IS A LARGER EQUITY MARKET MORE INFORMATION EFFICIENT? - EVIDENCE FROM INTERVALLING EFFECT

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Abstract

This paper investigates the impact of equity return autocorrelation on financial market efficiency via intervalling effect. A simple model is proposed to show that the degree of intervalling effect is related to the security return autocorrelation. A more general version of Levy and Levhari hypothesis is proposed to find that the degree of the autocorrelations of the security and the market returns determines the existence and the direction of the intervalling effect and the size of the intervalling effect are dependent on the degree of the security autocorrelations. Empirical evidence of the latter is presented.

Keywords: CAPM Beta, Information Efficiency, Intervalling Effect

1. INTRODUCTION

Intervalling effect is a phenomenon that the CAPM beta varies with respect to the length of the return measurement interval. Intervalling effect has an important indication on the speed of information spread. According to Pogue and Solnik (1974, P&S hereafter), adjustment lags result from the failure of stock prices to fully adjust to market changes during the trading period (can be days, weeks etc). Due to this, the stock price will "catch up" for what has happened in the market during later trading periods. This indicates a distributed lag model of a stock return. Considering the time dependence of the CAPM beta, the intervalling effect might result from lags in the adjustment of stock prices to new information. In an information efficient market, where adjustment lags were absent, the expected value of the CAPM beta should be invariant to the length of the return measurement interval. Thus the difference in the betas estimated with different return interval measures the relative importance of adjustment lags in a stock market. Therefore relative difference of the betas could be an indicator for market information efficiency.

Intervalling effect makes sense only if betas are time dependent. In this sense, it seems that the intervalling effect is due to time conditioning. Hong and Satchell (2013, H&S hereafter) theoretically shows that the beta is a function of the length of the return measurement interval when returns are serially correlated. The pattern of the intervalling effect (increasing or decreasing with respect to the length of the measurement interval) is determined by the degree of asset return autocorrelation. From the result of H&S and that of P&S, two logical conclusions could be drawn: [1] Stock market efficiency could be measured by the magnitude of intervalling effect. [2] Return serial correlation in a stock market is closely related to stock market efficiency. This paper theoretically and empirically investigates these two statements.

Therefore, the objectives and contributions of this paper are clear. [1] It provides a more generalized theoretical model to show the result of H&S. [2] It empirically investigates how intervalling effect is related to market efficiency; therefore the study updates the result of P&S with more recent and wide set of data. [3] It investigates the empirical relationship between the market efficiency and the security return autocorrelation.

To achieve the first objective, I propose a more general model to investigate the intervalling effect. From a theoretical model with AR(1) return process, the paper finds that the longer return measurement interval enhances the information efficiency in the stock market. This indicates that the longer investment horizon is recommended for the information efficiency in the stock market. In order to achieve the second and the third objectives, this paper empirically investigates the impact of stock return autocorrelation to market efficiency via intervalling effect. The data base consists of daily price and return data for 373 stocks from 13 countries. This paper can be considered as an empirical version of H&S with more generalized asset price process and it can also be seen as an extension to P&S with stock return autocorrelation.

This paper provides empirical evidence that the size of the intervalling effect depends on the security return autocorrelation. The empirical result supports the theoretical findings. Since other popular risk measures such as VaR may suffer the same problem, I hope this paper could build a theoretical foundation and empirical framework to investigate intervalling effect in risk measures in general. The rest of the paper is organized as follows. Section 2 presents a simple model to investigate the relationship between the market efficiency measure, the ratio of betas, and the security return autocorrelation. Section 3 discusses the data, potential methodological issues and the empirical linkage between the market efficiency and the intervalling effect. Section 4 empirically shows that the ratio of betas can be explained by the security return autocorrelation. Then Section 5 concludes the paper.

2. THE MODEL

2.1. The market model

The model is based on the hypothesis that the return on security i of a country j during interval *t*, $r_i^j(t)$, is a return on security *i* of country *j* during the time interval t and excess return of security *i* is a linear function of the excess market risk return of country *j*, $\left(r_m^j(t)-r_f^j(t)\right)$. Hence the security returns are generated by

$$r_i^j(t)-r_f^j(t)=\alpha_i^j+\beta_i^j\left(r_m^j(t)-r_f^j(t)\right)+\varepsilon_i^j(t) \qquad (1)$$

Where $r_i^j(t)=\dfrac{P_i^j(t+1)-P_i^j(t)}{P_i^j(t)}$ and $r_m^j(t)=\dfrac{P_m^j(t+1)-P_m^j(t)}{P_m^j(t)}$

