

PRIVATELY HELD OR PUBLICLY OWNED? LARGE SHAREHOLDERS AND CORPORATE CONTROL – EVOLUTIONARY GAME THEORETIC ANALYSIS

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Abstract

I use evolutionary game theoretic techniques to model the interaction between managers and shareholders and describe the equilibrium ownership structure arrived at in different legal environments. The decision to go public and the shape of the ownership structure itself depend on the particular combination of ownership that maximizes the initial owners' wealth. Owners/managers and large shareholders exert costly efforts to increase their share of the value of the public firm. The respective shares and the listing decision are affected by the efficiency of the judiciary and law enforcement system.

Keywords: Ownership Structure, Corporate Governance, Agency Costs, Monitoring, Managerial Conflict, Legal Protection, Investor Protection, Evolutionary Game Theory

JEL Classification: D23, D73, D74, G32, G34, K22

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1 Introduction

In Toukan (2014), I have used traditional game theory to analyze the conflict between owners/managers and outside shareholders. However, traditional game theory is a "static" theory, which reduces its usefulness in analyzing situations that typically involve a large population of agents interacting. I tried to capture some of the dynamics of the decision-making process by modeling the game in its extensive form, however for games of reasonable complexity (and usually of interest), the extensive form of the game quickly becomes unmanageable. The contribution of my paper is in the use of qualitative evolutionary game theoretic techniques developed in Saari (2002) to model the interaction between managers and shareholders and describe the equilibrium ownership structure arrived at in different legal environments. Different legal environments provide varying degrees of legal protection of outside shareholders from expropriation or "tunneling"⁴⁰ by the insiders.

Skyrms (2000) argues that good reasons can be given to why the replicator dynamics is a plausible candidate for a dynamics of cultural evolution. He believes that the replicator dynamics is a natural place to begin investigations of dynamical models of

cultural evolution, but does not believe that it is the whole story. A number of different models of social learning by imitation have been shown to yield the replicator dynamics. Binmore, Gale and Samuelson (1995) believe that the principles to which one must appeal when predicting actual behavior, in the laboratory or elsewhere, are almost always evolutive in character. That is to say, the outcomes we observe are not the product of careful reasoning but of trial-and-error learning. Their paper demonstrates that interactive learning processes readily lead to outcomes in the Ultimatum Game⁴¹ that are Nash equilibria but not subgame-perfect. They argue that game theorists were therefore wrong to put all their eggs in the subgame-perfect basket when predicting laboratory behavior in the Ultimatum Game. A case exists for predicting that interactive learning will result in the selection of one of the other Nash equilibria of the game.

Björnerstedt and Weibull (1995) discussed dynamic models of evolution by imitation. The authors argue that their analysis suggests that evolutionary predictions may be context dependent. The social, cultural, institutional etc. environment in which the interaction takes place presumably shapes the transmission mechanism by which behaviors spread in society. And different transmission

⁴⁰ Please see López de Silanes, Johnson, La Porta, and Shleifer (2000).

⁴¹ Please see Thaler (1988).

mechanisms induce different population dynamics, and hence possibly different dynamically stable sets. They also argue that their approach suggests that some, perhaps less precise, predictions can be made with only some qualitative knowledge about the dynamics in question.

Schlag (1998) considers the situation in which individuals in a finite population must repeatedly choose an action yielding an uncertain payoff. Between choices, each individual may observe the performance of one other individual. The author searches for rules of behavior with limited memory that increase expected payoffs for any underlying payoff distribution. The author shows that the rule that outperforms all other rules with this property is the one that specifies imitation of the action of an individual that performed better with a probability proportional to how much better she performed. When each individual uses this best rule, the aggregate population behavior can be approximated by the replicator dynamic⁴². In this dynamic, the growth rate of an action is equal to its relative payoff measured with respect to the average payoff in the population.

Saari (2002) questions if the replicator dynamics is appropriate for all situations, and introduces more general dynamics through the use of winding numbers. He argues that an important question is to understand the appropriate choice of dynamics. According to Saari (2002), "While many of the dynamical stories offered for evolutionary game theory are reasonable on a qualitative level, it is difficult to accept the precise equations." Saari (2002) shows that just knowing the local information and that the change is continuous provides considerable qualitative information about the global dynamics. Local information corresponds to the direction of dynamics, near a plausible equilibrium of the system.⁴³

I will use the winding numbers technique presented in Saari (2002) to understand the qualitative aspects of the analysis conducted in Toukan (2014). In analyzing the share ownership dynamics that pertain to the conflict between owners/managers and shareholders, each agent evaluates the current state of the game and chooses her ownership share in a manner that optimizes her net payoff. The paper is organized as follows. Section 2 outlines the model. Section 3 examines the case where the owners/managers decide to sell a share of the company to outside shareholders (in this case we do not make the distinction between large and dispersed shareholders). In section 4, I examine the case where the owners/managers decide to sell a share of the company to both large and dispersed shareholders. Section 5 concludes.

2 The model

We consider a private firm initially fully owned by its founders. At time 0, the owners decide whether to take their company public. Upon their decision to take their company public, the owners decide what fraction to sell to large shareholders, what fraction to keep and what fraction to sell to dispersed shareholders.

Assumption 1: Founders of the private firm will also act as managers of the public firm due to their special expertise in running the firm.

Assumption 2: All funds raised from selling equity will be reinvested in the firm.

Assumption 3: Dispersed shareholders act as free riders in our model due to their high opportunity cost of monitoring the owner/manager⁴⁴.

3 Going public by selling shares to only one type of shareholders

The owners/managers decide to take their privately held firm public and sell a share of the company to outside shareholders (we do not make the distinction between large and dispersed shareholders). θ represents the efficiency of the judiciary and law enforcement system in a country (or the degree of legal protection of outside shareholders from expropriation or "tunneling" by the insiders) and it varies between 0 and 1. An increase in θ towards 1 would indicate stronger law enforcement or a more efficient legal system. Conversely a movement of θ toward 0 would indicate weaker law enforcement or a less efficient legal system. We will examine the owner's decision for values of θ between 0 and 1.

