

MODELING AND FORECASTING UTILITY RESOURCES USAGE IN AN EMERGING COUNTRY

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Abstract

The purpose of this study is to compare the forecasting efficiency of two univariate time series models, the seasonal autoregressive integrated moving average (SARIMA) and the Holt-Winter's triple exponential smoothing. A monthly electricity and water consumption data used was sourced from the South African Reserve Bank. This data was available for the period Q3 2008 to Q1 2016. Upon subjecting the data to the diagnostic tests of normality, heteroscedasticity and stationarity, parameters of the selected model were estimated using the maximum likelihood method. Although the two models were found to be good estimators and globally significant, Holt Winter's triple exponential smoothing (HWTES) was selected as the best forecasting model based on the small forecast errors generated. The forecasts revealed that utility resources demand in South Africa are expected to be high for the period 2016 to 2017 and the trend extends to periods ahead. Using these findings, better strategies on the production and distribution of electricity and water can be formulated. Lives of people in South Africa could also be improved.

Keywords: Utility Resource Usage Forecasting, Holt-Winter's Exponential Smoothing, SARIMA, Seasonality, Time Series Analysis

1. INTRODUCTION

Forecasting time series data is an imperative piece of operations research because these data often provide the groundwork for decision models. Time series analysis tools are typically used to construct a model used to produce forecasts of future events. Consequently, modeling time series is a statistical problem. Forecasts are used in computational processes to estimate the parameters of a model being used to allocate limited resources. Forecasting may be used as decision risk minimization when the result of an action is of consequence, but cannot be known in advance with precision. This could be done by supplying additional information about the possible outcome. Time series forecasting further assumes that a series is a combination of a pattern and some random error. The analyst's goal is to separate the pattern from the error by understanding the trend pattern, its long-term increase or decrease, seasonality, the change caused by seasonal factors such as fluctuations in use and demand. Time series models assume that observations vary according to some probability distribution about an underlying function of time. Not only time series methods can be used to do forecasting. Non-scientific methods such as expert judgment are often used to predict long-term changes in the structure of a system. A very good example related to qualitative analysis such as the Delphi technique may be used to forecast major technological innovations and their effects. Another example related to quantitative analysis such as Causal regression models may be used to predict dependent variables as function of other correlated observable independent variables.

This study scratches the surface of the field, by restricting attention to using historical time series

data to develop a time-dependent forecasting model. The proposed methods are appropriate for automatic, short-term forecasting of frequently used information where the underlying causes of time variation are not changing markedly. Time series methods proposed in this study are assumed to be competent in handling the series which contain both the trend and seasonal components. Wei (2006) cautioned that many business and economic time series contain a seasonal phenomenon that repeats itself after a regular period of time. He refers to the smallest period of time for this repetitive phenomenon as seasonal period. Seasonal phenomenon may stem from factors such as weather which may have many effects in many business and economic activities. The current study compares less sophisticated but powerful methods to come up with a model that may be used in doing short term forecasts of electricity and water consumption, herein referred to as utility resources in South Africa. The predictive power of the seasonal autoregressive moving average (SARIMA) and HWTES methods will be evaluated on utility resource usage data.

SARIMA models are developed from the Box-Jenkins autoregressive moving average (ARMA) and characterize a very powerful and flexible class of models for time series analysis and forecasting. Over the years, ARMA models have been effectively applied to many problems in research and practice. However, literature provides evidence on certain situations where this class of model falls short on providing accurate answers. A typical example such as forecasting future values of a series primarily relies on the information about the past. This implicitly assumes that circumstances at which the data is collected will remain the same in the future as well. Nonetheless, the data used in this study is

collected on monthly time intervals and it is assumed that these data exhibit periodic patterns. Thus the study applies SARIMA and (HWTES) methods to cater for these seasonal and trend variations. These methods further place more emphasis on recent than distant past. Pankratz (1983) warned against the application of these methods when doing long term forecasting since they have a tendency of producing less reliable estimates. The application of Holt-Winters with general seasonality modelling in this paper is to offer broader spectrum of possibilities for seasonality treatment. These methods are additionally appropriate when time series exhibit irregular components and offers both the additive and multiplicative seasonality.

Another reason for using these less sophisticated and low computational intensive methods apart from the ability to handle both seasonal and trend features is due to lack of guarantee that more sophisticated methods perform better than the less sophisticated ones. In essence, the performance of methods is dependent on a particular data set. Some methods outperform others; however such display of supremacy may not be generalized. Such differences may not be evident when similar methods are applied on other data sets. It is of utmost importance to model monthly peak electricity and water demand as this will provide short-term forecasts. These forecasts may assist system operators in effective transmission of electrical energy and water in the country. Forecasts of load demands are also very vital for decision making processes in the electricity and water sectors. This may help eliminate or reduce the problem of water shortages in the country which we experience especially during dry seasons. Load shedding has also since the past two years been a pressing issue and remained to have effects in the operation of businesses in the country. This also discourages investors and prevents injection of funds in the country. Consequently this could have long term effects on the country's economic wellbeing. If the demand of these sectors is unknown, planning may be uncertain and decision making may also be almost wrong. Through the findings of this study, decision makers would be able to devise strategies for optimal month to month operation of power and water plants. They may even be encouraged to come up with strategies to do capacity expansion. As highlighted by Ismail et al. (2009), the demand of electricity forms the basis for power system planning, power security and supply reliability.

