariate and robust extention of the univariate median, having the highest depth), the bag (containing 50% of the observations) and the fence (the contour magnifying the bag by a factor 3). The observations outside the fence are outliers. Figure 1 shows a bagplot for bivariate data with and without outliers. Another possible tool is the ellipsoid as shown in Figure 2.

Figure 1. Bagplot with and without outliers



This graphical representation enables us to identify the outliers in an easy way and becomes the key to smooth the bivariate outliers. Unfortunately, this possibility to represent the data graphically disappears when higher dimensional data have to be examined. The Mahalanobis distance however leaves an option to measure how far away a data point is from a 'center'. The MCD method calculates a center and the Mahalanobis distances of all observations. To solve the problem of sensitivity of this measure towards outlying data, the FAST-MCD R-package developed by Rousseeuw calculates a robust version for the location and for the scatter. In using this method for an insurance company with 3 business lines, the results for the 'center' were very disappointing (the result is far away from the coordinatewise averages of the 3 business lines) for a majority of examples. Therefore, we were forced to examine these findings more closely. The reason for a visible deviation of the MCD estimate of the location towards the coordinatewise average 'center' is discussed in T. Bernholt and P. Fischer (2004). In this article T. Bernholt and P. Fischer explain why the MCD-algorithm fails to give acceptable results for the center when the dimension of the data set increases. The MCD-algorithm can be viewed as an application of the so-called 'clique number' problem in graph theory. In graph theory, the 'clique number' is a well known example of the family of NP-complete problems. This means that the algorithm to solve the clique number problem can run in polynomial time or in exponentional time and that the time to solve an NP-complete problem increases rapidly when the size of the problem grows. With this knowledge, it is clear that the MCD-algorithm will face similar problems. As a consequence the results with the MCD-algorithm will not be reliable for increasing dimensions. It also means that we have to search for new approximative methods to detect and to smooth the outliers in the run-off triangles when the dimension of the data is exceeding the number 2.

Figure 2. Ellipsoid with and without outliers



4. TWO DIFFERENT APPROACHES TO DETECT AND SMOOTH THE OUTLIERS WITH *N>2*

If we look back at bivariate data sets, a particular way to shrink the outliers (and to lower their influence on the outstanding claims reserve) can be applied. If we look back at the 2-dimensional representation of the data set, we easily discover a method to bring back the outliers to the fence of the bagplot or to the tolerance ellipse curve obtained with MCD method. These techniques of shrinking the outliers to the fence of the bagplot or to the border of the tolerance ellipse are discussed in Verdonck and Van Wouwe (2011).

In the previous section, we learned to make a distinction between bivariate data sets and multivariate data sets. We will discuss two possible approaches to detect and to correct the outliers in a multivariate data set.

As it was already mentioned before, the Mahalanobis distance is a possible tool to discover outliers in higher dimensions (Aggarwal (2001)). The Mahalanobis distance accounts for the variance of each variable and the covariance between variables. Geometrically, it does this by transforming the data into standardized uncorrelated data and computing the ordinary Euclidean distance for the transformed data. In this way, the Mahalanobis distance is like a univariate z-score: it provides a way to measure distances that takes into account the scale of the data. The outliers are detected by their large distance. Because the MCD-algorithm is not satisfying, we prefer to use the original Mahalanobis distance to discover the outliers (an open question remains about when the Mahalanobis distance is regarded to be large).

We develop an algorithm to detect and to smooth the outlying observations for multivariate data sets with more than two components. As always the process starts with calculating the residuals of the data. We will look at two possibilities to detect and to smooth the outlying data. A first solution method starts to detect the outlying data by the Mahalanobis distance for all data points. The outlying values are then replaced by the coordinatewise median (each component of an outlying data point is replaced by the corresponding univariate median). This process is repeated until successive results are close together. The following figures show this process.

A second solution is given by replacing the outliers by the L_1-median (Fritz (2010)).