β_i is the security beta, measures the sensitivity of the return on security i as a result of a change in the market index return, $r_m(t)$. α_i measures the change in $r_i(t)$ that is independent of a change in the index return. $\varepsilon_i(t)$ represents the non-market related component of security return and it is usually assumed to have zero expected value and iid. Using these assumptions, it is well known that the beta parameter is given by

$$\beta_i=\frac{Cov\left(r_i(t),r_m(t)\right)}{Var\left(r_m(t)\right)} \qquad (2)$$

According to this model, neither α_i nor β_i depend on the length of the differencing interval used to calculate the returns. The estimates of the both parameters of (1), obtained using an ordinary least square regression, are, however, strongly dependent on the length of the differencing interval as P&S, Hawawini (1980) and Cohen et al. (1983) have found. To gain some insight into the nature of α_i parameter, I rely on the equilibrium predictions of Capital Asset Pricing Model (CAPM). The CAPM relates the expected security risk premiums to their systematic risk coefficients, β_i and therefore α_i can be seen as a measure of the return on security *i* in excess of that predicted by the CAPM. Under CAPM assumptions the expected value of α_i is equal to zero.

2.2. Market Efficiency Indicator

Adjustment lags result when equity prices fail to fully reflect the market changes during the trading period. Instead the adjustment process of the stock price may extend over multiple periods. This can happen by non or infrequent trading of the stock, or not successfully following and reflecting market conditions when delays in index reporting exist. The result is that the price will "catch up" for previous market activity during later days. As P&S argue, this phenomenon would imply a distributed lag model for explaining security returns. They state, "This problem would also tend to disappear as the measurement interval increased. The adjustment lags will be more important in European markets, where trading volume is typically much lower than for U.S. stocks, and where reliable market information is more difficult to obtain."

Proposition 2 of this paper provides mathematical evidence of this statement. Although their statement regarding European financial markets would have been true in 1974, this does not necessarily hold true in 2009. The point is that if the market is less efficient, the intervalling effect would be more prominent. Therefore the relative size of betas of various return measurement interval would be a good proxy for the degree of market efficiency. For each security this ratio can be calculated as

$$\varphi_i^j=\frac{\overline{\beta}_{i,Monthly}^{\,j}}{\overline{\beta}_{i,Daily}^{\,j}} \qquad (3)$$

H&S show that the monthly beta can be larger or smaller than the daily beta, hence equation (3) does not necessarily be larger than 1.

2.3. Intervalling Effect and Autocorrelation

In this section, I present a general statistical model to investigate how the beta is affected by autocorrelation in market return and security return and also by cross correlation between market and security returns. Key difference of this section from H&S is that it no longer assumes a specific stochastic process for asset price. Instead I take a general approach, only assuming stable mean and variance of returns with respect to time.

We face two different kinds of return correlation when investigating intervalling effect in beta, time series correlation and cross correlation. We denote asset and market return autocorrelation as $\rho_i(h)$ and $\rho_m(h)$ and cross correlation between asset and market return as $\rho_{im}(h)$. In this section, I present a simple model to investigate the same relationship without assuming a specific stochastic process for market and asset prices. Proposition 1 generalizes the result of H&S. Investigating such relationship reveals that the security return autocorrelation causes intervalling effect.

The security return generating process, equation (1), is assumed again but with a specification of return measurement interval, *h*. Let $R_i(t,t+h)$ be asset *i* excess return over the risk free interest rate measured between time 0 and *h*, $R_t(t,t+h)$ be the market excess return measured between the same time periods. Assume that security return follows AR(1) process. The security return generating process can be rewritten as

$$\begin{cases} R_i(t,t+h)=\alpha_i+\beta_i R_m(t,t+h)+\varepsilon_i(h) \\ R_i(t,t+1)=\rho_i(1)R_i(t-1,t)+\eta_i(1) \end{cases} \qquad (4)$$

Where $\rho i(h)$ stands for *h* period autocorrelation within time series of security *i* and $\rho_{im}(h)$ stands for *h* period cross correlation between security *i* and market return. In this model, I make an assumption that the autocorrelation and the cross correlation of security and the market return are time variant. The CAPM postulates that the expected return on an asset above the risk-free rate is linearly related to the non-diversifiable risk as measured by the asset's beta. This leads to a testable hypothesis that the alpha, the return in excess of the compensation for the risk borne or active return on an investment is zero. Since the focus of this paper is to investigate the relationship between information efficiency and the autocorrelation, I exclude any systematic mispricing of risk. I set the expected value of alpha to be zero. The purpose is to separate the impact of autocorrelation to market efficiency from any other causes.