To model the interaction between shareholders and owners/managers, let the two groups be identified by S (shareholders) and M (original owners/managers) where s , $0 \leq s \leq 1$, describes the share of ownership in the public firm by shareholders and m , $0 \leq m \leq 1$, describes the share of ownership in the public firm by the original owners/managers. Figure 1 below describes the share of ownership in the public firm by shareholders and by original owners/managers where the $s = 1$, $m = 0$ endpoint, the left-hand endpoint,

⁴⁴ Shleifer and Vishny (1986) show that in a corporation with many small owners, it may not pay any one of them to monitor the performance of the management. They build on Grossman and Hart (1980) argument that opposite to what is often suggested the free rider problem cannot be avoided by the use of the takeover bid mechanism. Outsiders without a share in a diffusely held firm would never take over a diffusely held firm in order to improve it. The reason is that outsiders' improvement plan would be understood by atomistic incumbent shareholders and they will demand the value of the improvement in return for their shares or else they stay on. If the outsider can gain only on the shares they already own (which are few if any) but have to pay all the monitoring and takeover costs, the deal may not be worth their while. For the same reason, small shareholders do not have a big enough stake in the firm to absorb the costs of watching the management.

⁴² Please see Taylor, P. D. (1979).

⁴³ Please see Sandeshika, Sharma, (2004)

means that shareholders owns 100% of the shares in the public firm, and $s = 0$, $m = 1$ endpoint, the right-hand endpoint, means that original owners/managers

owns 100% of the shares in the public firm and in this case the firm will be private.

Figure 1. A shared ownership structure by the original owners/managers and by shareholders



According to Saari (2002), “While many of the dynamical stories offered for evolutionary game theory are reasonable on a qualitative level, it is difficult to accept the precise equations. The question is to determine whether the local, readily acceptable information suffices to provide global information.”

Following the argument in Saari (2002) I assume that the dynamic is continuous and I use the local, readily acceptable information to provide global information. I build my model based on the analysis in Saari (2002), where I represent “moving to the left” as negative and “moving to the right” as positive. Next, I will examine the dynamics of going public under three different levels of the efficiency of the judiciary and law enforcement system in a country: weak, strong, and moderate.

3.1 Case i: The case when the efficiency of the judiciary and law enforcement system is weak

I start by examining the dynamics for the case when the efficiency of the judiciary and law enforcement system is weak or θ is close to zero. When the going public decision is graphed, I start with the minimal information shown in Figure 2 below where the graph starts at a positive value on the left (reflecting the movement to the right which represents an increase in the share ownership of the original owners/managers) and ends at a positive value on the right (reflecting the movement to the right which again represents an increase in the share ownership of the original owners/managers).

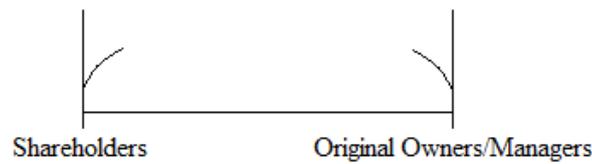
The reason that the graph starts at a positive value on the left is due to local, readily acceptable information. Shareholders have a very low valuation of the public firm due to the lack of protection provided to them by the legal system which will result in a high risk of expropriation by the original owners/managers. An ownership structure representing 100% ownership by shareholders will be resisted by shareholders. La Porta et al. (2002) find that in countries with better protection of shareholders that limit expropriation by the entrepreneur who controls the firm, entrepreneurs are able to finance their investments externally, leading to the expansion of financial markets.

An ownership structure representing 100% ownership by shareholders will also be resisted by the

original owners/managers since for low values of θ , shareholders have a very low valuation of the public firm due to the lack of protection provided to them by the legal system and any tempt to bring them in as shareholders will result in a loss of value to the original owners/managers. In legal regimes with the weakest shareholder protection, very low values of θ , the value of the private firm exceeds its value being public. La Porta et al. (2002) find evidence of higher valuation of firms in countries with better protection of shareholders. They argue that with better protection of outside investors, outside investors are willing to pay more for financial assets such as equity and debt. They pay more because they recognize that, with better legal protection, more of the firm’s profits would come back to them as interest or dividends as opposed to being expropriated by the entrepreneur who controls the firm. By limiting expropriation, the law raises the price that securities fetch in the marketplace. Castillo and Skaperdas (2005) examine how the legal protection of outside shareholders and the appropriative costs that they induce influence the incentives for private firms to go public. The authors find that overall, higher protection of outsiders increases the likelihood of going public. In both cases discussed above shareholders will want to sell their shares and the original owners/managers will want to consolidate outstanding shares and buy back the firm which implies that the dynamics is represented by the movement away from the $s = 1$, $m = 0$ endpoint, the left-hand endpoint.

Similarly the reason that the graph ends at a positive value on the right is due to local, readily acceptable information. As mentioned above, shareholders have a very low valuation of the public firm due to the lack of protection provided to them by the legal system and any tempt to bring them in as shareholders will result in a loss of value to the original owners/managers. In legal regimes with weakest shareholder protection, very low values of θ , the value of the private firm exceeds its value being public and the original owners will decide to keep the firm private. Castillo and Skaperdas (2005) find that for weak protection of shareholders the value of the public firm will be lower than the value of the private firm for all ownership structures and it would be optimal for the firm to remain private.

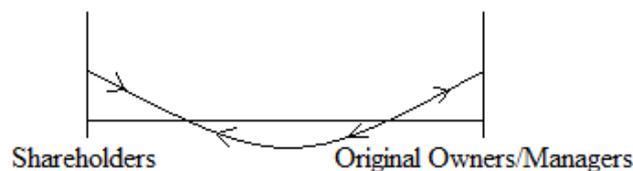
Figure 2. Local information in the case when the efficiency of the judiciary and law enforcement system is weak



The graph connecting the two endpoints shown in Figure 2 above may or may not cross the x-axis. When it does cross the x-axis it will cross it an even number of times. In the following argument we will show that in order to connect the endpoints, in general the slope of the connecting curve must be zero. Obviously the slope of the curve when it does not pass through the x-axis is equal to zero. This implies that the sum of the signs of the slopes of the curve when it

does pass through the x-axis must be equal to 0. The reason is that starting at the $s = 1, m = 0$ endpoint, the left-hand endpoint, whenever the curve crosses the x-axis downwards (with the sign of the slope equaling to -1) it has to go back upwards to cross the axis again (with the sign of the slope equaling to $+1$) as illustrated by the example shown in Figure 3 below. The sum of the signs from the pairs of crossings will always add up to the global value of zero.

