MEAN-GINI AND MEAN-EXTENDED GINI PORTFOLIO SELECTION: AN EMPIRICAL ANALYSIS

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Abstract

The purpose of this study was to examine Mean-Gini strategy (MG) and Mean-Extended Gini strategy (MEG) for optimum portfolio selection, in terms of the monthly Rate of Return, Standard Deviation, Sharpe Ratio, Treynor Ratio and Jensen's Alpha. This paper compared different optimum portfolio strategies, based on Moroccan financial market data taken from turbulent market periods between the years 2007 to 2015. Two distinct sub-periods were studied: (1) crisis period: 2007-2009; (2) post-crisis period: 2010-2015. The results show that both strategies were profitable for investors, but that the MEG strategy is the more appropriate and secure strategy for an individual investor.

Keywords: Mean-Gini, Mean-Extended Gini, Portfolio Selection, Performance Measures

1. INTRODUCTION

Investors seek to insure future returns on positions which requires them to choose their best strategies before investment. Since the birth of modern finance with the pioneering work of Markowitz (1952a, 1952b), the Mean-Variance (MV) theory has been a reliable response for investors confronted with the risk-return dilemma when choosing financial assets. The theory is based on the presumption that distribution of portfolio returns is normal and can be successfully described by two moments: mean and variance. In fact, empirical evidence has revealed that portfolio returns are neither normally nor symmetrically distributed. Consequently, several research works have attempted to find alternative strategies such as Markowitz (1959), Fish burn (1977) and Bawa and Lindenberg (1977), which proposed a semi-variance concept which considers downside risk. Yitzhak in (1982) and Shalit and Yitzhak (1984, 2005) suggested the Mean-Gini model, Konno and Yamazaki (1991) suggested the Mean-Absolute Deviation model, Young (1998) suggested Minimax Optimum , Sortino et al. (1999) proposed the Upside Potential Model Ratio which considered the return that exceeded target return as rewards, and Favre and Galeano (2002) presented the Mean-Modified Value-at-Risk Optimization Model.

The Mean-Gini (MG) Model was proposed by Shalit and Yitzhak (1984) as an alternative strategy to the Mean-Variance Model (MV) and has the merit of providing a simple model of portfolio selection which can outperform the Mean-Variance Model (MV) in the case of abnormally distributed returns, as shown by Jaaman and Lam (2012) and Agouram and Lakhnati (2015b). However, one of the factors to consider when selecting the optimum portfolio for a particular investor is their degree of risk aversion. This is related to the behavior of the individual in the face of future uncertainties. Different investors have different risk profiles: risk-averse, risk-neutral and risk-seeker.

The common answer to the problem of varying risk aversion was a generalization of the Gini index by Yitzhaki (1983), which makes the Gini index depend upon a specified degree of risk aversion. Later, Shalit and Yitzhak (1984, 2005) presented the Mean-Extended Gini (MEG) as a model that provides a measure that is flexible enough to embody the preferences of different investors regarding the degree of risk aversion. Therefore, this model can better reflect the perceived risk of an individual investor, as has been highlighted in recent study by Cardin et al. (2013). The problem is to ascertain the degree of risk aversion in order to compose optimum portfolios.

This study provides a comprehensive statistical analysis of two strategies: the Mean-Gini (MEG) strategy versus the Mean-Extended Gini (MEG) strategy.

Firstly, the portfolios were composed with shares listed on the Moroccan financial market according to the Mean-Gini (MEG) strategy and the Mean-Extended Gini (MEG) strategy. Secondly, the three traditional measures of financial performance were used; Sharpe Ratio, Treynor Ratio and Jensen's Alpha, in addition to the Rate of Return and Standard Deviation which was computed monthly to determine if any of the portfolios underperformed or outperformed others. The performance of portfolios was measured during the period from 2007 to 2015 with respect to two sub-periods: (1) crisis period: 2007-2009; (2) post-crisis period: 2010-2015.

The remainder of this paper is organized as follows: Section 2 contains a review of the related literature. Section 3 discusses the data and the methodologies, including the portfolio optimization of Mean-Gini (MEG) strategy and the Mean-Extended Gini (MEG) strategy on data retrieved from the Moroccan financial market. Section 4 examines the empirical results. The final section summarizes and concludes.

2. MODELS

We consider a market with n risky assets $i = 1, 2, \dots, n$, .We suppose our total wealth to be invested is 1, in some units.

Let x_i denote the portfolio weight of asset i, namely, the fraction of the investor budget allocated to asset i, Ri denote the random one-period return[9] on asset $i, i = 1, 2, \dots, n$, rf denote the risk-free return.

A portfolio is defined to be a list of weights x_i for assets $i, = 1, 2, \dots, n$, which represent the amount of capital to be invested in each asset.

The expected return of the portfolio is:

$$E(Rp) = \sum_{i=1}^{n} x_i \, E(Ri) = \sum_{i=1}^{n} x_i \mu_i \qquad (1)$$

Where μ_i is the expected return from asset i.

2.1. Mean-Gini Model

The MG analysis introduced by Shalit and Yitzhak (1984) defines the Gini coefficient as an index of variability of a variable random. Specifically, Dorfman (1979) and Shalit and Yitzhak (1984) retain the following formula of the Gini coefficient[10]:

$$\Gamma p = 2\mathrm{cov}(Rp, F(Rp)) \qquad (2)$$

Where R_P the return of portfolio and F is the cumulative distribution function.