Computing everything conditional on time t = 0, the security and market return generation AR(1) process can be written as

$$R_i(t,t+1) = \rho_i(1)^t R_i(0,1) + \eta_i(1) \tag{5}$$

Simplifying the excess return notation $R_t(t,t+h)$ as $R_t(h)$, we get Remark 1.

Remark 1: Based on equation (5), the h period return of the security i and the market m can be written as

$$R_i(h) = \prod_{l=1}^{h}\left(\rho_i(1)^{(l-1)} R_i(0)\right) - 1 \tag{6}$$

Proof of Remark 1 is omitted. The distribution of excess asset and market return can be written as

$$\begin{pmatrix} R_i(h) \\ R_m(h) \end{pmatrix} \sim \left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_i(h)^2 & \rho_{im}(h)\sigma_i(h)\sigma_m(h) \\ \rho_{im}(h)\sigma_i(h)\sigma_m(h) & \sigma_m(h)^2 \end{pmatrix} \right) \tag{7}$$

Where $\sigma(h)$ stands for standard deviation. Note that the model is time dependent and all computations are conditional on time t = 0. Subscript i indicates security i and m indicates market. If we assume volatility to be stable over time and let $\sigma_j(h)$ be asset and market volatility over one period, where again, j = i and m, $\sigma_j(h) = \sigma_j(1) \times \sqrt{h}$ would hold. Let h_1 and h_2 be two different return measurement intervals, where $h_1 < h_2$, then betas over h_1 and h_2 can be written as

$$\beta_i(h_1) = \frac{\rho_{im}(h_1)\sigma_i(h_1)\sigma_m(h_1)}{\sigma_m(h_1)^2} \quad \beta_i(h_2) = \frac{\rho_{im}(h_2)\sigma_i(h_2)\sigma_m(h_2)}{\sigma_m(h_2)^2} \tag{8}$$

Lemma 1: The ratio of the two betas, φ, can be expressed as

$$\varphi = \frac{\beta_i(h_2)}{\beta_i(h_1)} = \frac{\rho_{im}(h_2)}{\rho_{im}(h_1)} \tag{9}$$

The proof of Lemma 1 can be found in Appendix 1. Proposition 1 shows that the fundamental reason for intervalling effect is that $\rho_{im}(h_1)$ is different from $\rho_{im}(h_2)$. The gap between the two determines the size of adjustment lags therefore have significant influence on market efficiency.

Lemma 2: The relationship between time series correlation and cross correlation between market and a security return under current setting can be expressed as

$$\rho_{im}(h_2) = H \cdot \rho_i(1)^{(h_2-h_1)} \cdot \rho_{im}(h_1) \tag{10}$$

Where $H = (h_1/h_2)$^1/2

The proof of Lemma 2 can be found in Appendix 2.

Proposition 1: The Levi-Levhari Hypothesis applied to the AR(1) Model

If $\rho_i(1) > H^{\frac{1}{h_2-h_1}}$, CV > 1, then $\beta_i(h_2) > \beta_i(h_1)$

If $\rho_i(1) < H^{\frac{1}{h_2-h_1}}$, CV < 1, then $\beta_i(h_2) < \beta_i(h_1)$

If $\rho_i(1) = H^{\frac{1}{h_2-h_1}}$, CV = 1, then $\beta_i(h_2) = \beta_i(h_1)$

Where $CV = H \cdot \rho_i(1)^{(h_2-h_1)}$ and $H = (h_1/h_2)$^1/2

The proof of Proposition 1 can be found in Appendix 3. This is a generalized version of the critical value (CV) in H&S (See Proposition 5 of H&S). The result shows that the intervalling effect on the beta depends on the magnitude of the security and the market return autocorrelation. We see that when there is no return autocorrelation, hence $\rho_i(1) = 1$ and $\rho_m(1) = 1$, we recover the result of Levy and Levhari (1977) where the beta of aggressive stocks are over-estimated with the opposite occurring for defensive stocks. We get the consistent result with H&S when $\beta i(h_1)$ corresponds to the instantaneous beta hence h_1 converges to 0. Proposition 1 also indicates monotonicity of time dependent beta with respect to the length of the return measurement interval.

Proposition 2: Combining Lemma 1 and Lemma 2, we can show that the value of φ is determined by time series correlation within a security and market return

$$\varphi = \begin{cases} H \cdot \rho_i(1)^{(h_2-h_1)} & \text{if } \beta_i(h_2) \neq \beta_i(h_1) \\ 1 & \text{if } \beta_i(h_2) = \beta_i(h_1) \end{cases} \tag{11}$$

The proof of Proposition 2 is omitted. Proposition 1 and 2 are the main findings of the theoretical part of this paper and there are several implications.

[1] Proposition 1 show that the intervalling effect can cause the beta to be over or underestimated depending on the magnitude of the return autocorrelations.