In South Africa, electricity and water are regarded as basic needs. The government has made it a point that these sectors are top of their priority list and their availability to residents of the country will help in eradicating poverty. Electricity is lately essential for economic development more specifically for the industrial sector. Lepojević and Andelković-Pešić (2011) highlighted that authorities use energy demand forecasts as one of the most important policy tools, but they are faced with a dilemma of forecasting this demand. To ensure good planning and controlling, accurate forecasts must be available. If accurate forecasts are known, proper amount of utility resources will be supplied and this will guard against underestimation or

overestimation of these supplies. More money could be saved and be allocated to other important and basic sectors. Better budget planning, maintenance, scheduling and fuel used for producing these sectors could be better managed. Unnecessary interruption of services could be dealt away with and more businesses could also be restored and as a result. High unemployment rates and pointless strikes could be reduced. This study would serve as point of reference for other researchers who wish to embark studies on electricity and water or scholars who wish to evaluate forecasting models.

The remainder of the paper is planned as follows; Section 2 gives a brief review of studies on the subject and Section 3 provides data description and also presents the SARIMA and HWTES, model selection and forecasting criteria. Empirical results are covered and discussed in Section 4 whereas concluding remarks and recommendations are given in Section 5.

2. LITERATURE REVIEW

In modelling and forecasting of univariate and multivariate time series data, several methods are available. Among those available are simple and sophisticated methods. Simple methods include the Decomposition, exponential smoothing and the Box and Jenkins Seasonal Autoregressive Integrated Moving Average (SARIMA). Tong's Threshold Autoregressive (TAR), Teräsvirta's Smooth Transitional Autoregressive (STAR), Artificial Neural Networks and others are regarded as sophisticated models. This study limits its application to only two linear univariate methods to model and forecast utility resources demand in South Africa.

A number of studies used SARIMA and HWTES methods in variety of data sets. Studies by Bolarinwa (2005, 2010) did a comparative analysis of Decomposition, SARIMA and Holt-Winters forecasting methods on meteorological and temperature data in Nigeria. The authors concluded based on the findings that a Decomposition method is best in producing a short term, *i.e.* 1-step ahead forecasts. SARIMA model was reported to be the best in producing longer forecasts, 7-step forecasts. A follow-up study by Bolarinwa and Ayoolwa (2014) compared the efficiency of the mentioned time series models in modelling Nigeria's monthly minimum temperature. The data used for this study was collected for the period January 2000 to August 2013. The Root mean square error (RMSE) was used to make selection between the three models and a Decomposition model was found to be the best in producing out-of-sample forecasts despite its simplicity. The study could have used more than one error metrics in selecting the models so as to avoid biasness or better still power analysis could have been performed to assess if the difference between the calculated RMSE is of significance or not.

An empirical study by Taylor et al. (2006) compared the efficiency of six univariate methods to model and forecast short-term electricity demand. The study used an hourly time series electricity demand for Rio de Janeiro and half-hourly time series electricity demand for England and Wales. Despite the use of the so called sophisticated models, the exponential smoothing method produced better forecasts compared to others. This

study lent the use of the mean absolute percentage error (MAPE) to measure accuracy in load forecasting. Different conclusion could have been reached had the authors used more than one error metric as a measure of forecast accuracy.

Chikobvu and Sigauke (2012) developed the SARIMA and regression with SARIMA errors models to predict the South African daily peak electricity demand for the period 1996 to 2009. The study further introduced the HWTES model to evaluate the performance of these two models. While the MAPE and RMSE were in favour of SARIMA model as far as short term forecasts are concerned, the regression-SARIMA model captured important drivers of electricity demand in the country. Inversely, the study by Sumer et al. (2009) reported the regression model with seasonal latent variable to be more efficient than ARIMA and SARIMA.

Ismail et al. (2015) used double seasonal autoregressive integrated moving average (DSARIMA) to estimate and forecast Egypt's electricity demand. Daily data used covered the period 1 June 2012 to 28 June 2012. Bayesian information criterion selected DSARIMA (1,1,1)₂₄(2,1,3) as the best model among other tentative models. The selected model produced lower error forecasts according to MAPE than others.

Molla et al. (2016) assessed the performance of ARIMA and Holt-Winter's additive trend and seasonality smoothing methods in forecasting electricity generation in Australia. Monthly electricity productions data covering the period from January, 1980 to August, 1995 was used in this study. Among the four forecast error metrics, Theil's U-statistic proved that SARIMA (0, 1, 1) (0, 1, 2)₁₂ is a good forecaster of Australia's electricity demand than its counterpart.

Holt Winters exponential smoothing and Box-Jenkins SARIMA methods have also been used in many other areas to model and produce short term forecasts. For instance, Olowe (2009) and Etuk (2012, 2013) used these methods to forecast exchange rates, and Junttila (2001), Pufnik and Kunovac (2006), Schulze and Prinz (2009) forecasted inflation rates using these methods.