The portfolio allocation problem would be to choose the x_i subject to the constraints: $\sum_{i=1}^{n} x_i = 1$, the x_i sum to unity, called weights in the portfolio allocation problem. In addition, we restrictive than the x_i are positive, so that the weights of assets can only be positive.

In Agouram and Lakhnati (2015a, 2015b), the following optimization program was used:

$$\text{Minimize}: \ \Gamma p$$

$$\text{Subject to} \begin{cases} E(Rp) \geq \mu \\ \sum_{i=1}^{n} x_i = 1 \\ x_i \geq 0, \ 1 \leq i \leq n \end{cases} \qquad (OP1)$$

2.2. Mean-Extended Gini Model

A generalization of the Gini coefficient was proposed by Yitzhaki (1983) that makes the Gini index dependent on the specified degree of risk aversion. The generalized Gini coefficient (or extended Gini coefficient) can also be expressed as a covariance similar to its definition in equation (2):

$$\Gamma p(\nu) = -\nu * \mathrm{cov}\left(Rp, \left(1 - F(Rp)\right)\right)^{\nu - 1} \qquad (3)$$

Where ν is a parameter tuning the degree of aversion to risk. The standard Gini corresponds to $v = 2$[11].

So that the optimization problem of MEG model becomes:

$$\text{Minimize}: \ \Gamma p(\upsilon)$$

$$\text{Subject to} \begin{cases} E(Rp) \geq \mu \\ \sum_{i=1}^{n} x_i = 1 \\ x_i \geq 0, \ 1 \leq i \leq n \end{cases} \qquad (OP2)$$

3. PERFORMANCE MEASURES

Several measures to compare portfolio returns can be used[12]. A simple comparison is to compare their returns or their risk. But traditional measures of risk-adjusted performance, including the Sharpe Ratio, Treynor Ratio and Jensen's Alpha would be preferable because returns by themselves do not account for the risk taken. If two portfolios have the same return, but one has lower risk, then that would be the preferable, more efficient portfolio.

3.1. The Sharpe Ratio

In 1966 William Sharpe conceived a measure of portfolio performance called the Sharpe Ratio. It measures the return earned in excess of the risk-free rate on a portfolio relative to the portfolio's total risk, measured by the Standard Deviation.

It quantifies the reward per unit of total risk. The Sharpe Ratio formula is as follows:

$$S_p = \frac{E(R_p) - r_f}{\sigma_{(R_p)}} \qquad (4)$$

Where $\sigma_{(R_p)}$ is the portfolio Standard Deviation.

A high Sharpe Ratio shows a portfolio's superior risk-adjusted performance, while a low Sharpe Ratio is an indication of unfavorable performance.

[9] If you buy at price P1 and sell at price P2, the return is the dimensionless number $R = \frac{P_2 - P_1}{P_1}$.

[10] For the method of calculating the Gini index, see Cheung et al. (2007).

[11] Note that with v=2, equation (3) collapses to the standard Gini Index (equation (2)).

[12] Shalit (2014) presents a methodology for using the Lorenz curve to define a partial ordering of investment opportunities. But if Lorenz curves intersect the clear dominance between risky assets cannot be established.

3.2. Treynor Ratio

In 1965 Jack Treynor conceived an index of portfolio performance measure called Reward to Volatility Ratio, based on systematic risk. It is similar to the Sharpe Ratio, except it uses the beta[13] instead of the Standard Deviation. Hence, his performance measure denoted as T is the excess return over the risk-free rate per unit of systematic risk; it indicates risk premium per unit of systematic risk. The Treynor Ratio is calculated as:

$$T_p = \frac{E(R_p) - r_f}{\beta_p} \tag{5}$$

Where β_p is the beta of the portfolio.

Generally, higher Treynor Ratios indicate higher or superior performance, and vice versa.

3.3. Jensen's Alpha

In 1968 Jensen developed a statistical measurement called Jensen's Alpha which is the Rate of Return that exceeds what was expected or predicted by models like the Capital Asset Pricing Model (CAPM)[14] . To understand how it works, consider the CAPM formula:

$$E(R_p) = r_f + \beta_p(E(R_m) - r_f) + \alpha \tag{6}$$

Jensen's Alpha can be defined as:

$$\alpha = E(R_p) - (r_f + \beta_p(E(R_m) - r_f) \tag{7}$$

Where $E(R_p)$ is the expected market return. Note that two similar portfolios might carry the same amount of risk (same beta) but because of differences in Jensen's Alpha, one might generate higher returns than the other. The higher alpha, signifies that the portfolio has earned above the level predicted.

4. METHODOLOGY

4.1. Data

In this section, the performance of MG and MEG models for different degrees of risk aversion using the historical data of daily returns of 14 stocks from the Moroccan financial market from Jan 2, 2004 to Jun 5, 2015 was compared. To deduce an optimum portfolio selection rule, the past data of 3 years from Jan 2, 2004 to Nov 30, 2006 was used to calculated the MG and MEG portfolios with different degrees of risk aversion[15] v=4, v=6, v=8, v=10, v=12, v=16 and v=20 and these portfolios were held from 2007 to 2015. This period was divided into two sub-periods: (1) Crisis period (2007-2009); (2) Post-crisis period (2010-2015).