Figure 3. Dynamics in the case when the efficiency of the judiciary and law enforcement system is weak



Following the argument in Saari (2002), we can see that by knowing the local information and that the change is continuous, we do know that there are at least two equilibria and, generically, there is an even number of them. Also, because the sign of the curve

must alternate at each crossing of the x-axis, we know that, generically, the stability of the equilibria must alternate between being stable and unstable. The above argument is illustrated in Figure 4 below.

Figure 4. Two examples of the dynamics in the case when the efficiency of the judiciary and law enforcement system is weak

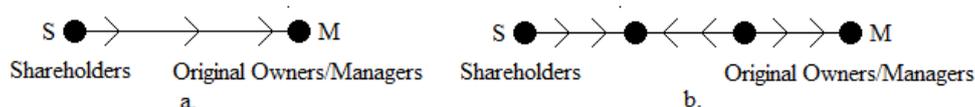
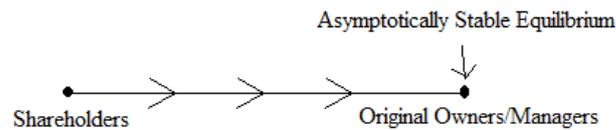


Figure 4 above tells us that the model of interaction of original owners/managers and shareholders has at least two plausible equilibria: one with all shareholders and second with all owners/managers. Examining Figure 4a above we can observe two pairs of equilibria where one is an attractor (original owners/managers) and the other is a repeller (shareholders). According to Saari (2002), “The local information can be accompanied by more interesting kinds of market dynamics.” By examining Figure 4 above, we see that regardless of the form the dynamics assumes; it has to introduce new pairs of equilibria where one is an attractor and the other is a repeller. The simple case offered in Figure 4b introduces two more equilibria and a threshold effect. Even though we do not know where the threshold point is, knowing that it exists is important for further analysis. For example the threshold in the going public example tells us that in order to increase the valuation

of the company by outside shareholders it is necessary to increase their monitoring and control in the public company through increasing their share of ownership. An increase in the share of ownership by outside shareholders compensates for the weakness in the legal protection of outside shareholders. Shleifer and Vishny (1986), show that large shareholders raise expected profits and the more so the greater their percentage of ownership. Holmström and Tirole (1993) argue that on the benefit side of going public, there are benefits of some outside monitoring.

Returning to the case when the efficiency of the judiciary and law enforcement system is weak and considering a private firm initially fully owned by its founders, it is easy to prove that “Original Owner/Manager” is a globally asymptotically stable equilibrium since every possible initial state in the interior of the state space is carried to it by the dynamics [Skyrms, 2000].

Figure 5. The asymptotically stable equilibrium in the case when the efficiency of the judiciary and

Examining Figure 5 above we see that small changes in ownership structure away from the equilibrium “Original Owner/Manager” will be carried back to the equilibrium by the dynamics. As argued above, shareholders have very low valuation of the public firm due to the lack of protection provided to them by the legal system and any tempt to bring them in as shareholders will result in a loss of value to the owners/managers. Large and small changes in ownership structure away from the equilibrium “Original Owner/Manager” will be resisted by outside shareholders due to the large increase in expropriation risk by the original owners/managers and will be carried back to the equilibrium by the dynamics as shown in Figure 5 above.

Castillo, R., & Skaperdas, S. (2005) examine how the legal protection of outside shareholders and the appropriative costs that they induce influence the incentives for private firms to go public. Their results indicate that a higher degree of protection of outside shareholders increases the likelihood of going public. Pagano, M., & Roell, A. (1998) find that a stronger legal protection of minority shareholders promotes cooperation among external shareholders and hinders collusion between the entrepreneur and potential monitors. Pagano, M., & Roell, A. (1998) model predicts that improved legal protection of minority shareholders increases companies' propensity to go public. This prediction is consistent with the cross-country evidence reported by La Porta et al. (1997), who show that better legal protection of shareholders' rights is associated with a larger value and number of publicly listed companies and of initial public offering. La Porta et al. (2000) argue that legal protection of outside investors makes the expropriation technology less efficient. At the extreme of no investor protection, the insiders can steal a firm's profits perfectly efficiently. Without a strong reputation, no outsider would finance such a firm.

3.2 Case ii: The case when the efficiency of the judiciary and law enforcement system is strong

Next I examine the dynamics for the case when the efficiency of the judiciary and law enforcement system is strong or θ is close to one. When the going public decision is graphed, I start with the minimal information shown in Figure 6 below where the graph starts at a negative value on the left (reflecting the movement to the left which represents a decrease in the share ownership of the original owners/managers) and ends at a negative value on the right (reflecting

the movement to the left which again represents a decrease in the share ownership of the original owners/managers).

The reason that the graph starts at a negative value on the left is due to local, readily acceptable information. Shareholders have a very high valuation of the public firm due to the strong protection provided to them by the legal system which will result in a small risk of expropriation by the original owners/managers. An ownership structure representing 100% ownership by shareholders will be welcomed by shareholders. La Porta et al. (2002) find that in countries with better protection of shareholders that limit expropriation by the entrepreneur who controls the firm, entrepreneurs are able to finance their investments externally, leading to the expansion of financial markets.