3. DATA AND METHODS

Data analysed in this study is based on the South African electricity consumption collected from the South African reserve bank website. The Statistical Analysis Software (SAS) version 9.3, registered to the SAS Institute Inc. Cary, NC, USA is used for data analysis. A quarterly data covers the period 2008 Q3 to 2016 Q1. A total of 100 observations is used. Since electricity consumption is collected over time and thus violates the assumption of unit root, the original series is transformed to obtain uniform variability. Transformation is done using the natural logs as suggested by Sadowski (2010). The author advocates for this type when the standard deviation of the original time series increases with the series mean in a linear fashion.

One other general class of transformations is the Box-Cox transformation suggested by Chatfield (1996). This transformation can be shown in a time series with y_1, \dots, y_t ,

$$\begin{cases} (Y_t^\lambda - 1)/\lambda & \lambda \neq 0 \\ \log Y_t & \lambda = 0 \end{cases} \quad (1)$$

where, λ according to Chatfield denotes the transformation parameter which may be estimated by subjective judgement or by a formal statistical procedure. Scholars advice against transformation of the series and suggest the use of the original values. Further, there is little evidence that the use of a non-linear transformation improves forecasts (Nelson and Granger, 1979). The use of Box-Cox transformation does not guarantee that the properties of the series will be constant. On the same breath, Brockwell and Davis (2002) warned against the use of non-stationary series. The authors suggested that for a series to be considered stationary and be a candidate for Box-Jenkins modelling, it must not have any parameter estimates inside the unit circle, *i.e.* parameter estimates with complex roots. They recommended that this series must oscillate around a constant mean with a constant variance. Literature identifies some of the reasons leading to non-stationarity as random walk, drift, or trend. To avoid complicated results, the study nonetheless applies the Box-Cox transformation to the series prior to primary data analysis.

Seasonal stationarity testing

A seasonal series can also be non-stationary and requires seasonal differencing to render it stationary. If, say, Y_t is a seasonal time series with seasonal period s , Wei (2006) suggested a seasonal differencing $(1 - B^s)Y_t = Y_t - Y_{t-s}$. In such a case, the associated time series model contains seasonal unit root. Say the time series model $(1 - \Phi B^s)Y_t = a_t$ has equivalence;

$$Y_t = \Phi Y_{t-s} + a_t \quad (2)$$

Wei (2006) defines $Y_{1-s}, Y_{2-s}, \dots, Y_0$ as initial conditions and a_t are identically and independently distributed (*i.i.d*) random variables with zero mean and constant variance σ_a^2 . The ordinary least squares estimator of Φ is given by;

$$\hat{\Phi} = \frac{\sum_{t=1}^n Y_{t-s} Y_t}{\sum_{t=1}^n Y_{t-s}^2} \quad (3)$$

which is also a maximum likelihood estimator when a_t is Gaussian. The studentized statistic for testing the null hypothesis $H_0: \Phi = 1$ is;

$$T = \frac{\hat{\Phi} - 1}{se_{\hat{\Phi}}} \quad (4)$$

If the observed absolute T value is greater than the critical value, no simple differencing is required since the series has been rendered stationary. Alternatively, a spectral density function is used to confirm the stationarity of the series. The desire is to have a plot revealing the stochastic properties revolving around a zero line to help decide if the series is stationary or not. This plot should further reveal periodic movements, indicative of high peaks.

Holt-Winters method

This section reviews forecasting exponential smoothing method that captures the trend and seasonal cycles developed by Holt in 1957. Suppose a trend plus noise (non-seasonal) model given as;

$$X_t = m_t + Y_t \quad t = 1, \dots, n \quad (5)$$

where $EY_t = 0$ has observations Y_1, Y_2, \dots, Y_n . Further, suppose the exponential smoothing recursions obtained from;

$$(1 - \alpha)\hat{m}_{t-1} \quad t = 2, \dots, n \quad (6)$$

allowed us to compute estimates \hat{m}_t of the trend at times $t = 1, 2, \dots, n$. Equation (6) is often referred to as exponential smoothing, since the recursions imply that for $t \geq 2$, $\hat{m}_t = \sum_{j=0}^{t-2} \alpha(1 - \alpha)^j X_{t-j} + (1 - \alpha)^{t-1} X_1$ a weighted moving average of X_t, X_{t-1}, \dots , with weights decreasing exponentially with the exception of the last one. If the series is stationary, m_t becomes constant and the exponential smoothing forecast of Y_{n+h} based on the observations Y_1, Y_2, \dots, Y_n becomes;

$$P_n Y_{n+h} = \hat{m}_n, \quad h = 1, 2, \dots \quad (7)$$

In the presence of a trend, then a natural generalization of the forecast function (7) that takes this into account is;

$$P_n Y_{n+h} = \hat{a}_n + \hat{b}_n h, \quad h = 1, 2, \dots, \quad (8)$$

\hat{a}_n and \hat{b}_n are regarded as estimates of the "level" a_n and "slope" b_n of the trend function at time n . Holt (1957) suggested a recursive scheme for computing the quantities \hat{a}_n and \hat{b}_n in (8). Denoting by \hat{Y}_{n+1} the one-step forecast $P_n Y_{n+h}$, we have from (8) $\hat{Y}_{n+1} = \hat{a}_n + \hat{b}_n$.