[2] Proposition 1 generalizes the result of Levy and Levhari (1977) and H&S. It shows that the degree of the autocorrelations of the security and the market returns determines the existence and the direction of the intervalling effect. When there is no return autocorrelation, we recover the result of Levy and Levhari (1977).

[3] Proposition 2 shows why the intervalling effect disappears as the measurement interval increases. (See the quote from Pogue and Solnik the

previous section) As the gap between h_2 and h_1 becomes large, φ becomes 0 and the intervalling effect disappears.

[4] Proposition 2 shows that the size of the intervalling effect is dependent on the degree of the security autocorrelations.

Implication [1] is to confirm what H&S finds with more generalized framework. Implication [2] is an extension of Levy and Levhari (1977) and H&S. Implication [3] theoretically shows what has been qualitatively argued in many of the previous literatures including P&S. Therefore, implication [2] and [3] are the main theoretical findings of this paper. Implication [4] indicates the relationship between degree of the security return autocorrelations and the size of the intervalling effect. In order to provide some empirical evidence to the implication [4], I empirically investigate following hypothesis in section 4: The size of intervalling effect depends on the security and market return autocorrelation.

3. EMPIRICAL EVIDENCE

3.1. The data

The data base consists of daily and monthly prices for 373 common stocks of 13 countries. The time period covered is from January 2009 to December 2011. The list of the countries, indices and risk free rate used are shown in Table 1. There were two sampling criteria. [1] Within each country, the companies in current sample tend to be the largest in terms of index composition weight as of September 2012. [2] The security price data should be available during the entire sample period. This is to select large, liquid stocks that can represent the market index. 30 might seem to be very little, but this usually comprises a significant portion of in total market. 30 largest stocks represent at least 60% and sometimes up to 90% of the sample equity market indices. For simplicity I use generic government bond rate, provided by Bloomberg as proxies for risk free rates. In such low interest rate period with not much volatility in interest rates, I believe they sufficiently represent risk free rates.

Table 1. Summary of sample data

Country (j)	Sample Size (= Nj)	Market Index
US	29	Dow Jones
UK	30	FTSE 100 Index
France	30	CAC 40 Index
Germany	30	DAX 30 Index
Japan	30	NIKKEI 225 Index
Brazil	30	Bovespa Index
Korea	30	KOSPI Index
Indonesia	30	Jakarta Composite Index
Singapore	21	FTSE Straits Time Index
Italy	30	FTSE MIB Index
Spain	30	IBEX 35 Index
Netherlands	23	AEX Index
Thailand	30	SET Index

Security log returns are computed on a daily and monthly basis using adjusted price data provided by Bloomberg. Table 2 presents the annualized descriptive statistics for the sample.

Table 2. Annualized sample descriptive statistics: daily return intervals (all figures in %)

Country (j)	Securities Return		Market Index		Risk Free Rate
	Mean	Std Dev	Mean	Std Dev	Mean
US	1.548	31.533	8.191	20.608	0.101
UK	2.108	33.717	4.571	20.749	0.656
France	-2.615	35.692	-5.148	26.056	0.555
Germany	-1.163	35.752	2.383	25.388	0.431
Japan	-6.375	34.693	-5.104	24.360	0.133
Brazil	7.210	39.249	8.685	25.595	10.495
Korea	10.396	39.838	13.485	22.489	2.487
Indonesia	26.826	42.341	36.099	22.584	6.669
Singapore	12.431	29.971	10.840	19.759	0.377
Italy	-12.553	37.300	-12.750	30.086	1.165
Spain	-9.670	34.613	-6.873	27.507	0.968
Netherlands	11.157	36.128	9.391	23.624	0.504
Thailand	25.272	35.649	26.841	21.746	1.863

As expected, average returns and standard deviations on securities returns and market index returns tend to be higher for emerging markets. As the sample period includes the European financial crisis, France, Italy and Spain exhibited negative mean return on both securities and the market index. S&P500 performed relatively well compare to other developed markets during the sample period, having annual expected return of 8.191%. Most of countries experienced extremely low level of risk free interest rate during the sample period. During the sample period, three out of big fives (US, UK, Germany, France and Japan) reported negative mean security returns. For 9 out of 13 sample countries, the average performance of the large individual stocks was worse than that of the market index. The index volatility of all the sample countries' is lower than the average volatility of the large individual stocks. This indicates the diversification is effective during the sample period.

3.2. The Methodological Issues

As P&S point out, there are two methodological issues that need to be addressed when the relative magnitude of the market model parameter (beta) is estimated.