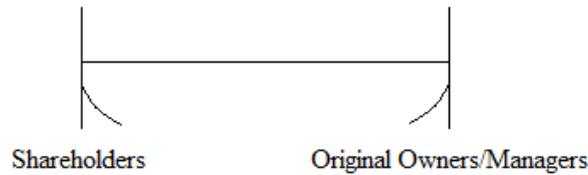
An ownership structure representing 100% ownership by shareholders will also be welcomed by the original owners/managers since for high values of θ , shareholders have a very high valuation of the public firm due to the strong protection provided to them by the legal system and any tempt to bring them in as shareholders will result in an increase of value to the original owners/managers. In legal regimes with the strongest shareholder protection, very high values of θ , the value of the public firm exceeds its value being private. La Porta et al. (2002) find evidence of higher valuation of firms in countries with better protection of shareholders. They argue that with better protection of outside investors, outside investors are willing to pay more for financial assets such as equity and debt. They pay more because they recognize that, with better legal protection, more of the firm's profits would come back to them as interest or dividends as opposed to being expropriated by the entrepreneur who controls the firm. By limiting expropriation, the law raises the price that securities fetch in the marketplace. Castillo and Skaperdas (2005) examine how the legal protection of outside shareholders and the appropriative costs that they induce influence the incentives for private firms to go public. The authors find that overall, higher protection of outsiders increases the likelihood of going public.

Similarly the reason that the graph ends at a negative value on the right is due to local, readily acceptable information. As mentioned above, shareholders have a very high valuation of the public firm due to the strong protection provided to them by the legal system and any tempt to bring them in as shareholders will result in an increase of value to the original owners/managers. In legal regimes with

strongest shareholder protection, very high values of θ , the value of the public firm exceeds its value being private and the original owners will decide to take the firm public. Castillo and Skaperdas (2005) find that for strong protection of shareholders the value of the public firm will be higher than the value of the private firm for all ownership structures and it would be optimal for the firm to go public.

In both cases discussed above shareholders will want to invest in the public company by buying shares and the original owners/managers will want to take their company public and sell shares to outside investors which implies that the dynamics is represented by the movement away from the $s = 0, m = 1$ endpoint, the right-hand endpoint.

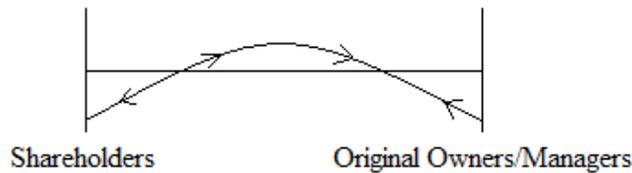
Figure 6. Local information in the case when the efficiency of the judiciary and law enforcement system is strong



The graph connecting the two endpoints shown in Figure 6 above may or may not cross the x-axis. When it does cross the x-axis it will cross it an even number of times. In the following argument we will show that in order to connect the endpoints, in general the slope of the connecting curve must be zero. Obviously the slope of the curve when it does not pass through the x-axis is equal to zero. This implies that the sum of the signs of the slopes of the curve when it

does pass through the x-axis must be equal to 0. The reason is that starting at the $s = 1, m = 0$ endpoint, the left-hand endpoint, whenever the curve crosses the x-axis upwards (with the sign of the slope equaling to +1) it has to go back downwards to cross the axis again (with the sign of the slope equaling to -1) as illustrated by the example shown in Figure 7 below. The sum of the signs from the pairs of crossings will always add up to the global value of zero.

Figure 7. Dynamics in the case when the efficiency of the judiciary and law enforcement system is strong



Following the argument in Saari (2002), we can see that by knowing the local information and that the change is continuous, we do know that there are at least two equilibria and, generically, there is an even number of them. Also, because the sign of the curve

must alternate at each crossing of the x-axis, we know that, generically, the stability of the equilibria must alternate between being stable and unstable. The above argument is illustrated in Figure 8 below.

Figure 8. Two examples of the dynamics in the case when the efficiency of the judiciary and law enforcement system is strong

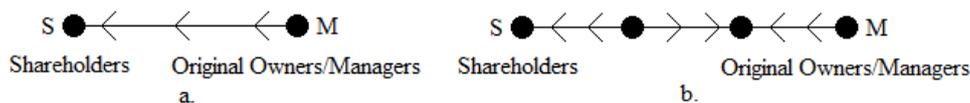


Figure 8 above tells us that the model of interaction of original owners/managers and shareholders has at least two plausible equilibria: one with all shareholders and second with all original owners/managers. Examining Figure 8a above we can observe two pairs of equilibria where one is an attractor (shareholders) and the other is a repeller (original owners/managers). According to Saari

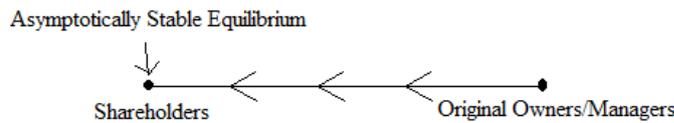
(2002), “The local information can be accompanied by more interesting kinds of market dynamics.” By examining Figure 8 above, we see that regardless of the form the dynamics assumes; it has to introduce new pairs of equilibria where one is an attractor and the other is a repeller. The simple case offered in Figure 8b introduces two more equilibria and a threshold effect. Even though we do not know where

the threshold point is, knowing that it exists is important for further analysis. For example the threshold in the going public example tells us that in order to increase the valuation of the company by original owners/managers it is necessary to increase their control in the public company through increasing their share of ownership. McConnell and Servaes (1990) find a significant curvilinear relation between Tobin's Q and the fraction of shares owned by corporate insiders. Q first increases, then decreases, as share ownership becomes concentrated in the hands of

managers and members of the board of directors. The authors also find a significant positive relation between Q and the proportion of shares held by institutional investors.

Returning to the case when the efficiency of the judiciary and law enforcement system is strong and considering a private firm initially fully owned by its founders, it is easy to prove that "Shareholders" is a globally asymptotically stable equilibrium since every possible initial state in the interior of the state space is carried to it by the dynamics [Skyrms, 2000].

Figure 9. The asymptotically stable equilibrium in the case when the efficiency of the judiciary and law enforcement system is strong



Examining Figure 9 above we see that small changes in ownership structure away from the equilibrium "Shareholders" will be carried back to the equilibrium by the dynamics. As argued above, shareholders have very high valuation of the public firm due to the strong protection provided to them by the legal system and any tempt to bring them in as shareholders will result in an increase of value to the owners/managers. Large and small changes in ownership structure away from the equilibrium "Shareholders" will be resisted by the original owners/managers due to the decrease in financing available to the firm and by outside shareholders due to the small expropriation risk by the original owners/managers and will be carried back to the equilibrium by the dynamics as shown in Figure 9 above.