For a data set $\boldsymbol{X} = \{x_1, ..., x_n\}$ with each $x_i \in \mathbb{R}^p$, the L_1-median $\hat{\mu}$ is defined as

$$\hat{\mu}(X) = \mathrm{argmin}_{\mu} \sum_{i=1}^{n} \|x_i - \mu\|$$

Where $\|\cdot\|$ denotes the Euclidean norm. In words, the L_1-median is the point for which the sum of the Euclidean distances to n given data points is minimal.

The L_1-median has several further attractive statistical properties, like:

(a) Its breakdown point is 0.5. Only in the case that at least 50% of the data points are contaminated, the L_1-median can take values beyond all bounds.

(b) It is location and orthogonally equivariant.

For both solutions, the adjusted residuals are brought back to the original data and on the 'smoothed' data set a multivariate chain-ladder method is applied.

Remark: alternative techniques are possible and bring back the data onto lower dimensional subspaces. This idea was already exploited by Aggarwal and Yu (2001). Aggarwal and Yu use lower dimensional projections and assume that combinations of attributes in the projection correlate to the attributes that are deviant. This could be done in several ways. For a start, in treating higher dimensional data sets, we could restrict ourselves to 3-dimensional data and explore the following possibility. A first possible algorithm could be to treat the run-off triangles in the following way: the run-off triangles are summed up two by two and the bivariate method (T. Verdonck and M. Van Wouwe (2011)) is applied on this sum and once more the same method is used on this result and the third run-off triangle. This procedure is very time consuming and is not retained for that reason.

Figure 3. Density.default (x=b, bw=0,5)

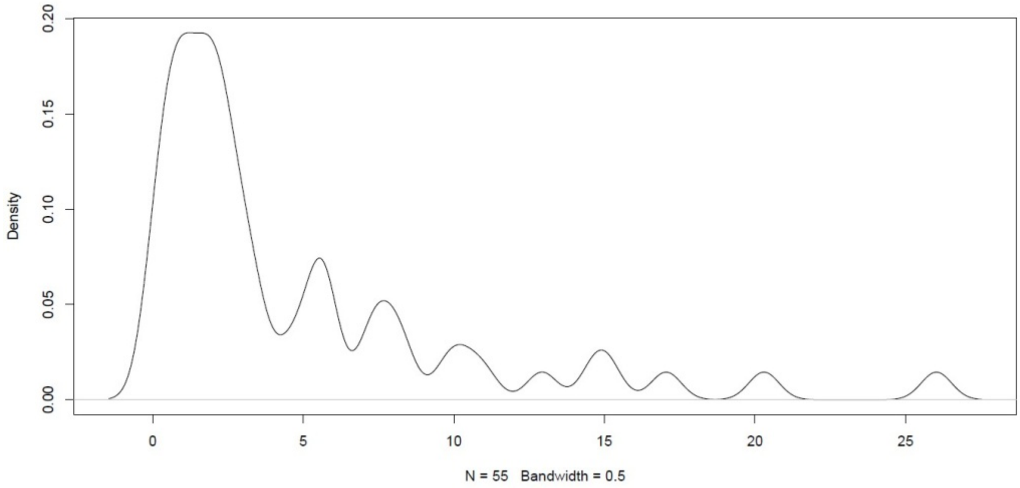


Figure 4. Density.default (x=bb, bw=0,5)