Table 1 represents the descriptive statistics of the sample data for each stock. The strong results for the normality test (Jarque-Bera) for each stock, led to a rejection of the null hypothesis of the normality test at 99% confidence level. These results indicate a well-known property of financial data series: returns are usually not normally distributed. In addition, skewness and kurtosis, other properties of risky assets, were discovered in the data series. Since both properties are apparent in the data, it is assumed that using the MG and MEG strategies should provide the best portfolios due to the fact that they exceed normal return distribution assumptions.

4.2. Portfolio Optimization

The portfolio optimization programs (OP1 and OP2) were adopted to deteminee fraction $x_i (i = 1, \ldots, n)$ of a given capital invested in asset i of portfolio P with its Gini coefficient (or extended Gini coefficient), Γ_P and $\Gamma_P(v)$ being maximized subject to obtaining a predetermined level of its expected return $E(R_P)$. It was assumed that there are no risk-free assets in the market and investors required a Rate of Return of 0.15.

After the resolution of the optimization programs, their optimum portfolios were obtained. Table 2 and table 3 present the summary statistics of the optimum portfolios.

5. RESULTS AND DISCUSSION

The performance of these portfolios for 8.5 years (97 months) was evaluated by five criteria: Rate of Return; Standard Deviation; Sharpe Ratio; Treynor Ratio; Jensen's Alpha.

It was decided that the ranks of the portfolios needed to be calculated in order to observe their consistency during the investment period. Consequently, each month we calculate Rate of Return, Standard Deviation, Sharpe Ratio, Treynor Ratio and Jensen's Alpha was calculated for data that corresponds to the immediately preceding 3 years (36 months).

Therefore, The Borda-Kendall (BK)[16] method was used to construct a ranking of portfolios. The BK method assigns the first ranking place a mark of 1, the second ranking place a mark of 2, and so on. The total score (Z_i) each portfolio receives can be computed by aggregating the results from the simple equation:

$$Z_i = \sum_{i=1}^{p} j\, v_{ij} \tag{8}$$

Where j is the Ranks, v_{ij} are the votes that each portfolio receives j^{th} ranking place. The optimum portfolio will be the one with the lowest total score.

[13] Beta signifies the sensitivity of the portfolio returns in comparison to the movement of the stock market index, namely: $\beta_p = \frac{cov(R_P - R_m)}{var(R_m)}$

[14] The bulk of the CAPM formula (everything but the alpha factor) calculates what the Rate of Return on a certain portfolio ought to be under certain market conditions. So if CAPM model predicts that your portfolio should return 10%, but it actually returns 15%, we would call the 5% difference alpha, in Jensen's measure.

[15] Assigning different values to v can change the value of the Gini index by weighting returns differently in different parts of their distribution.

[16] This is the well-known Kendall scores method (Kendall, 1962), or the method of marks due to Borda (1781).