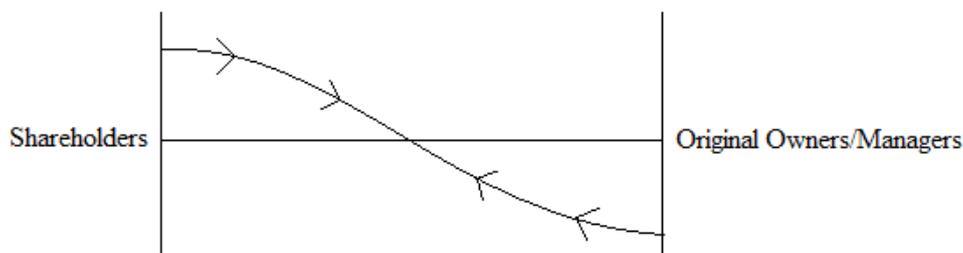
Castillo, R., & Skaperdas, S. (2005) examine how the legal protection of outside shareholders and the appropriative costs that they induce influence the incentives for private firms to go public. Their results indicate that a higher degree of protection of outside shareholders increases the likelihood of going public. Pagano, M., & Roell, A. (1998) find that a stronger legal protection of shareholders promotes cooperation among external shareholders and hinders collusion between the entrepreneur and potential monitors.

Pagano, M., & Roell, A. (1998) model predicts that improved legal protection of shareholders increases companies' propensity to go public. This prediction is consistent with the cross-country evidence reported by La Porta et. al. (1997), who show that better legal protection of shareholders' rights is associated with a larger value and number of publicly listed companies and of initial public offering. La Porta et. al. (2000) argue that legal protection of outside investors makes the expropriation technology less efficient. At the extreme of no investor protection, the insiders can steal a firm's profits perfectly efficiently. Without a strong reputation, no outsider would finance such a firm.

3.3 Case iii: The case when the efficiency of the judiciary and law enforcement system is moderate

Figure 10 below shows the dynamics for the case when the efficiency of the judiciary and law enforcement system is moderate or θ is between zero and one. We can see that the graph starts at a positive value on the left (reflecting the movement to the right) and ends at a negative value on the right (reflecting the movement to the left).

Figure 10. Dynamics in the case when the efficiency of the judiciary and law enforcement system is moderate



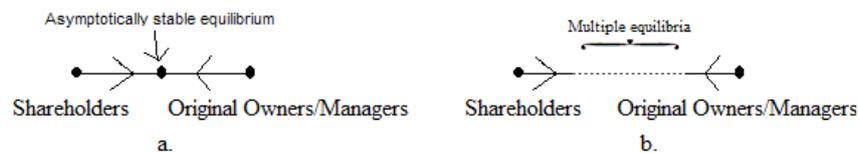
The reason that the graph starts at a positive value on the left is due to shareholders favoring a smaller ownership stake in an effort to align original owners/managers objectives with their own. McConnell and Servaes (1990) find a significant curvilinear relation between Tobin's Q and the fraction of shares owned by corporate insiders. Q first increases, then decreases, as share ownership becomes concentrated in the hands of managers and members of the board of directors. The reason that the graph starts at a negative value on the right is due to original owners/managers desire to avoid a possible decrease in payoff due to the erosion of their powers to influence firm policies. Zingales (1995) argues that if the original owners/managers choose to retain too few cash flow rights, then they might lose the incentive to sell control to a more efficient buyer later on. This will completely eliminate any surplus from trade, decreasing the value of the company and the wealth of the original owners/managers.

As illustrated in Figure 11 below, I will argue that a shared ownership structure by both the original owners/managers and by shareholders is the only globally asymptotically stable equilibria. To prove that the equilibria points shown in Figure 11 below are globally asymptotically stable equilibria, we need to

prove that every possible initial state in the interior of the state space is carried to it by the dynamics [Skyrms, 2000]. Starting at the asymptotically stable equilibrium shown in Figure 11a below, any move to increase shareholders ownership share in the public firm, will be resisted by both original owners/managers and by shareholders. Original owners/managers would want to avoid a possible decrease in payoff due to the erosion of their powers to influence firm policies. Shareholders on the other hand, would favor a smaller ownership stake in an effort to align original owners/managers objectives with their own.

Similarly starting at the asymptotically stable equilibrium shown in Figure 11a below any move to decrease shareholders ownership share in the public firm, will be resisted by both original owners/managers and by shareholders. There is an incentive for both owners/managers and shareholders to increase the ownership share by shareholders. Increased ownership share by shareholders will increase the amount of equity financing and increase the size of the firm. Similar arguments can be made for any of the many asymptotically stable equilibria shown in Figure 11b below.

Figure 11. Two examples of the dynamics in the case when the efficiency of the judiciary and law enforcement system is moderate

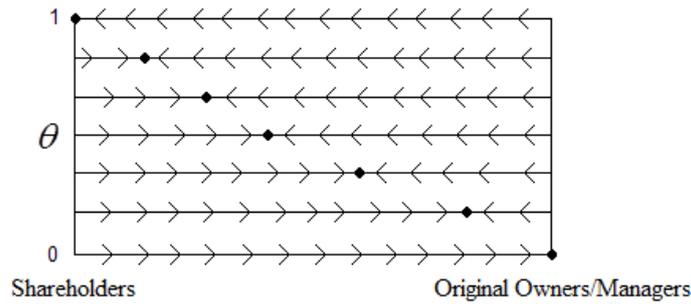


From the three cases explained above, we can now represent an example describing the dynamics for the range of θ between 0 and 1. As shown in Figure 12 below, the dynamics will be a function of both the ownership share and the degree of legal protection of outside shareholders from expropriation by the insiders, θ . For low values of θ or when the legal protection of outside shareholders is weakest, it is optimal for the original owners/managers to keep the firm private. For moderate values of θ or when the legal protection of outside shareholders is moderate, it is optimal for the original owners/managers to sell a fraction of the private firm to outside shareholders. The share of ownership sold to outside shareholders is increasing the legal protection of outside shareholders. Finally for high values of θ or when the legal

protection of outside shareholders is strongest, it is optimal for the original owners/managers to sell the majority of the shares to outside shareholders while staying on as managers of the public firm.