Brockwell and Davis (2002) suggested that as in exponential smoothing, the estimated level at time $n + 1$ is a linear combination of the observed value at time $n + 1$ and the forecast value at time $n + 1$. As a result,

$$\hat{a}_{n+1} = \alpha Y_{n+1} + (1 - \alpha)(\hat{a}_n + \hat{b}_n) \quad (9)$$

The slope at time $n + 1$ can be estimated as a linear combination of $\hat{a}_{n+1} - \hat{a}_n$ and the estimated slope \hat{b}_n at time n . Consequently,

$$\hat{b}_{n+1} = \beta(\hat{a}_{n+1} - \hat{a}_n) + (1 - \beta)\hat{b}_n \quad (10)$$

In the presence of both the trend and seasonal variations, the model with period d becomes;

$$X_t = m_t + s_t + Y_t, \quad t = 1, \dots, n, \quad (11)$$

where, $EY_t = 0$, $s_{t+d} = s_t$, and $\sum_{j=1}^d s_j = 0$. Further generalization of the forecast function (11) that takes this into account is;

$$P_n Y_{n+h} = \hat{a}_n + \hat{b}_n h + \hat{c}_{n+h} \quad h = 1, 2, \dots, \quad (12)$$

Where, \hat{a}_n, \hat{b}_n and \hat{c}_n are thought of as estimates of the "trend level" a_n , "trend slope" b_n and "seasonal component" c_n at time n . If k is the smallest integer such that $n + h - kd \leq n$, then;

$$\hat{c}_{n+h} = \hat{c}_{n+h-kd}, \quad h = 1, 2, \dots, \quad (13)$$

The values of \hat{a}_i, \hat{b}_i and \hat{c}_i , $i = d + 2, \dots, n$ are calculated from recursions analogous to (9) and (10) given as;

$$\hat{a}_{n+1} = \alpha(Y_{n+1} - \hat{c}_{n+1-d}) + (1 - \alpha)(\hat{a}_n + \hat{b}_n) \quad (14)$$

$$\hat{b}_{n+1} = \beta(\hat{a}_{n+1} - \hat{a}_n) + (1 - \beta)\hat{b}_n, \text{ and,} \quad (15)$$

$$\hat{c}_{n+1} = \gamma(Y_{n+1} - \hat{a}_{n+1}) + (1 - \gamma)\hat{c}_{n+1-d} \quad (16)$$

on condition that $\hat{a}_{d+1} = Y_{d+1}$, $\hat{b}_{d+1} = \frac{(Y_{d+1} - Y_1)}{d}$ and $\hat{c}_i = Y_i - (Y_1 - \hat{b}_{d+1}(i - 1))$, $i = 1, \dots, d + 1$. Brockwell and Davis (2002) recommended that equations (14) to (16) can be solved successively for \hat{a}_i, \hat{b}_i and $\hat{c}_i, i = d + 1, \dots, n$ and the predictors $P_n Y_{n+h}$ found from (12). The forecasts of (14) to (16) depend entirely on the parameters α, β and γ .

Seasonal ARIMA

Suppose the fitted ARIMA model has seasonal behaviour present and it needs to be accounted for. Box and Jenkins (1976) proposed that seasonal differencing could render the series stationary. Use letters d and D as the degrees of non-seasonal and seasonal differencing respectively, to make the series stationary. Box *et al.*, (2008); Cryer and Chan (2008) proposed the following generalised form of the model to account for seasonal variations;

$$\Phi_P(B^s)Z_t = \theta_Q(B^s)a_t \quad (17)$$

where, $s = 12$ if data is collected on monthly basis and 4 if quarterly data is used. A proposed SARIMA model given by:

$$\Phi_P(B^s)\phi(B)\nabla_s^D\nabla^d Z_t = \alpha + \theta_Q(B^s)\theta a_t \quad (18)$$

This model is denoted as a multiplicative $ARIMA(p, d, q) \times (P, D, Q)_s$, where Φ and Θ are polynomials of order P and Q respectively. The non-seasonal AR and MA characteristics operators are;

$$\phi(B) = 1 - \phi_1 B - \dots - \phi_p B^p \quad (19)$$

and

$$\theta(B) = 1 - \theta_1 B - \dots - \theta_q B^q \quad (20)$$

The the seasonal AR and MA characteristics operators are:

$$\Phi_P(B^s) = 1 - \phi_1 B^s - \dots - \phi_p B^{Ps} \quad (21)$$

$$\Theta_Q(B^s) = 1 - \theta_1 B^s - \dots - \theta_q B^{Qs} \quad (22)$$

$$\nabla_s^D \nabla^d Z_t = (1 - B)^d (1 - B^s)^D Z_t \quad (23)$$

where B = the backward shift operator, d and D = the non-seasonal and seasonal order of differences respectively. This operation is usually abbreviated as $SARIMA(p, d, q)(P, D, Q)_s$. In the absence of seasonal effect, a $SARIMA$ model reduces to pure $ARIMA(p, d, q)$ and in case of stationary time series dataset, a pure $ARIMA$ reduces to $ARMA(p, q)$.