Table1. The Descriptive Statistics of The Sample data

Period January 2004-Novembre 2006.							
	Mean	***Standard deviation***	***Gini***	***Skewness***	***Kurtosis***	***JB***	***Prob***
Afriquia Gaz	0.1862	2.2928	1.1673	-0.0800	2.3718	172.14	0.0000
Auto Hall	0.2229	2.0914	0.8929	1.1443	17.7353	9799.92	0.0000
Ciments Du Maroc	0.0929	1.6077	0.7338	-0.0340	5.1616	814.66	0.0000
Cosumar	0.0373	1.6997	0.7573	-0.1455	5.8472	1048.27	0.0000
Dari Couspate	0.0228	1.2321	0.4551	0.4914	15.4891	7380.09	0.0000
Disway	0.1231	2.5039	1.0784	0.4339	4.7398	709.96	0.0000
Holcim Maroc	0.0838	1.7909	0.8488	-0.5897	5.0536	823.80	0.0000
Itissalat Al-Maghrib	0.0557	1.3510	0.6261	0.0795	7.0251	1510.85	0.0000
Lafarge Ciments	0.0936	1.4862	0.6871	-0.1378	5.2091	831.96	0.0000
Lesieur Cristal	-0.0151	1.8477	0.8036	0.0033	5.3290	868.30	0.0000
Lydec	0.0514	1.3750	0.5492	0.7243	9.2240	2669.75	0.0000
Med Paper	0.0734	2.0073	0.7646	0.7302	8.3781	2214.57	0.0000
Samir	0.1476	1.7727	0.9145	-0.3345	3.2162	329.50	0.0000
Wafa Assurance	0.1712	1.5393	0.7926	0.2415	2.7367	235.5687	0.0000
Period January 2007-Novembre 2009.							
	Mean	***Standard deviation***	***Gini***	***Skewness***	***Kurtosis***	***JB***	***Prob***
Afriquia Gaz	0.1058	2.1919	1.0896	0.0154	5.6291	195.87	0.0000
Auto Hall	0.0101	3.2050	1.6017	-0.0498	5.4355	168.34	0.0000
Ciments Du Maroc	-0.0504	2.4629	1.2386	-0.1666	5.2807	150.53	0.0000
Cosumar	0.1501	2.1557	1.0650	0.0902	5.7132	209.49	0.0000
Dari Couspate	0.0551	1.8916	0.9130	0.8149	7.0762	546.03	0.0000
Disway	-0.0028	2.3080	1.1160	0.2433	5.4776	180.63	0.0000
Holcim Maroc	-0.0513	3.5723	1.7945	0.0824	6.0681	2674.79	0.0000
Itissalat Al-Maghrib	0.0113	1.6292	0.8172	-0.2338	9.5084	120.64	0.0000
Lafarge Ciments	0.0599	2.6415	1.3590	-0.0521	5.5461	183.99	0.0000
Lesieur Cristal	0.0899	2.1424	1.0705	-0.1169	5.5965	192.56	0.0000
Lydec	-0.0036	2.7720	1.4440	0.0880	5.6949	206.64	0.0000
Med Paper	0.0237	1.9510	0.8335	0.2528	7.4816	576.31	0.0000
Samir	0.0358	1.5679	0.8073	0.3316	5.6880	217.18	0.0000
Wafa Assurance	0.0586	2.0995	1.0735	-0.0285	5.0567	119.94	0.0000
Period January 2010-July 2015.							
	Mean	***Standard deviation***	***Gini***	***Skewness***	***Kurtosis***	***JB***	***Prob***
Afriquia Gaz	0.0599	1.9721	0.8033	0.2766	9.3884	2256.3610	0.0000
Auto Hall	0.0205	4.3631	1.6259	3.4526	75.0935	287828.0000	0.0000
Ciments Du Maroc	0.0437	2.3169	1.1433	-0.0123	5.4893	340.0620	0.0000
Cosumar	0.0319	1.9152	0.7958	-0.0097	10.1400	2797.5570	0.0000
Dari Couspate	0.0373	1.6806	0.6869	-0.1403	12.3542	4805.9360	0.0000
Disway	-0.0008	2.2745	1.1478	0.4594	7.3979	1107.6790	0.0000
Holcim Maroc	0.0620	2.6419	1.3407	-0.0163	5.7255	407.6759	0.0000
Itissalat Al-Maghrib	-0.0099	1.7057	0.7711	-2.4698	43.7539	92479.5700	0.0000
Lafarge Ciments	0.0501	3.1442	1.5214	-0.0362	10.0595	2735.0420	0.0000
Lesieur Cristal	-0.8154	31.0717	1.0915	-36.1656	1310.9660	94165860.0000	0.0000
Lydec	0.0582	2.6103	1.2029	-0.0248	9.3806	2234.2130	0.0000
Med Paper	-0.0639	2.4949	1.2823	0.3745	5.5030	374.5953	0.0000
Samir	-0.0733	2.0885	1.0896	-0.1122	6.4224	645.4917	0.0000
Wafa Assurance	0.0987	2.0288	0.9796	0.2655	5.4216	337.2847	0.0000

Note: This table reports the summary statistics of 14 stocks from the Moroccan financial market from Jan 2, 2004 to Jun 5, 2015, including Mean, Standard Deviation, Skewness, Kurtosis Coefficients and the Jarque-Bera test

Table 2. Percentage of Stocks in Optimum Portfolios

	MG	***MEG***	***MEG***	***MEG***	***MEG***	***MEG***	***MEG***	***MEG***
	ν=2	***ν=4***	***ν=6***	***ν=8***	***ν=10***	***ν=12***	***ν=16***	***ν=20***
Afriquia Gaz	4.65	3.13	3.29	3.73	4.11	4.40	4.80	5.18
Auto Hal	30.74	33.33	31.42	29.61	28.45	27.39	25.79	24.67
Ciments Du Maroc	7.99	7.41	7.59	7.29	7.03	6.93	7.21	7.49
Cosumar	1.74	1.48	0.65	0.00	0.00	0.00	0.00	0.00
Dari Couspate	8.16	10.53	11.11	11.20	11.22	11.12	11.06	10.31
Disway	3.38	3.29	3.92	4.45	4.81	4.98	5.34	5.64
Holcim Maroc	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Itissalat Al-Maghrib	4.99	5.55	4.39	3.39	2.35	1.65	0.21	0.00
Lafarge Ciments	4.36	1.79	0.16	0.00	0.00	0.00	0.00	0.00
Lesieur Cristal	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Lydec	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Med Paper	4.88	4.94	6.30	7.44	8.17	8.65	9.34	9.82
Samir	6.48	5.35	5.82	5.87	5.66	5.56	5.25	5.19
Wafa Assurance	22.64	23.21	25.34	27.02	28.21	29.32	31.00	31.70

Note: This table reports the percentage of stocks of 8 optimum portfolios, MG (or MEG with $\nu = 2$) to MEG with $\nu = 20$

Table 3. Summary Statistics of Optimum Portfolios

Statistical data of the empirical distribution over the period January 2010-July 2015								
	MG	*MEG*	*MEG*	*MEG*	*MEG*	*MEG*	*MEG*	*MEG*
		v=4	*v=6*	*v=8*	*v=10*	*v=12*	*v=16*	*v=20*
Mean	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15
Standard deviation	0.93	0.96	0.96	0.99	0.97	1.02	1.09	1.18
Skewness	0.48	0.66	0.83	1.69	1.15	2.43	4.31	6.58
Kurtosis	7.83	8.84	9.08	18.09	11.72	28.72	62.32	110.40
Jarque-Bera	755.71	1117.58	1235.65	7442.25	2534.17	21322.88	111845.40	364397.60
Proba-bility	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Note: This table reports the summary statistics of 8 optimum portfolios, MG (or MEG with v = 2) to MEG with v = 20, including Mean, Standard Deviation, Skewness, Kurtosis Coefficients and the Jarque-Bera test. The construction of MG to MEG is described in Section 2