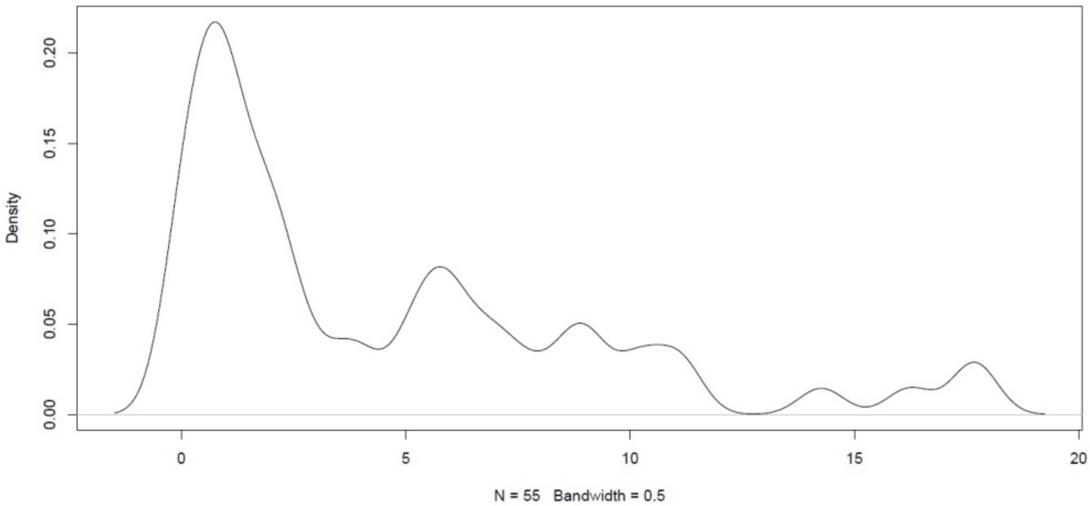
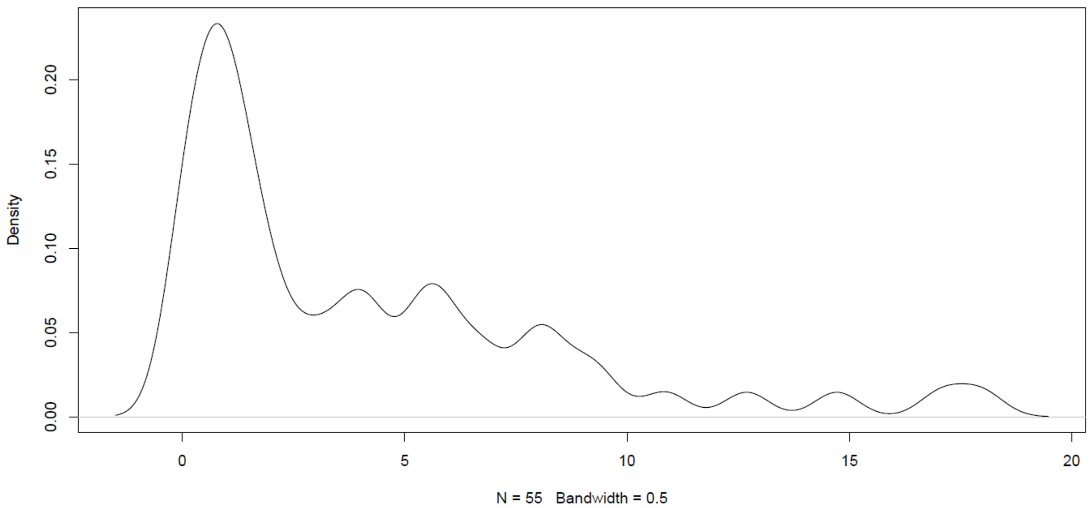


Figure 5. Density.default (x=bbb, bw=0,5)



5. NUMERICAL RESULTS

The numerical results will be illustrated by a real example. To do this we start from three run-off triangles representing three business lines of the same non-life insurance company. The three triangles are:

Table 2. Observed incremental claims business line 1

524914	285647	244175	129698	138796	110650	93308	69315	8797	4512
696729	380888	286367	227269	188542	57207	55881	58608	39453	0
772188	581335	547795	565772	357396	295440	284133	294873	0	0
767974	418593	304174	147959	144680	118461	65479	0	0	0
402505	287498	240137	216582	207624	168963	0	0	0	0
390650	325253	278633	222955	134545	0	0	0	0	0
556262	359657	288889	249878	0	0	0	0	0	0
414878	238414	192568	0	0	0	0	0	0	0
654057	392432	0	0	0	0	0	0	0	0
536550	0	0	0	0	0	0	0	0	0