Forecasting $SARIMA$ model is analogous to the forecasting of $ARIMA$. Following Brockwell and Davis (2002) approach, the trick is to use operator (16) and setting $t = n + h$ to obtain;

$$X_{n+h} = Y_{n+h} + \sum_{j=1}^{d+Ds} a_j X_{n+h-j} \quad (24)$$

Under the assumption that the first $d + Ds$ observations $X_{-d-Ds+1}, \dots, X_0$ are uncorrelated with $\{Y_t, t \geq 1\}$ the best linear predictors of (18) can be determined based on $\{1, X - d - Ds + 1, \dots, X_n\}$ by applying P_n to each side of (18) to obtain the expression;

$$P_n X_{n+h} = P_n Y_{n+h} + \sum_{j=1}^{d+Ds} a_j P_n X_{n+h-j} \quad (25)$$

The first term on the right is just the best linear predictor of the ARMA process $\{Y_t\}$ in terms of $\{1, Y_1, \dots, Y_n\}$. The predictors $P_n X_{n+h}$ can then be computed recursively for $h = 1, 2, \dots$ from (25) provided that $P_n X_{n+1-j} = X_{n+1-j}$ for each $j \geq 1$. This therefore gives a prediction mean squared error as;

$$\sigma_n^2(h) = E(X_{n+h} - P_n Y_{n+h})^2 + \sum_{j=0}^{h-1} \left(\sum_{r=0}^j X_r \theta_{n+h-r-1, j-r} \right) v_n + h - j - 1 \quad (26)$$

where, θ_{nj} and v_n are obtained by applying the innovations algorithm to the differenced series $\{Z_t\}$ (23).

Model assumptions evaluation

The study assesses the model residuals for normality and heteroscedasticity. These assumptions concern the mean and variance of the distribution. The proposed test of normality is the Kolmogorov-Smirnov which requires a minimum sample of size $N \geq 50$. The selected sample used in this study perfectly fit in the supremum class of empirical distribution function statistic (Conover, 1999). The proposed null hypothesis is that the residuals are normally distributed. This hypothesis is rejected if the p -value associated with the test statistic is less than an appropriate level of significance.

Heteroscedasticity assumption is evaluated using the Lagrange multiplier test based on the autoregressive conditional heteroscedasticity (ARCH) disturbances based on the ordinary least squares residuals. The observed probabilities associated with the Lagrange multiplier (LM) test from this procedure must be greater than a conventional level of significance to render the residuals homoscedastic. Once these assumptions have been assessed, the study uses the model(s) to produce the forecasts of the series. Both the in-sample and out-of-sample forecasts are obtained using the model.

Choosing between candidate models

A couple of models are anticipated from the SARIMA framework. This implies that only one optimal model should be selected and be used for further analyses. The main idea behind this process is better described using the Occam's razor (Blumer et al. 1987) stating that given a set of models all of which explain data equally well, the simplest should be chosen. A substantive number of approaches are available based on this trade-off between the increase in data likelihood and model over fitting when adding parameters to a model, the Akaike information criterion (AIC) (Sakamoto et al., 1986) and Bayesian information criterion (BIC). The latter is most preferred in this study. This approach provides a framework for estimating the optimal model order by penalising models with larger

number of parameters more heavily as does the AIC. As a result, a model with relatively lower complexity is selected. For a given data set D with parameters θ , the BIC is defined according to (Bishop, 2006; and Cavanaugh and Neah, 2012) as;

$$BIC = \ell(\theta|D) - \frac{1}{2} N_\theta \log N_D \quad (27)$$

where, N_θ and N_D are the number of model parameters and data points in respect. A model that maximises the BIC is chosen, and it should be noted that this criterion selects a model which maximises the log-likelihood of the data, $\ell(\theta|D)$ with respect to the model's complexity, $\frac{1}{2} N_\theta \log N_D$.

Forecast error model selection

It is not surprising to realize that several models representing a series may be found adequate. The ultimate choice of a model depends on a goodness-of-fit. Suppose the main intention of the analysis is to apply the model to produce forecasts of a series, the criteria for model selection can be based on forecast errors. Assuming the l -step ahead forecast error is;

$$e_l = Z_{n+1} - \hat{Z}_l \quad (28)$$

with n defined as forecast origin larger than or equal to the length of the series so that the evaluation is based on out-sample forecasts. This section compares the forecasting capability of the time series models, HWTES and SARIMA. In a broad spectrum, the forecasting capability and accuracy of the model is evaluated with reference to their forecast errors (29). To be precise, the study uses the mean absolute percentage error (MAPE), suggested by Khan (2011) which according to Makridakis et al. (1997) is less sensitive to outlier distortion than the mean square forecast error (MSFE). To avoid bias, the study further uses the mean absolute error (MAE), mean square error (MSE) and the root of MSE as described below;