5.1. Rate of Return

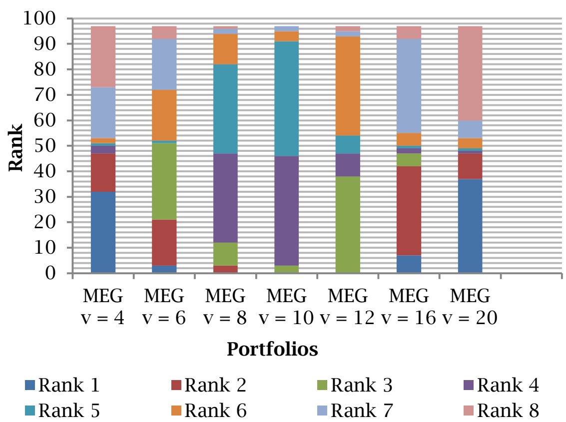
Table 4 shows the results of the monthly Rate of Return evaluation from the Moroccan financial market. MEG with v=8 optimum portfolio was the best based on the Borda-Kendall (BK) method with 109 points. However, for the post-crisis period 2010-2015, MEG with v=4 occupies first place with 213 points. Although the results of ranks of portfolios are different for each period, the best optimal portfolio is MEG with v=4 and its total points for the period 2007-2015 was 326 followed by MG with 334 points. The ranks of the various portfolios according to the monthly Rate of Return RP on the entire sample period are plotted in figure 1. This figure provides an overview of the ranks of the 8 optimum portfolios for 97 months of the analysis.

Table 4. Ranking of Portfolio's Performance by Rate of Return

		MG	*MEG*	*MEG*	*MEG*	*MEG*	*MEG*	*MEGG*	*MEG*
		v=2	*v=4*	*v=6*	*v=8*	*v=10*	*v=12*	*v=16*	*v=20*
Crisis period: 2007-2009	Average rank	4,61	4,42	4,33	4,3	4,55	4,61	4,55	4,64
	Median rank	5	4	3	4	5	5	5	6
	Borda points	119	113	110	109	117	119	117	120
	Rank	6	3	2	1	4	6	4	8
Post-crisis period: 2010-2015	Average rank	4,36	4,33	4,55	4,69	4,59	4,64	4,44	4,41
	Median rank	3,5	3	4	5	5	5	3,5	3
	Borda points	215	213	227	236	230	233	220	218
	Rank	2	1	5	8	6	7	4	3
The entire sample period: 2007-2015	Average rank	4,44	4,36	4,47	4,56	4,58	4,63	4,47	4,48
	Median rank	4	4	3	5	5	5	4	5
	Borda points	334	326	337	345	347	352	337	338
	Rank	2	1	3	6	7	8	3	5

Note: This table reports the results of the evaluation of the performance of 8 optimal portfolios, MG (or MEG with v = 2) to MEG with v = 20, including crisis period: 2007-2009, post-crisis period: 2010-2015 and the entire sample period: 2007-2015

Figure 1. A schematic illustration of the ranks of the various portfolios according to the Rate of Return over the entire sample period (2007-2015)



5.2. Standard Deviation

Table 5 shows the results of the Standard Deviation evaluation. These results differ to those relating to Rates of Return. The best optimum portfolio is the MEG with v = 20 with 81 points for the crisis period: 2007-2009 and MG with 192 points for the post-crisis period: 2010-2015, but the best optimal portfolio is MEG with v = 12, and its total points on the period (2007-2015) are 326, followed by MEG with v = 20, and MG comes in 4th place. The ranks of the various portfolios according to the Standard Deviation over the entire sample period (2007-2015) are plotted in Figure 2.

5.3. Sharpe Ratio

The results of the Sharpe Ratio evaluation from table 6 show that the optimal portfolio is MEG with *v*=12 for the crisis period: 2007-2009, with 14 points. But, for the post-crisis period of 2010-2015, MEG with *v*=16 occupies first place with 110 points. The ranks of the various portfolios according to the Sharpe Ratio over the entire sample period (2007-2015) are plotted in Figure 3, and the optimal portfolio is MEG *v*=12, while MG comes in 5^{th} place.

Table 5. Ranking of portfolio's performance by Standard Deviation

		MG v=2	*MEG v=4*	*MEG v=6*	*MEG v=8*	*MEG v=10*	*MEG v=12*	*MEG v=16*	*MEG v=20*
Crisis period: 2007-2009	Average rank	4,27	6,3	5,39	4,52	3,94	3,85	4,27	3,45
	Median rank	5	8	6	5	4	3	2	1
	Borda points	108	175	145	116	97	94	108	81
	Rank	4	8	7	6	3	2	4	1
Post-crisis period: 2010-2015	Average rank	4	5,45	5,03	4,42	4,13	4,11	4,53	4,33
	Median rank	5	7,5	6	4,5	4	3	4,5	3,5
	Borda points	192	285	258	219	200	199	226	213
	Rank	1	8	7	5	3	2	6	4
The entire sample period: 2007-2015	Average rank	4,09	5,74	5,15	4,45	4,06	4,02	4,44	4,03
	Median rank	5	8	6	5	4	3	4	2
	Borda points	300	460	403	335	297	293	334	294
	Rank	4	8	7	6	3	1	5	2