Consistent with my results, Boubakri, Cosset, & Guedhami (2005) investigate the role of ownership structure and investor protection in post-privatization corporate governance. Using a sample of 209 privatized firms from 39 countries over the period 1980 to 2001, they find that investor protection explain the cross-firm differences in ownership concentration. The positive effect of ownership concentration on firm performance matters more in countries with weak investor protection.

Figure 12. An example describing the dynamics for the range of the efficiency of the judiciary and law enforcement system is moderate



4 Going public by selling shares to large and dispersed shareholders

The owners/managers decide to take their privately held firm public and sell a share of the firm to both large and dispersed shareholders. Consistent with Saari (2002), we will assume that it makes sense to

talk about any pair of the aforementioned agents without the third. Again, we will build our model based on the analysis in Saari (2002), where the description of the dynamic can be captured on an equilateral triangle representing the simplex:

$$\{(DS, LS, M) \mid DS + LS + M = 1; DS, LS, M \geq 0\} \tag{1}$$

Where M represents the share of original owners/managers, DS represents the share of dispersed shareholders and LS represents the share of large shareholders.

Figure 13. A shared ownership structure by the original owners/managers, large shareholders, and dispersed shareholders

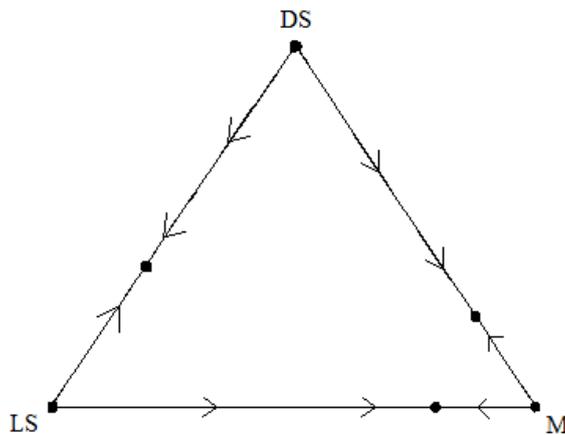


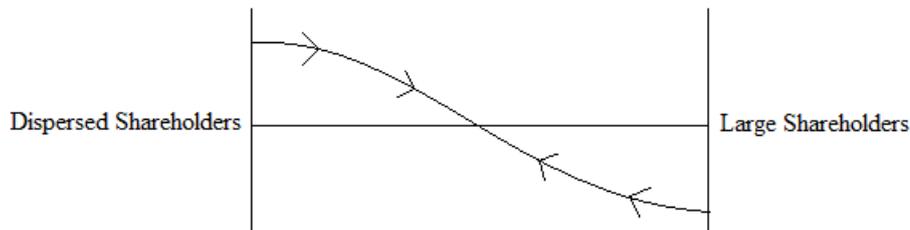
Figure 13 above provides a picture of the global interactions of the pairs where the dynamics of the bottom edge, $\{(LS, M) \mid LS + M = 1; LS, M \geq 0\}$, and right edge, $\{(DS, M) \mid DS + M = 1; DS, M \geq 0\}$, are discussed in section 3 above. The description of the dynamic between large and dispersed shareholders can be captured on left edge and can be represented by $\{(DS, LS) \mid DS + LS = 1; DS, LS \geq 0\}$. In order to illustrate the left edge dynamics, we assume that there is an infinitesimal amount of expropriation effort exerted by owners/managers.

zero and one. The reason that the graph starts at a positive value on the left is due to the fact that dispersed shareholders free ride⁴⁵ on the monitoring efforts exerted by large shareholders and the absence of large shareholders would mean a great opportunity for original owners/managers to expropriate the ownership share of dispersed shareholders. Shleifer and Vishny (1986), show that large shareholders raise expected profits and the more so the greater their percentage of ownership. McConnell and Servaes (1990) find a significant positive relation between Tobin's Q and the proportion of shares held by institutional investors.

Figure 14 above shows the left edge dynamics for the case when the efficiency of the judiciary and law enforcement system is moderate or θ is between

⁴⁵ Hart, O. (1995).

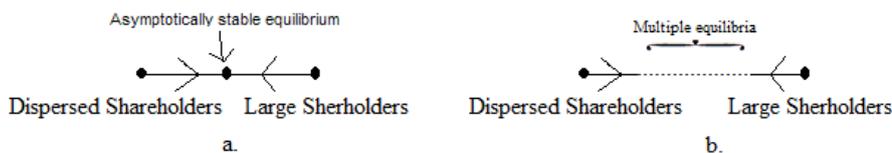
Figure 14. Dynamics between large and dispersed shareholders in the case when the efficiency of the judiciary and law enforcement system is moderate



The reason that the graph starts at a negative value on the right is due to large shareholders desire to increase the ownership share by dispersed shareholders in order to increase the amount of equity financing available to the public firm and hence increase investment in the public firm. Dispersed shareholders on the other hand would have the opportunity to invest in a business venture with more than optimal monitoring efforts exerted by large shareholders. Burkart, Gromb, and Panunzi (1997) propose that dispersed outside ownership and the resulting managerial discretion come with costs but

also with benefits. For example, a dispersed ownership structure assures the manager that shareholders will interfere little, inducing him to show initiative. According to the authors, this gain has to be weighed against the loss in control due to inadequate monitoring. Conversely, a concentrated ownership induces high levels of monitoring and control but renders management less active. Hence, the ownership structure is an instrument to solve the trade-off between control and initiative because it determines the shareholders' incentives to monitor.