Table 3. Observed incremental claims business line 2

407586	307487	328409	239292	230888	258932	253436	247088	242088	11600
258404	238027	189391	179202	62368	55280	32347	31001	27754	0
369757	228335	141125	102985	83655	67003	30822	18822	0	0
114626	78458	63058	54985	50901	21185	18956	0	0	0
176922	194786	149293	38915	15925	9538	0	0	0	0
200351	122767	100273	36339	27686	0	0	0	0	0
172429	80351	30787	23967	0	0	0	0	0	0
315270	277844	157231	0	0	0	0	0	0	0
180920	211137	0	0	0	0	0	0	0	0
235813	0	0	0	0	0	0	0	0	0

Table 4. Observed incremental claims business line 3

4717438	3016789	2484066	1904194	1447276	1252992	824851	709612	508993	467512
5867515	4048002	2544032	1965870	1587106	1350246	899017	999375	1050118	0
5390147	3300851	2483372	1857682	1354058	1144486	593458	521315	0	0
6157650	4184517	3302699	2618703	2164798	1662522	1413788	0	0	0
5066275	3707332	2463467	1962620	1867330	1559849	0	0	0	0
6968353	4067230	2714697	2174128	1518695	0	0	0	0	0
6076115	4080323	2847006	2388172	0	0	0	0	0	0
5384700	3976736	3224352	0	0	0	0	0	0	0
6816917	4929496	0	0	0	0	0	0	0	0
8434600	0	0	0	0	0	0	0	0	0

To start we introduce outliers in the three triangles in different ways.

Firstly, outliers are introduced at the same place in the three triangles (this situation could be related to an event that influenced the claim amounts in the different triangles at the same place at the same time, f.i. currency movements, macro-economic movements). The corresponding claim amounts (at the same position in the three triangles) are therefore replaced by outliers. This procedure is applied to any of the positions in the triangles except for the extreme corners. This procedure was initially introduced for the univariate chain-ladder method (Verdonck et al. (2009)) and is now used in a similar manner for the multivariate chain-ladder method.

Secondly, another possible situation could be the appearance of outlying data in two of the three triangles at the same place and at the same time. This possibility reflects the possible correlation between two of the three triangles. This corresponds to an event of which the influence is restricted to two of the three triangles and leaves the third triangle unchanged.

Thirdly, the claim amounts in one particular triangle are replaced by an outlier (this could correspond for instance to a realistic situation of mistyping a claim amount).

The following tables show the results for the outstanding claim amounts regarding the different possibilities:

5.1. Case 1

Outliers at the same place and at the same time in the three triangles

- the outstanding claim reserves for the situation where outliers are introduced simulteanously in the three triangles
- the outstanding claim reserves where the outliers are replaced by the coordinatewise median
- the outstanding claim reserves where the outliers are smoothed by means of the L_1-median method

Table 5. Results for the deviation of the outstanding claims reserves towards the original outstanding claims reserves when an outlier is introduced at the same place and the same time in the 3 triangles

	with outliers	***coordinatewise***	***L_1***
Average	61236103.7	133211793.1	130350797.9
Median	598832179	12957652	14231932
MSE	4,66693E+17	1,83837E+17	1,78446E+17
Root MSE	683149268.3	428762235	422428627.6

5.2. Case 2

Outliers in two of three triangles

- the outstanding claim reserves where outliers are introduced simulteanously in two of the three triangles
- the outstanding claim reserves where the outliers are replaced by the coordinatewise median
- the outstanding claim reserves where the outliers are smoothed by means of the L_1-median method

Table 6. Results for the deviation of the outstanding claims reserves towards the original outstanding claims reserves when an outlier is introduced in two of the three triangles

	with outliers	*coordinatewise*	L_1
Average	569551778.7	125313937.1	123206934.8
Median	556087324	13619933	14231932
MSE	4,02434E+17	1,61716E+17	1,51494E+17
Root MSE	634376582.9	402139690.7	389222201.8