$$MSE = \frac{1}{n} \sum_{i=1}^n e_t^2 \quad (29)$$

and

$$Root\ MSE = \sqrt{\frac{1}{n} \sum_{i=1}^n e_t^2} \quad (30)$$

where, $e_t = Y_t - F_t$. Y_t and F_t are observed and forecasted values at time t , n is the sample size. The MSE is calculated as;

$$MAE = \frac{1}{n} \sum_{i=1}^n |e_t| \quad (31)$$

Finally, the MPE is the mean of the relative or percentage error and is given by;

$$MAPE = \frac{1}{n} \sum_{i=1}^n |PE_t| \quad (32)$$

where, $PE_t = \frac{Y_t - F_t}{Y_t} \times 100\%$ is the relative or percentage error at time t . The model that gives the

minimum measures of these errors will be the expected model for further forecasting (Khan, 2011).

4. EMPIRICAL FINDINGS

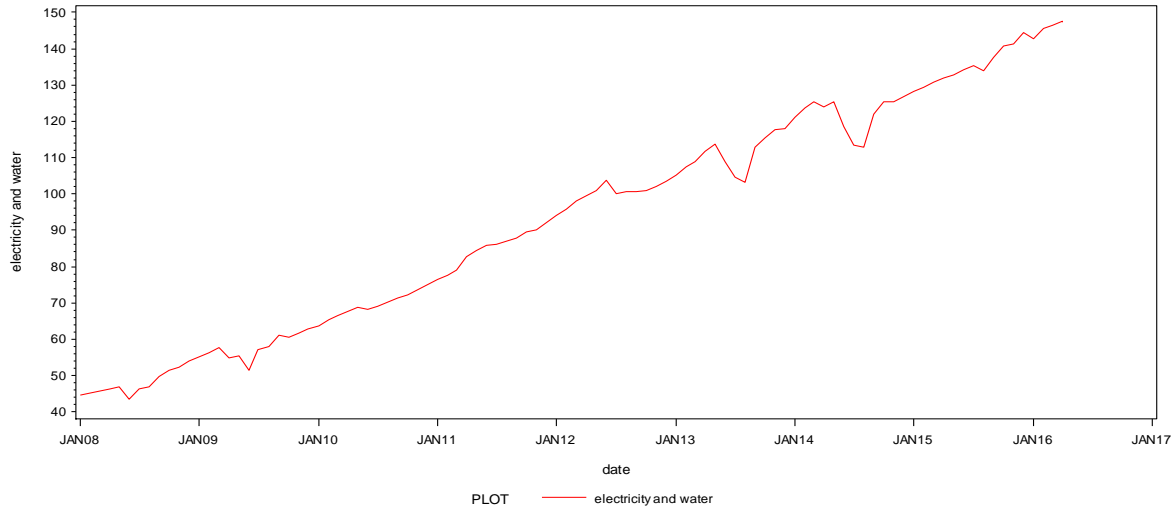
4.1. Initial analyses results

This section provides results for investigation into properties of time series data used in the study. The initial plot representing log transformed electricity

and water consumption in South Africa for the period 2008 Q3 to 2016 Q1 is given as Figure 1. This plot gives a reasonable understanding of the time series properties unveiled by the data.

Figure 1 shows a positive linear trend and some seasonal fluctuations during some of the months. This series suggests that there has been a steady increase in electricity and water consumption in South Africa over the years.

Figure 1. Original electricity and water plot

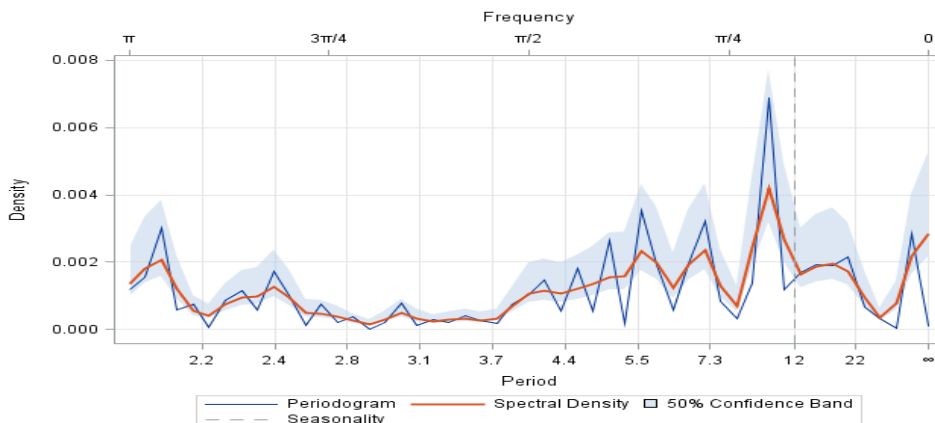


However, there is no solid pattern shown by the data on monthly basis. Generally, it could be deduced from the data that electricity and water demand will be high in years to come. This general increase in consumption implies that the stochastic properties of the series vary with time and as a result the series have unit root. To iron out the differences, the Box-Cox transformation was imposed and by observation, the estimations of the lambdas implied

that transformation of the actual values is appropriate.