Note: This table reports the results of the evaluation for the performance of 8 optimum portfolios, MG (or MEG with v = 2) to MEG with v = 20, including crisis period: 2007-2009, post-crisis period: 2010-2015 and the entire sample period: 2007-2015

Figure 2. A Schematic Illustration of the Ranks of the Various Portfolios According to the Standard Deviation Over the Entire Sample Period (2007-2015)

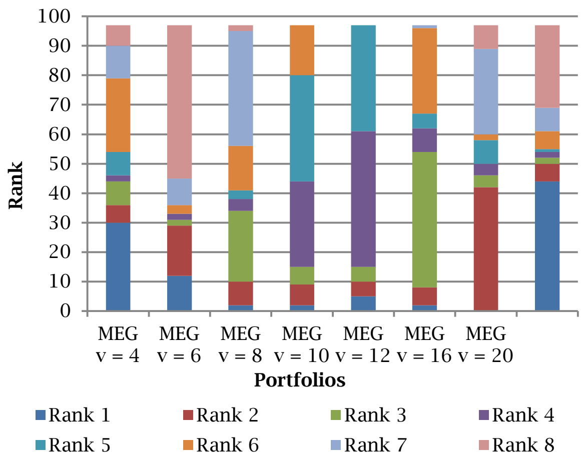


Figure 3. A Schematic Illustration of the Ranks of the Various Portfolios According to the Sharpe Ratio Over the Entire Sample Period (2007-2015)

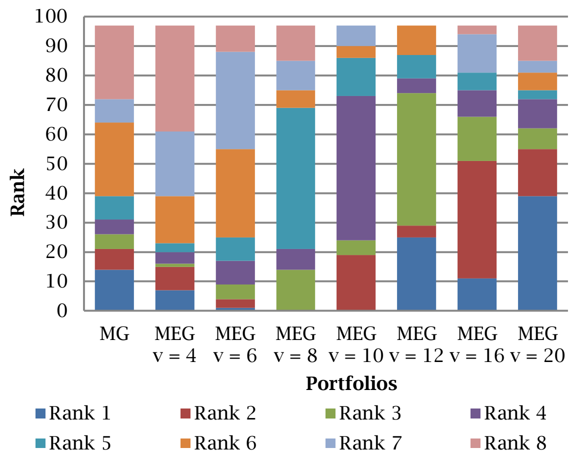


Table 6. Ranking of Portfolio's Performance by Sharpe Ratio

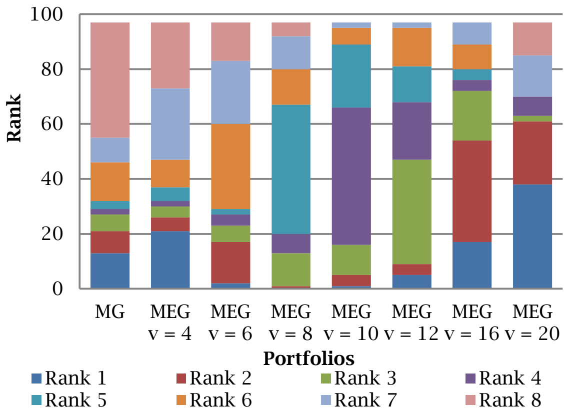
		MG v=2	*MEG v=4*	*MEG v=6*	*MEG v=8*	*MEG v=10*	*MEG v=12*	*MEG v=16*	*MEG v=20*
Crisis period: 2007-2009	Average rank	5,64	7,15	6,27	4,39	2,85	1,42	4,33	3,94
	Median rank	6	8	7	5	2	1	4	4
	Borda points	153	203	174	112	61	14	110	97
	Rank	6	8	7	5	2	1	4	3
Post-crisis period: 2010-2015	Average rank	5,05	5,61	5,78	5,73	4,58	3,77	2,72	2,77
	Median rank	6	7	6	5	4	3	2	1
	Borda points	259	295	306	303	229	177	110	113
	Rank	5	6	8	7	4	3	1	2
The entire sample period: 2007-2015	Average rank	5,25	6,13	5,95	5,28	3,99	2,97	3,27	3,16
	Median rank	6	7	6	5	4	3	2	2
	Borda points	412	498	480	415	290	191	220	210
	Rank	5	8	7	6	4	1	3	2

Note: This table reports the results of the evaluation for the performance of 8 optimal portfolios, MG (or MEG with v = 2) to MEG with v = 20, including crisis period: 2007-2009, post-crisis period: 2010-2015 and the entire sample period: 2007-2015

5.4. Treynor Ratio

The results of the Treynor Ratio evaluation in table 7 show that the optimal portfolio is MEG with v=16 for the crisis period: 2007-2009, and the entire sample period: 2007-2015, while MEG with v=20 is the best for the post-crisis period: 2010-2015. MG comes in last place for the crisis period: 2007-2009 and the entire sample period: 2007-2015 and comes in 4^{th} place for the post-crisis period: 2010-2015. The ranks of the various portfolios according to the Treynor Ratio over the entire sample period (2007-2015) are plotted in Figure 4.