[1] Estimator Efficiency. The parameter estimated over shorter periods should have more information on an asset's risk profile, although it suffers from frictions in the trading process. Any grouping of the short interval data into longer interval data increases the standard errors of the estimate market model parameters. Therefore, ceteris paribus, the shortest interval data available would give the most efficient estimates of the coefficients. Hence estimator efficiency alone would suggest the use of an interval of minimum length.

[2] Measurement Errors. The measurement errors in reporting the price data of a security will generate noise in the security returns. This will cause the variance of the residual term to increase relative to when correct returns are used. This measurement error will reduce R^2 in the estimated equation. However P&S argue that "There will be no expected attenuation in the estimated beta coefficient as long as the measurement errors are not correlated with the market returns (a plausible assumption). Other than simple reporting errors of lags, a common source of error results from the rounding of prices to fractional values (e.g. quarters). This small source of error can be important for short return intervals. The effect of measurement errors will diminish as the length of the return interval increases. Thus, measurement errors alone would suggest the use of an interval of maximum length."

As these two issues contradict to each other in terms of usage of the interval length, it is not easy to conclude on the best interval to use. It is not the objective of this paper to suggest the best measurement interval. Instead, this paper shall investigate the empirical market efficiency in various equity markets. Measurement errors will only be addressed by investigating the ratio of average R^2 figures.

3.3. Market efficiency and intervalling effect

Table 3. Average estimated daily and monthly betas

Country	Average Daily Beta	Average Monthly Beta	Larger Beta
US	1.132	1.175	Monthly
UK	1.047	1.006	Daily
France	1.031	1.017	Daily
Germany	0.967	0.992	Daily
Japan	0.99	0.981	Daily
Brazil	0.994	1.105	Monthly
Korea	1.026	1.016	Daily
Indonesia	1.029	1.113	Monthly
Singapore	0.933	0.971	Monthly
Italy	0.846	0.857	Monthly
Spain	0.901	0.967	Monthly
Netherlands	1.062	1.156	Monthly
Thailand	0.96	0.988	Monthly

From Table 3, we can see that the average daily beta does not necessarily be smaller than the monthly beta hence the ratio of daily and monthly beta is not always larger than 1. P&S states that, "The estimated betas are lowest for daily returns, highest for monthly returns." Empirical results in Table 2 do not support this statement. Instead, this result is consistent with the result of Hong and Satchell (2013, H&S hereafter) where one factor model beta can be increasing or decreasing with respect to the length of the return measurement interval.

Levy and Lehvari (1977) theoretically argue that, the systematic risk of defensive stocks tends to decline while that for aggressive stocks tends to increase with increases in the investment horizon. According to this, if the beta of a stock is less than 1 it increases as the length of the interval increases while the opposite happens for stocks where the beta is more than 1. Therefore their result basically calls for convergence to 1 as the length of the interval increases. If this were true, equity markets in France, UK, Japan, Italy and Korea should be considered defensive while the rest should be considered aggressive based on the sample φ and we see no clear empirical evidence of Levy and Lehvari (1977) in Table 2. This is not surprising since Levy and Lehvari (1977) could not reconcile for this either. They state that "the second conclusion has rather negative implications. By trying various horizons we could not reconcile the gap between the capital asset pricing model and empirical evidence".

The objective of this and the following section is to test the result of H&S that the degree of return autocorrelation has significant influence on the 'implied' market efficiency and link it to market efficiency. I take both theoretical and empirical approaches. I investigate 373 individual securities to see whether return autocorrelation has significant influence on φ, the relative size of betas of various return measurement interval, empirically.

Table 4. Average Absolute φ

This table reports the average absolute value of factor exposure ratio, φ and the 2011 market capitalization of sample indices in 2011 US Dollars. The data is ordered by the size of φ. The sample data is from January 2009 to December 2011.

Country	φ	Market Capitalization (2011)*
France	0.951	1,568,729
UK	0.953	1,202,031
Japan	0.976	3,540,684
Italy	0.983	431,470
Korea	0.991	994301
US	0.996	15,640,707
Germany	1.039	1,184,458
Singapore	1.043	308,320
Netherlands	1.068	594,731
Indonesia	1.081	390,106
Spain	1.082	1,030,951
Brazil	1.083	1,228,969
Thailand	1.201	268,488

*Note: *in million US$*
Market Capitalization data source: World Bank

Table 4 presents the average beta ratios for various countries in the sample. We can observe that the ratio of average factor exposures is not always larger than 1. This means that the average daily factor exposure is not necessarily smaller than the monthly factor exposure. P&S state that, "the estimated betas are lowest for daily returns, highest for monthly returns." Empirical results in Table 4 do not support this statement. Instead, this result is consistent with the result of H&S where one factor model factor exposure can be increasing or decreasing with respect to the length of the return measurement interval.