A visual display of the transformed series revealed a nearly smoothed linear upward trend with variations during particular months. The natural logarithm transformation further helps to control heteroscedasticity that may be present as a result of the large data set (Hekkenberg et al., 2009).

Figure 2. Spectral density of log transformed data



Over and above the Box-Cox transformation, differencing was applied to render the series stationary. This is due to a linear trend exhibited by the data implying that the associated stochastic properties have unit root. The results from first seasonal differencing are presented in Figure 2.

Revealed by spectral analysis is periodicity in the data. Figure 2 shows the spectral density of electricity and water consumption. The first major peak in the spectral density is evident during the period 12, the first month indicating the presence of a periodic movement on monthly basis. The plot further reveals that the stochastic properties of the

series oscillate around a zero line suggesting that the series is stationary. This revelation concludes that the proposed methods for this study could be applied.

The observed probabilities associated with the tests for ARCH disturbances based on OLS residuals shown in Table 1 shows residuals from lag 1 are greater than 0.05 level of significance, suggesting that the residuals are homoscedastic. Further, the fact that the observed *p-values* (0.1500) of the Kolmogorov-Smirnov test (see Table 4 on the appendix) for normality exceed the 0.05 level of significance implies that the normality assumption is not violated. The histogram as shown on Figure 6 (see appendices) also confirms the validity of this assumption.

Table 1. Tests for ARCH Disturbances Based on OLS Residuals

Order	LM	Pr > LM
2	2.3228	0.1275
3	2.3231	0.3130
4	4.9189	0.1778
5	8.2237	0.0837
6	8.4842	0.1315
7	10.9972	0.0885
8	10.9979	0.1387
9	11.6551	0.1673
10	13.0657	0.1597
11	13.0709	0.2197
12	13.6833	0.2510

Note: Model estimation and diagnostics results

The purpose of this section is to estimate and select the best model from candidate models. Firstly, HWTES technique was used to produce the initial and final results summarized in Table 2.

Table 2. Holt-Winters Triple exponential smoothing results

Parameter	Estimates	Standard error	t value	Approx Pr > t
LEVEL	0.6329	0.0491	12.8941	0.0000
TREND	0.0010	0.0159	0.0631	0.9498
SEASON				
SBC: -238.6713	0.9990	0.1817	5.4980	0.0000
LEVEL	0.8333	0.0651	112.7950	0.0000
SEASON	0.9990	0.4058	2.4618	0.01557
SBC: -261.79258				

The results from SARIMA modelling are presented in Table 3.

The maximum likelihood method was used to estimate model parameters. Both the signs of the initial (on the upper tier) and final Holt-Winter's model (on the lower tier) are positive. This could suggest an increased demand in electricity and water in the country over time. The appropriate Holt-Winters smoothing factors ($\alpha = 0.6329, \beta = 0.0010$ and $\gamma = 0.9990$) are shown for the initial model. The presence of seasonality in the series is evident from the gamma value. A glance at the *p-values* ((0.0000) less than 0.05 level of significance) of the initial model confirm that both level and seasonal models have significant parameters. The associated p-value of the trend (0.9498) is however insignificant. Consequently, the trend had to be removed from the model and re-

estimation be done without a trend (lower tier). The smoothing equations according to [13] and [15] convert to;

$$\hat{a}_{n+1} = 0.6329(Y_{n+1} - \hat{c}_{n+1-d}) + 0.3671(\hat{a}_n + \hat{b}_n), \quad (33)$$

and

$$\hat{c}_{n+1} = 0.9990(Y_{n+1} - \hat{a}_{n+1}) + 0.001\hat{c}_{n+1-d}. \quad (34)$$

The final HWTES model parameters were found significant at 0.05 significance. The minimum information criterion (SBC) was used to select the best fitted-smoothing model and the re-estimated (on lower tier) was found favourable by these criteria and was a candidate model to SARIMA. Presented next are the results produced from SARIMA procedure. The autocorrelation and partial autocorrelation plots were used to identify tentative models. The parameters of identified models were estimated using the maximum likelihood method and the results are presented in Table 3.

Table 3. SARIMA maximum likelihood estimates

Parameter	Estimate	Standard Error	t Value	Approx Pr > t	Lag
AR1,1	0.51978	0.09397	5.53	<.0001	1
AR2,1					
Standard error: 0.030929	-0.18493	0.12401	-1.49	0.1359	12
SBC: -346.209					
MA1,1	-0.87097	0.06057	-14.38	<.0001	1
MA2,1		0.3402			
Standard error: 0.02577		0.1107	3.07	0.0021	12
SBC: -375.298					
MA1,1	-0.75979	0.08817	-8.62	<.0001	1
MA2,1	0.99981	0	Infity	<.0001	12
AR1,1	0.15011	0.12369	1.21	0.2249	1
AR2,1					
Standard error: 0.023851	0.63321	0.12404	5.11	<.0001	12
SBC: -372.197					