Table 7. Results for the deviation of the outstanding claims reserves towards the original outstanding claims reserves when an outlier is introduced in one triangle

	with outliers	*coordinatewise*	L_1
Average	546954253.9	119579732.1	117625663.2
Median	530419505	10761599	15656173
MSE	3,71207E+17	1,51833E+17	1,39951E+17
Root MSE	609267019.6	389657243.7	374100220.8

5.3. Case 3

Outliers in one particular triangle

- the outstanding claim reserves where outliers are introduced in one particular triangle
- the outstanding claim reserves where the outliers are replaced by the coordinatewise median
- the outstanding claim reserves where the outliers are smoothed by means of the L_1-median method

The following table 8 for case 1 illustrates where the L_1-median method gives better results than the coordinatewise median method, figures in bold are these where the L_1-median method gives a better result (this is a result for the outstanding claims reserve that is closer to the outstanding claims reserve for the original data) while the figures in italics indicate the situations where the coordinatewise median method should be elected.

Table 8. Results quality

(1;1)	**(1;2)**	*(1;3)*	**(1;4)**	**(1;5)**	**(1;6)**	**(1;7)**	*(1;8)*	**(1;9)**	(1;10)
(2;1)	*(2;2)*	**(2;3)**	**(2;4)**	**(2;5)**	**(2;6)**	**(2;7)**	**(2;8)**	**(2;9)**	
(3;1)	*(3;2)*	*(3;3)*	**(3;4)**	**(3;5)**	**(3;6)**	*(3;7)*	**(3;8)**		
(4;1)	*(4;2)*	*(4;3)*	**(4;4)**	**(4;5)**	*(4;6)*	**(4;7)**			
(5;1)	**(5;2)**	*(5;3)*	*(5;4)*	**(5;5)**	*(5;6)*				
(6;1)	**(6;2)**	**(6;3)**	*(6;4)*	*(6;5)*					
(7;1)	**(7;2)**	*(7;3)*	*(7;4)*						
(8;1)	**(8;2)**	*(8;3)*							
(9;1)	*(9;2)*								
(10;1)									

The MSE is a reliable measure in order to decide which of the two methods gives the best corrective results. This is the method for which distance between the outstanding claim reserve with the adjusted data and the original outstanding claim reserve is the smallest and this for all possible situations regarded as higher up indicated.

The results can be summarised as follows:

– *Case 1.* The smoothing with the L_1-median method gives better results than the coordinatewise method. This can be explained by the fact that the L_1-median method focuses on the multivariate data and treats the outlying data simultaneously unlike the coordinatewise median method that concentrates on the univariate data.

– *Case 2.* When the outliers are introduced in two possible correlated triangles, neither method is the better of the other method.

– *Case 3.* When outliers only appear in one particular triangle, the coordinatewise median method gives the better corrective results. This can easily be explained by the fact that the coordinatewise median method focuses on the individual triangle and has no major influence on the behavior of the other triangles where no outliers are present.

6. CONCLUSION

In this article, we concentrate on the detection and correction of outlying data for an non-life insurance company that has activities in more than two branches of the non-life insurance sector. As a consequence the non-life insurance company will have to manage more than two run-off triangles. We explain why the algorithm that was previously developed for the univariate and the bivariate data (two business lines) does not work for the multivariate data set with more than two run-off triangles.

As a consequence of this fact, we have to search for alternative methods to detect and to smooth the outliers in the run-off triangles. The two possible corrective methods that are taken into consideration and that are discussed in this article are the coordinatewise median method and the L_1-median method.

The numerical results do not reveal a substantial advantage of one of the two methods for the totality of the possible presence of outliers in more than two run-off triangles. The results are very depending on the different possibilities of the presence of outliers in the run-off triangles. The L_1-median method gives better results than the coordinatewise median method when outliers are introduced in the three run-off triangles at the same place. However, when outliers are detected in two of the three run-off triangles, the results for two methods are very similar. In introducing an outlier in a single run-off triangle, we prefer the coordinatewise median method to the L_1-median method.

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