This effect most likely results from lags in the adjustment of stock prices to market shocks, as previously discussed. In an efficient market, where such lags do not exist, the expected value of beta should not vary with respect to the return measurement interval. The faster the price adjustments speed, the smaller the range between daily and monthly estimated betas. We have to be careful in interpreting Table 4. The stock market is more efficient if φ is closer to one. $\varphi = 1$ indicates there is no intervalling effect, hence no adjustment lag. In Table 4, developed countries have close to φ and this conforms comfortably to the general expectation that price information spreads faster in developed markets. With a liquid and efficient market, the capital may be invested and withdrew quickly to more profitable securities or projects (Bencivenga and Smith, 1991). This promotes the efficient capital allocation, helping long-term growth. This indicates that more efficient market helps economic growth and hence the size of the total market capitalization would be higher. Therefore higher market capitalization is expected if φ is close to one.

The market capitalization is the highest when φ is close to one and the relationship is hyperbolic. The t statistics for all the betas are significant at 1% level. Spain might seem like an outlier because it is generally considered as a developed market. However considering that the sample period includes the Euro zone crisis, Spain is not necessarily an outlier. Spanish stock market might have been actually information-inefficient due to the dominant negative market sentiment. If Spain is not an outlier, then Italy might look like one since they have experienced the same crisis, perhaps more intensely However, we can see that the market capitalization for the Italian stock market is significantly smaller than that of Spain. Considering that the total GDP of Italy is 1.5 times greater than Spain's, this would indicate that large numbers of Italian companies are privately owned and Italian investors have less domestic investment opportunities. Those Italian investors who have invested in the Italian stock market would be more experienced and professional. This may keep Italian stock market relatively more efficient than Spain's.

3.4. Measurement error

As discussed in section 2.3, measurement error is examined via ratio of average R^2. The average R^2 shows the percentage of variation in stock returns explained by the market movements in the sample countries.

$$\psi = \frac{\overline{R}^2_{Monthly}}{\overline{R}^2_{Daily}} \tag{12}$$

Table 5. Measurement errors

Country	Average Daily R2	Average Monthly R2	Ψ
UK	0.404	0.354	0.878
France	0.556	0.492	0.884
Japan	0.476	0.429	0.9
US	0.548	0.504	0.92
Spain	0.523	0.499	0.954
Italy	0.465	0.444	0.954
Brazil	0.446	0.427	0.957
Netherlands	0.465	0.462	0.995
Germany	0.475	0.503	1.058
Thailand	0.367	0.409	1.114
Indonesia	0.366	0.409	1.117
Korea	0.34	0.384	1.128
Singapore	0.396	0.52	1.314

The result in Table 5 shows that the larger countries tend to have smaller Ψ. I also find that the spreads between daily and monthly R^2 are larger than for beta. Hence the result is consistent with P&S, although there is almost 40 year gap. As P&S states, the reason for the greater gap spreads is related to the fact that R^2 are affected by stock price measurement errors, as well as price adjustment lags. The measurement error is expected to decrease in importance as the return interval increases.

4. AUTOCORRELATION AND THE MARKET EFFICIENCY

As previously discussed, in this section, I test 373 individual stock beta ratios, φ, to see whether the return autocorrelation has significant influence on the ratios. Therefore the hypothesis that I test is

H_0: The size of intervalling effect depends on the security return autocorrelation

H_1: Otherwise

The security autocorrelation takes the value between -1 and 1 and $\rho_i(1)^{22}$ would be a very small number. Also Proposition 2 indicates that the relationship between the independent and the dependent variable is polynomial, non-linear. A usual method to overcome this is to take log to both sides of the equation. However $\rho_i(1)$ can take negative value hence taking log is not a plausible option. By reorganizing the equation (11) we can see that $\varphi_i^{(h_2-h_1)}$ has a linear relationship with $\rho_i(1)$. Based on this, Remark 2 states the relationship between the security autocorrelation and the φ.

Remark 2: If $\beta_i(h_2) \neq \beta_i(h_1)$, $\varphi^{\frac{1}{22}}$ has statistically significant positive and linear relationship with $\rho_i(1)$

Remark 2 can be empirically tested by investigating below regression equation

$$\left(\varphi_i^j\right)^{\frac{1}{22}} = \alpha^j + \lambda^j \rho_i^j(1) + \varepsilon_i^j \tag{13}$$

where i is the number of securities in the sample and j is the country. I argued that if there is no adjustment lag, there would be no intervalling effect. Therefore if ρ_i is zero, φ would be one. Therefore the expected value of the estimated α is one.