Figure 15. Two examples of the dynamics between large shareholders and dispersed shareholders in the case when the efficiency of the judiciary and law enforcement system is moderate



To prove that the equilibrium point shown in figure 15a above is a globally asymptotically stable equilibrium, we need to prove that every possible initial state in the interior of the state space is carried to it by the dynamics [Skyrms 2000]. Any move to increase the ownership of dispersed shareholders to the left of the equilibrium point, will be resisted by both large and dispersed shareholders. As discussed above, dispersed shareholders free ride on the monitoring efforts exerted by large shareholders and any decrease in the ownership share of large shareholders would mean a greater opportunity for original owners/managers to expropriate part of the ownership share of dispersed shareholders. So starting at 100% ownership by dispersed shareholders, it would be necessary for dispersed shareholders to bring in large shareholders.

For ownership structures to the right of the equilibrium, smaller ownership share by dispersed shareholders, there is an incentive for both large and dispersed shareholders to increase the ownership share by dispersed shareholders. As discussed above, starting at 100% ownership by large shareholders, large shareholders can increase the value of their investment in the public firm by selling to dispersed shareholders which will increase the amount of equity

financing available to the firm. Dispersed shareholders on the other hand would have the opportunity to invest in a business venture with higher than optimal monitoring efforts exerted by large shareholders. Similar arguments can be made for any of the many asymptotically stable equilibria shown in Figure 15b above. La Porta et al. (1998) argue that heavily concentrated ownership results from, and perhaps substitutes for, weak protection of investors in a corporate governance system. This leads us to conclude that for the case when the efficiency of the judiciary and law enforcement system is weak or θ is close to zero we will be closer to the LS = 1, DS = 0 endpoint while for the case when the efficiency of the judiciary and law enforcement system is strong or θ is close to one we will be closer to the LS = 0, DS = 1 endpoint.

Next, we need a local analysis to indicate what happens near the three equilibria located on the edges. In Figure 16 below for example, the replicator dynamics near the equilibrium on the bottom edge, a small ownership share by dispersed shareholders would be welcomed by both original owners/managers (dispersed shareholders act as free riders in our model and they exert no effort in monitoring the manager) and by large shareholders (dispersed shareholders

increase the amount of financing available to the firm and consequently increase the value of the firm). The share of dispersed shareholders would flourish and this is denoted by an arrow pointing upwards rather than downwards.

The replicator dynamics near the equilibrium on the right edge, a small ownership share by large shareholders would be welcomed by the original owners/managers who own the majority of the firm (original owners/managers need large shareholders for self-monitoring otherwise dispersed shareholders will hold a low valuation of the firm). The share of large shareholders would flourish and this is denoted by an arrow pointing outwards rather than inwards.

The replicator dynamics near the equilibrium on the left edge, a small share by the original owners/managers would be welcomed by large and dispersed shareholders (shareholders would favor an ownership share by original owners/managers in an effort to align original owners/managers objectives with their own). In this setting the share of original owners/managers would flourish and this is denoted by an arrow pointing outwards rather than inwards.

According to Saari (2002), in order for us to characterize all possible choices of the global dynamics inside the simplex, we need a measure that

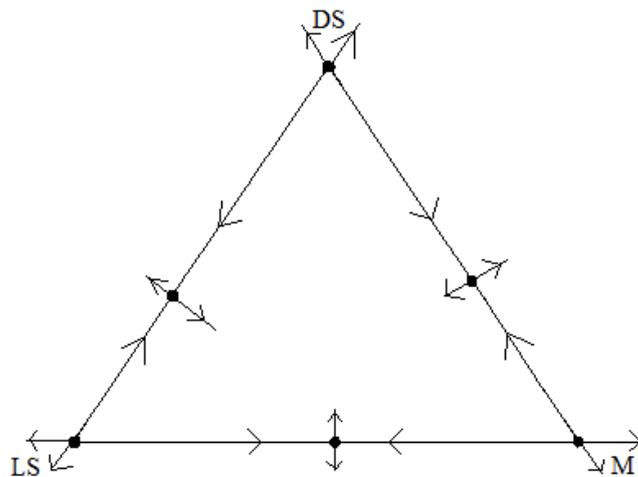
replaces the slope measure we used in the analysis above. According to Saari (2002), “The “slope” analysis fails if only because such a representation requires replacing the line with a two-dimensional surface and the two-dimensional setting for the graph with a four-dimensional one. Fortunately, something called the “winding number” (see Milnor (1969) for a description) captures what we need.”

Our next step is to calculate the “winding number” where we will use the methods described in Saari (2002). For purposes of the next step, the arrows at each of the equilibria described in Figure 13 above are symmetrically extended outside of the triangle as shown in Figure 16 below. The arrows are symmetrically extended according to the following criteria:

- i) If a solid arrow points toward an equilibrium from the simplex, we place a symmetric and companion solid arrow from outside the simplex pointing toward the same equilibrium
- ii) If the original arrow points away from an equilibrium from the simplex, we place a symmetric and companion solid arrow from outside the simplex pointing away from the same equilibrium.

The above is shown in Figure 16 below. All local information now is provided.

Figure 16. Local information in the case when the efficiency of the judiciary and law enforcement system is moderate



The next step is to characterize all possible choices of the global dynamic inside the simplex using the “winding number” method. To use the “winding number” method we need to add dashed arrows as shown in Figure 17 below. According to Saari (2002), “Everything takes place along a curve outside of the triangle. When traveling along this curve in a counterclockwise direction, compute the number of times the “arrows” make a complete revolution in a counter-clockwise direction. Since we start and end at the same place, this number of revolutions must be an integer. Of course, if the rotations are in a clockwise direction, the answer will be negative.”