Figure 4. A Schematic Illustration of the Ranks of the Various Portfolios According to the Treynor Ratio over the Entire Sample Period (2007-2015)



5.5. Jensen's Alpha

The results of the Jensen's Alpha evaluation in table 8 show that the optimal portfolio is MEG with v=12 for the crisis period: 2007-2009, but the best optimal portfolio is MEG with v=20 for the post-crisis period: 2010-2015. Although the results of ranks of portfolios are different for each period, the best optimal portfolio is MEG with v=16 over the entire period (2007-2015), followed by MEG with v=12, and MG comes in last place for the period 2007-2015. The ranks of the various portfolios according to the monthly Jensen's Alpha, over the entire sample period (2007-2015) are plotted in figure 5. This figure provides an overview of the ranks of the 8 optimal portfolios for 97 months of the analysis.

Table 7. Ranking of Portfolio's Performance by Treynor Ratio

		MG v=2	*MEG v=4*	*MEG v=6*	*MEG v=8*	*MEG v=10*	*MEG v=12*	*MEG v=16*	*MEG v=20*
Crisis period: 2007-2009	Average rank	7,97	4,09	4,39	4,39	3,42	3,82	3,18	4,73
	Median rank	8	4	6	5	4	4	3	4
	Borda points	230	102	112	112	80	93	72	123
	Rank	8	4	5	5	2	3	1	7
Post-crisis period: 2010-2015	Average rank	4,53	5,86	6,09	5,59	4,59	3,88	2,91	2,55
	Median rank	5,5	7	7	5	4	3	2	1
	Borda points	226	311	326	294	230	184	122	99
	Rank	4	7	8	6	5	3	2	1
The entire sample period: 2007-2015	Average rank	5,7	5,26	5,52	5,19	4,2	3,86	3	3,29
	Median rank	7	7	6	5	4	4	2	2
	Borda points	456	413	438	406	310	277	194	222
	Rank	8	6	7	5	4	3	1	2

Note: This table reports the results of the evaluation for the performance of 8 optimal portfolios, MG (or MEG with v = 2) to MEG with v = 20, including crisis period: 2007-2009, post-crisis period: 2010-2015 and the entire sample period: 2007-2015

Table 8. Ranking of Portfolio's Performance by Jensen's Alpha

		MG v=2	*MEG v=4*	*MEG v=6*	*MEG v=8*	*MEG v=10*	*MEG v=12*	*MEG v=16*	*MEG v=20*
Crisis period: 2007-2009	Average rank	7,58	4,61	5,64	4,76	3,21	2	3,7	4,52
	Median rank	8	5	6	5	3	2	3	4
	Borda points	217	119	153	124	73	33	89	116
	Rank	8	5	7	6	2	1	3	4
Post-crisis period: 2010-2015	Average rank	5,7	6,16	5,88	5,33	4,36	3,63	2,53	2,42
	Median rank	6	7	6,5	5	4	3	2	1
	Borda points	301	330	312	277	215	168	98	91
	Rank	6	8	7	5	4	3	2	1
The entire sample period: 2007-2015	Average rank	6,34	5,63	5,79	5,13	3,97	3,07	2,93	3,13
	Median rank	7	7	6	5	4	3	2	1
	Borda points	518	449	465	401	288	201	187	207
	Rank	8	6	7	5	4	2	1	3

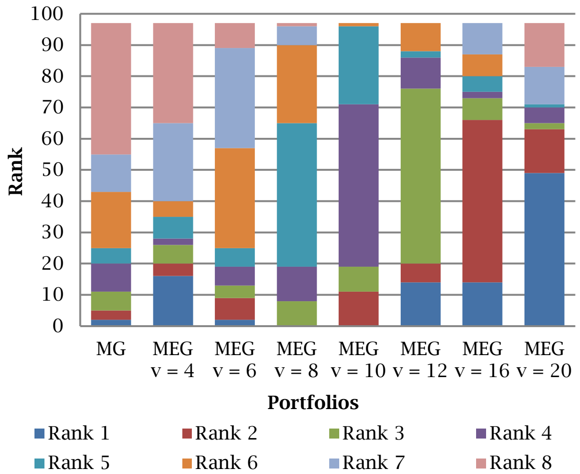
Note: This table reports the results of the evaluation for the performance of 8 optimal portfolios, MG (or MEG with v = 2) to MEG with v = 20, including crisis period: 2007-2009, post-crisis period: 2010-2015 and the entire sample period: 2007-2015

6. CONCLUSION

Since the normality hypothesis is rejected for all stocks, the results drawn from The Mean-Variance (MV) model may be misleading. To circumvent this limitation, Mean-Gini and the Mean-Extended Gini portfolio optimization was used. This study discusses and compares analytical results obtained with MG and MEG on Moroccan financial markets from 1 January 2007 to 5 June 2015. Eight optimal portfolios were used and their performance was compared by applying the Rate of Return, their Standard Deviation, their Sharpe Ratio, their Treynor Ratio and Jensen's Alpha for 8.5 years (97 months).

In this study, the returns on assets are not normally distributed in common for each country. Our empirical study shows that the results of ranks of portfolios are different for each period and criteria, but the best optimal portfolio is MEG with v= 4 for Rate of Return, MEG with v= 12 for Standard Deviation, Sharpe Ratio and Jensen's Alpha, while MEG with v =16 is the best optimal portfolio for Treynor Ratio over the entire sample period: 2007-2015.