Table 6 reports regression estimated coefficients of equation (13) and the statistical significance. All λ do not need to be statistically significant because actual historical return distribution does not need to be exact AR(1).

Table 6. Autocorrelation vs. φ: Number of $\rho(1)$ that is statistically significant at 5%

Country	α	λ	Country	α	λ
US	1.004**	0.091**	Italy	0.994**	0.09
	(0.000)	(-0.003)		(0.000)	(-0.235)
UK	0.996**	0.106**	Thailand	0.999**	0.038
	(0.000)	(-0.018)		(0.000)	(-0.688)
Germany	0.999**	0.055	Korea	0.995**	0.078**
	(0.000)	(-0.226)		(0.000)	(-0.041)
France	0.997**	0.148**	Brazil	1.003**	0.119**
	(0.000)	(0.000)		(0.000)	(-0.011)
Netherlands	1.001**	0.023	Singapore	1.000**	-0.007
	(0.000)	(-0.749)		(0.000)	(-0.851)
Japan	0.998**	-0.002	Indonesia	0.999**	0.045
	(0.000)	(-0.954)		(0.000)	(-0.229)
Spain	1.005**	0.001			
	(0.000)	(-0.986)			
Total Sample	0.999**	0.047**			
	(0.000)	(0.000)			

*Note: ** Significant at 5% confidence level; * Significant at 10% confidence level*

However, all λ are positive as expected by the model except two, Japan and Singapore. The p-values of the two countries are extremely high, 0.954 and 0.851. This indicates that the negative relationship does not necessarily be statistically meaningful. Although the empirical result is not exactly match the theoretical findings due to the AR(1) security return process assumption, results in Table 6 generally support the theoretical findings. More importantly, the λ coefficient based on the entire sample is statistically significant and positive with the p-value zero. Therefore we cannot reject the null hypothesis. All the constant terms are significant and they are very close to one. This tells us that the security return autocorrelation explains the deviation of the longer period beta from that of the shorter period. This result is another empirical support to the theoretical result represented in Proposition 2. And this also confirms with the model expectation.

5. CONCLUSION

This paper investigates the impact of equity return autocorrelation on financial market efficiency via intervalling effect. A simple model is proposed to show that the degree of intervalling effect is related to the security return autocorrelation. With generalized Levy and Levhari hypothesis from the suggested model, the theoretical findings of the paper can be summarized as follows. 1) The intervalling effect can cause the beta to be over or underestimated depending on the magnitude of the return autocorrelations. 2) The degree of the autocorrelations of the security and the market returns determines the existence and the direction of the intervalling effect. 3) The intervalling effect disappears as the measurement interval increases. 4) The size of the intervalling effect is dependent on the degree of the security autocorrelations. When there is no return autocorrelation, we recover the result of Levy and Levhari (1977)

The paper investigates its theoretical findings with the data of 373 common stocks from January 2009 to December 2011. It tests the hypothesis "The size of intervalling effect depends on the security autocorrelation" and concludes that the hypothesis cannot be rejected, therefore in the sample the security autocorrelation seems to have significant on the market efficiency.

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Appendix 1

A. Proof of Lemma 1

Equation (8) states

$$\beta_i(h_1) = \frac{\rho_{im}(h_1)\sigma_i(h_1)\sigma_m(h_1)}{\sigma_m(h_1)^2} \quad \beta_i(h_2) = \frac{\rho_{im}(h_2)\sigma_i(h_2)\sigma_m(h_2)}{\sigma_m(h_2)^2}$$

Substituting this to equation (3), we get

$$\varphi = \frac{\beta_i(h_2)}{\beta_i(h_1)} = \frac{\rho_{im}(h_2)\sigma_i(h_2)\sigma_m(h_2)}{\rho_{im}(h_1)\sigma_i(h_1)\sigma_m(h_1)} \cdot \frac{\sigma_m(h_1)^2}{\sigma_m(h_2)^2} = \frac{\rho_{im}(h_2)}{\rho_{im}(h_1)} \cdot \frac{\sigma_i(h_2)\sigma_m(h_1)}{\sigma_i(h_1)\sigma_m(h_2)}$$

Since $\sigma_j(h) = \sigma_j(1) \times \sqrt{h}$ where $j = i$ and m,

$$\varphi = \frac{\rho_{im}(h_2)}{\rho_{im}(h_1)} \cdot \frac{\sigma_i(1)\sqrt{h_2}\sigma_m(1)\sqrt{h_1}}{\sigma_i(1)\sqrt{h_1}\sigma_m(1)\sqrt{h_2}} = \frac{\rho_{im}(h_2)}{\rho_{im}(h_1)}$$