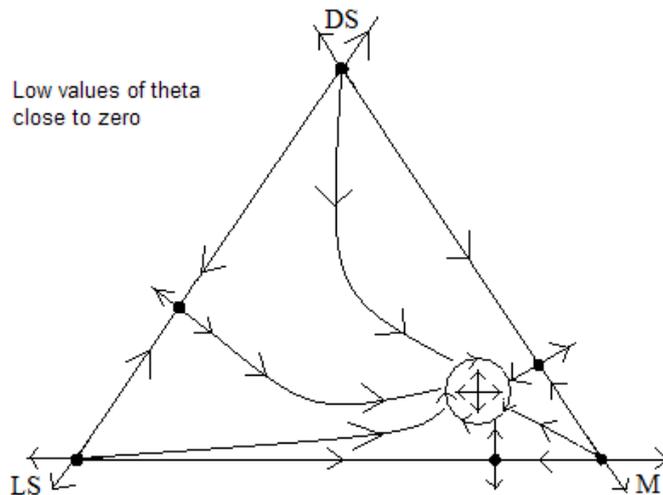
We will use Figure 17 to compute the “winding number.” According to Saari (2002), we need to imagine a path very close to the outside of the triangle. At the starting point shown in Figure 17 above, the dynamic is moving outwards to the right. A way to compute the “winding number” is to put a pencil on this dashed line with the point in the direction of the arrowhead. We then move the pencil around the triangle with the pencil’s point always pointing in the direction indicated by the arrows. The objective is to count the number of times the pencil rotates when it completes a whole circle around the triangle. The dashed arrows indicate the directions the pencil is pointing as it moves around the triangle.

4.1 Case i: The case when the efficiency of the judiciary and law enforcement system is weak

law enforcement system is weak, the ownership share sold to outside shareholders is small and the equilibrium will be close to the lower right vertex where the company will stay private or the majority of the ownership share will stay with the original owners.

As shown in Figure 18 below and as discussed above, in the case when the efficiency of the judiciary and

Figure 18. Dynamics in the case when the efficiency of the judiciary and law enforcement system is weak

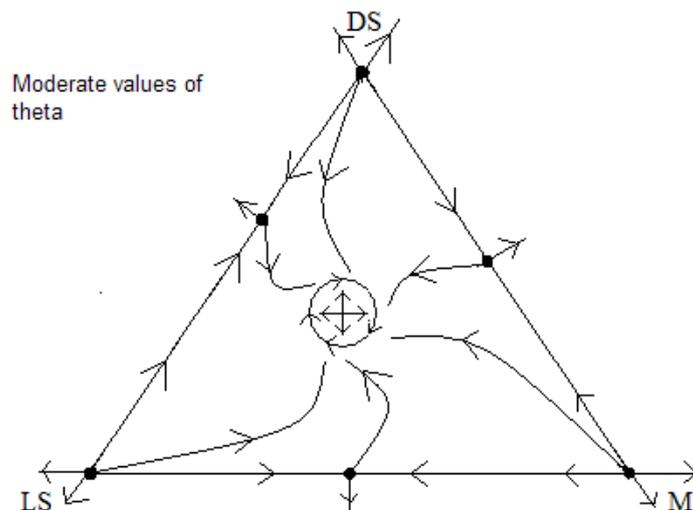


4.2 Case ii: The case when the efficiency of the judiciary and law enforcement system is moderate

share sold to outside shareholders will increase as compared to case i above and the equilibrium will be close to the center of the simplex where the public company will have a more balanced share ownership with the original owners practicing some influence over the firm's decisions.

As shown in Figure 19 below and as discussed above, in the case when the efficiency of the judiciary and law enforcement system is moderate, the ownership

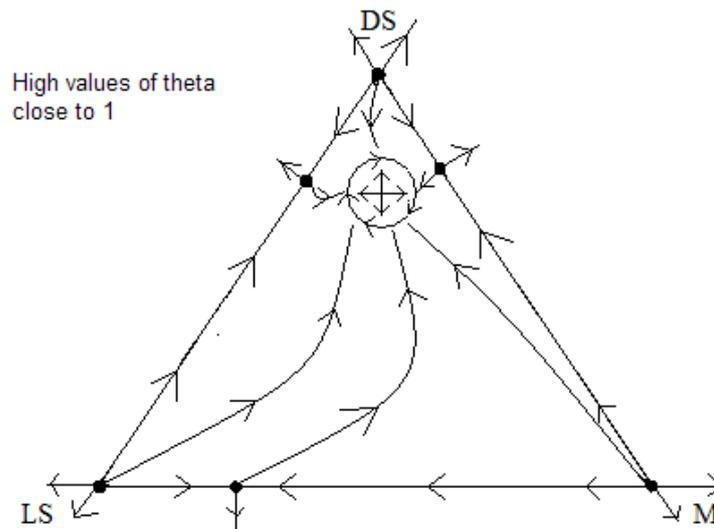
Figure 19. Dynamics in the case when the efficiency of the judiciary and law enforcement system is moderate



4.3 Case iii: The case when the efficiency of the judiciary and law enforcement system is strong

and law enforcement system is strong, the ownership share sold to dispersed shareholders will increase as compared to case i and case ii above and the equilibrium will be close to the upper vertex where the public company will have a more dispersed ownership share with the original owners staying on as managers.

As shown in Figure 20 below and as discussed above, in the case when the efficiency of the judiciary

Figure 19. Dynamics in the case when the efficiency of the judiciary and law enforcement system is strong

Using an attractor for the inner equilibrium instead of a repeller will not change the results qualitatively. In the attractor case, all motion moves inwards toward an equilibrium forcing a position of coexistence of original owners/managers, large shareholders and dispersed shareholders.

Consistent with Burkart, Panunzi, & Shleifer (2003) we have shown that, in legal regimes that successfully limit the expropriation of minority shareholders, the widely held professionally managed corporation emerges as the equilibrium outcome. In legal regimes with intermediate protection, the original owners stay on as large shareholders to protect their interest in the public firm. In legal regimes with the weakest protection, the ownership and control remains with the original owners and the firm remains private.

López de Silanes, La Porta, Shleifer and Vishny (1998) find that the concentration of ownership of shares in the largest public companies is negatively related to investor protections. The authors argue that their finding is consistent with the hypothesis that small, diversified shareholders are unlikely to be important in countries that fail to protect their rights.

5 Conclusion

I have used qualitative evolutionary game theoretic techniques presented in Saari (2002) to model the interaction between managers and shareholders and describe the equilibrium ownership structure in public firms arrived at in different legal environments. By knowing the local, readily acceptable information and that the change is continuous I was able to provide considerable qualitative information about the global dynamics. The above analysis is based on examples and scenarios of minimum complexity. More work need to be done to examine the effect of added inner equilibria and prove the implausibility of other equilibrium strategies.

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