Figure 5. A Schematic Illustration of the Ranks of the Various Portfolios According to the Jensen's Alpha Over the Entire Sample Period (2007-2015)

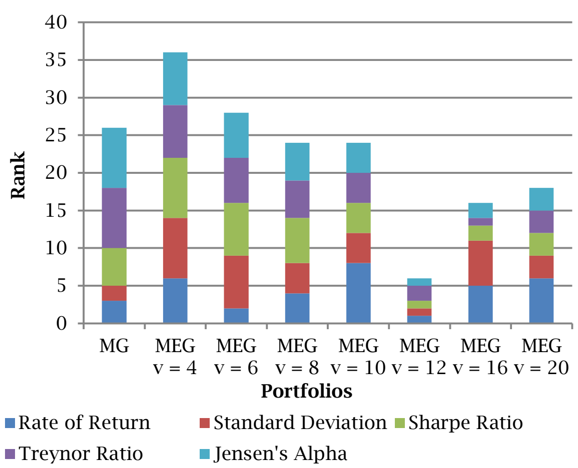


The results showed that the performance of Mean-Variance (MV) is inferior to that of alternative models in the actual stock markets in which the return on asset was not normally distributed.

A more detailed analysis of the performance of portfolios as shown in Figure 6 in which the rankings on the different criteria are aggregated, confirms that most MEG portfolios outperformed MG portfolios. This study's results show that for investors willing to take more risk, a MEG strategy is a better choice when selecting the optimal portfolio.

In view of these results, we conclude that the Mean-Gini and the Mean-Extended Gini strategy outperform the MV strategy in our real-world examples taken from the Moroccan Financial Market.

Figure 6. A Schematic Illustration of the Sum of the Rankings on Various Criteria of the Various Portfolios Over the Entire Sample Period (2007-2015).



REFERENCES

1. Agouram, J. and Lakhnati, G.(2015a), Mean-Gini portfolio selection: Forecasting VaR using GARCH models in Moroccan financial market, Journal of Economics and International Finance, 7(3), pp.
2. Agouram, J. and Lakhnati, G. (2015b), A comparative study of mean-variance and mean Gini portfolio selection using VaR and CVaR, Journal of Financial Risk Management,04(02), pp. 72-81.
3. Bawa, V. S. and Lindenberg, E. B. (1977), Capital market equilibrium in a mean-lower partial moment framework, Journal of Financial Economics, 5(2), pp. 189-200.
4. Borda, J. C. (1781), Mémoire sur les élections au scrutin.
5. Cardin, M., Eisenberg, B. and Tibiletti, L. (2013), Mean extended Gini portfolios per sonalized to the investor's profile, Journal of Modelling in Management, 8(1), pp. 54-64.
6. Cheung, C. S., Kwan, C. C. and & Miu, P. C. (2007), Mean-Gini portfolio analysis: A pedagogic illustration, Spreadsheets in Education (eJSiE), 2(2), 3.
7. Dorfman, R. (1979), A formula for the Gini coefficient, The Review of Economics and Statistics, 61(1), p. 1-46.
8. Favre, L. and Galeano, J.-A. (2002), Mean-modified value-at-risk optimization with hedge funds, The Journal of Alternative Investments, 5(2), pp. 2125.
9. Fishburn, P. C. (1977), Mean-risk analysis with risk associated with below-target returns, The American Economic Review, 116-126.
10. Konno, H. and Yamazaki, H. (1991), Mean-absolute deviation portfolio optimization model and its applications to Tokyo stock market, Management Science, 37(5), pp. 519-531.
11. Kendall, M. G. (1962), Ranks and measures, Biometrika, 49(1-2), pp. 133137.
12. Jaaman, S. H. and Lam, W. H. (2012), Mean-variance and mean-Gini analyses to portfolio optimization in Malaysian stock market, Economic and Financial Review, 2(2), 60-64.
13. Lien, D. and Shaffer, D. R. (1999), A note on estimating the minimum extended Gini hedge ratio", Journal of Futures Markets, 19(1), pp. 101-113.
14. Markowitz, H. (1952a), Portfolio selection, The Journal of Finance, 7(1), p. 77.
15. Markowitz, H. (1952b), The utility of wealth, Journal of Political Economy, 60(2), pp. 151-158.
16. Markowitz, H. (1959), Portfolio selection: efficient diversification of investments, Cowies Foundation Monograph, (16).
17. Shalit, H. (2014), Portfolio risk management using the Lorenz curve, The Journal of Portfolio Management, 40(3), pp. 152-159.
18. Shalit, H. and Yitzhaki, S. (1984), Mean-Gini, portfolio theory, and the pricing of risky assets, The Journal of Finance, 39(5), pp. 14491468.
19. Shalit, H. and Yitzhaki, S. (2005), The Mean-Gini efficient portfolio frontier, Journal of Financial Research, 28(1), pp. 59-75.
20. Sortino, F. A., Meer, R. van der and Plantinga, A. (1999), The Dutch triangle, Journal of Portfolio Management, 26(1), pp. 50-57.
21. Yitzhaki, S. (1982), Stochastic dominance, mean variance, and Gini's mean difference, The American Economic Review, 178-185.
22. Yitzhaki, S. (1983), On an extension of the Gini inequality index, International Economic Review, 24(3), p. 6-17.
23. Young, M. R. (1998), A Minimax portfolio selection rule with linear programming solution, Management Science, 44(5), pp. 673-683.