Therefore,

$$\tilde{\varphi} = \frac{\rho_{im}(h_2)}{\rho_{im}(h_1)}$$

B. Proof of Lemma 2

Correlation between X and Y is defined as

$$\rho_{XY} = \frac{E[XY] - E[X]E[Y]}{\sqrt{E[X^2] - E[X]^2}\sqrt{E[Y^2] - E[Y]^2}} = \frac{E[XY] - \mu_X \mu_Y}{\sigma_X \sigma_Y}$$

Therefore under current setting,

$$\rho_{im}(h) = \frac{E[r_i(h) r_m(h)]}{\sigma_i(h)\sigma_m(h)}$$

From equation (6), we get

$$R_i(h_2) = \rho_i(h_2)R_k(0) + \eta_i(h_2) \text{ and } R_i(h_1) = \rho_i(h_1)R_i(0) + \eta_i(h_1)$$

$$R_m(h_2) = \frac{R_i(h_2) - \varepsilon_i(h_2)}{\beta_i(h_2)} = \frac{\rho_i(h_2)R_i(0) - \alpha_i + \eta_i(h_2) - \varepsilon_i(h_2)}{\beta_i(h_2)}$$

$$R_m(h_1) = \frac{R_i(h_1) - \varepsilon_i(h_1)}{\beta_i(h_1)} = \frac{\rho_i(h_1)R_i(0) - \alpha_i + \eta_i(h_1) - \varepsilon_i(h_1)}{\beta_i(h_1)}$$

From this we get,

$$R_i(h_2) = \frac{\rho_i(h_2)}{\rho_i(h_1)} R_i(h_1) + \eta_i'(h_1, h_2)$$

$$R_m(h_2) = \frac{\rho_i(h_2)}{\rho_i(h_1)} \cdot \frac{\beta_i(h_1)}{\beta_i(h_2)} R_m(h_1) + \frac{\alpha_i}{\beta_i(h_2)}\left(\frac{\rho_i(h_2)}{\rho_i(h_1)} - 1\right) + \eta_m'(h_1, h_2)$$

Using $\rho_i(h) = \rho_i(1)^h$ we get

$$R_i(h_2) = \rho_i(1)^{(h_2-h_1)} R_i(h_1) + \eta_i'(h_1, h_2)$$
$$R_m(h_2) = \frac{\rho_{im}(h_1)}{\rho_{im}(h_2)} \rho_i(1)^{(h_2-h_1)} R_m(h_1) + \frac{\alpha_i \sigma_m(h_2)}{\rho_{im}(h_2)\sigma_i(h_2)} \left(\rho_i(1)^{(h_2-h_1)} - 1\right) + \eta_m'(h_1, h_2)$$

Since the error term $\eta'(h)$ is independent to the returns $R(h)$ and has the expected value 0, all cross terms associated with $\eta'(h)$ gets eliminated when expectation is taken. Also I assumed that $E(a_i)$=0. Therefore we get,

$$E[R_i(h_2) R_m(h_2)] = \frac{\rho_{im}(h_1)}{\rho_{im}(h_2)} \rho_i(1)^{2(h_2-h_1)} E[r_i(h_1) r_m(h_1)]$$

Using equation (A3) and $\sigma(h) = \sigma(1) \times \sqrt{h}$ we get,

$$\rho_{im}(h_2) = H \cdot \rho_i(1)^{(h_2-h_1)} \cdot \rho_{im}(h_1)$$

where $H = (h_1/h_2)$^1/2

C. Proof of Proposition 1

$$\beta_i(h_1) = \beta_i(h_2) \Rightarrow \frac{\rho_{im}(h_1)\sigma_i(h_1)\sigma_m(h_1)}{\sigma_m(h_1)^2} = \frac{\rho_{im}(h_2)\sigma_i(h_2)\sigma_m(h_2)}{\sigma_m(h_2)^2}$$

Reorganizing standard deviation terms as in Appendix 1 we get,

$$\rho_{im}(h_1) = \rho_{im}(h_2)$$

Applying Proposition 2,

$$\frac{h_1}{h_2} \left(\rho_i(1)\rho_m(1)\right)^{(h_2-h_1)} = 1$$

Re organizing this we get

$$\rho_i(1) = \left(\frac{h_2}{h_1}\right)^{\frac{1}{h_2-h}} \rho_m(1)^{-1}$$

Therefore we get the condition,

$$\beta_i(h_2) > \beta_i(h_1) \quad \text{if CV} > 1$$
$$\beta_i(h_2) = \beta_i(h_1) \quad \text{if CV} = 1$$
$$\beta_i(h_2) < \beta_i(h_1) \quad \text{if CV} < 1$$