It is notable that the SBC (-375.298) is in favour of the first model. This therefore implies that seasonal $ARIMA(0,2,1)(0,2,1)_{12}$ fits the data well and will be used as a competing model against the smoothed Holt-winters triple exponential model. The standard error of this model is also the lowest compared to those of the counterparts. The estimated parameters of the selected model were diagnosed for statistical significance and stability. These parameter estimates must not be close to 1 to be rendered significant (Yaffee and MacGee, 2000). It is clear that the stability condition is satisfied when taking a quick glance at the parameters of seasonal $ARIMA(0,2,1)(0,2,1)_{12}$. Therefore the SARIMA model is written according to (21) as;

$$\Phi_p(B^{12}) = (1 + 0.8709B)(1 - 0.3402B)^{12} \quad (35)$$

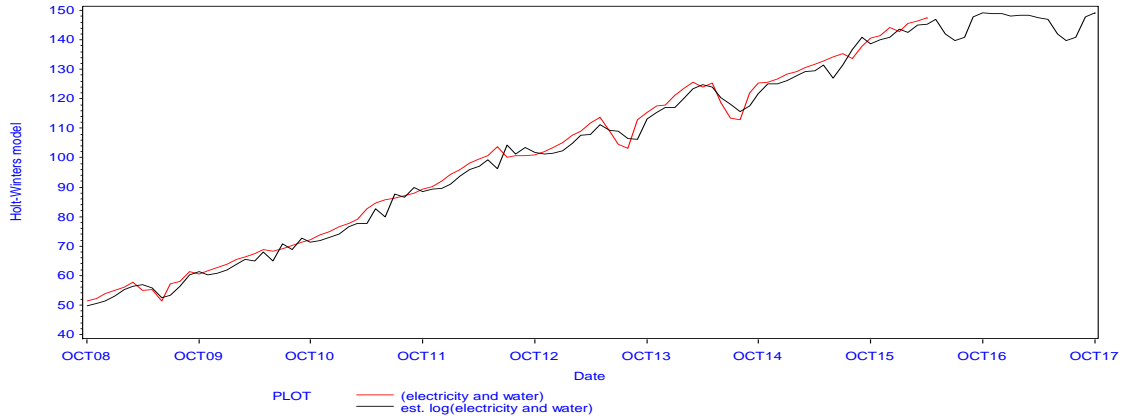
The models were subjected to a battery of diagnostics testing to assess their overall fit. A visual examination of the histograms (see Figure 6 on the appendix) for smoothed Holt-Winters's and SARIMA confirmed that the residuals of these models are normally distributed. The associated observed probabilities for Kolmogorov-Smirnoff

tests are greater than the significance level of 0.05 providing more evidence to conclude that models' residual are explained by a normal distribution. The selected models were later used for making projections into the future and summary of the results are presented in the form of Figures 3 and 4 for Holt-Winters and SARIMA respectively.

4.2. Forecast results

The purpose of this section is to assess the accuracy of the forecasts produced. The forecasts shown in Tables 4 are obtained using the Holt-Winters and SARIMA models respectively. Figure 3 and Figure 4 are graphical representations of the forecasts for the period May 2016 to April 2017.

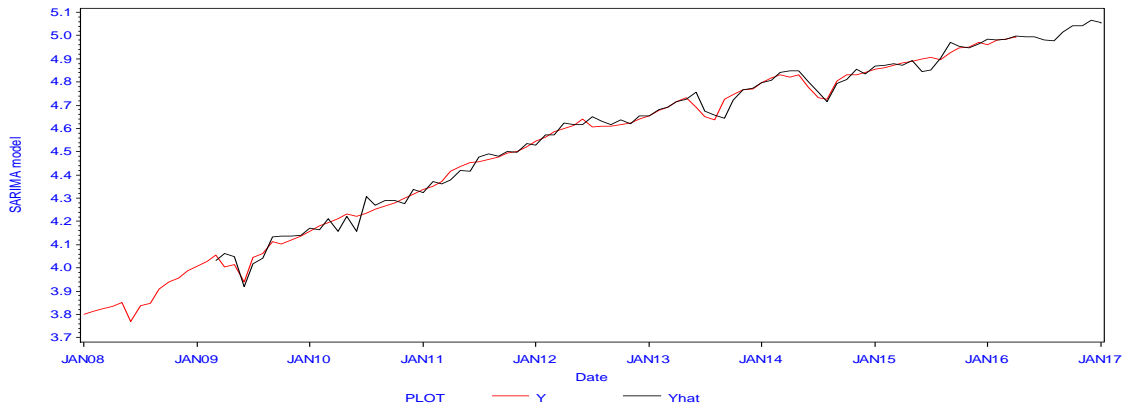
Figure 3. Holt-Winters forecasts



Figures 3 and 4 reveal that the models mimic the data very well. Original and estimated values are parallel to each other. The figures display that the

forecasted values are in an increasing fashion implying that utility resource demand in South Africa is expected to be high in the next period.

Figure 4. SARIMA (0,2,1) (0,2,1), forecasts



Since the process is non-stationary, the 95% confidence limits tend to widen with an increased number of forecasts. These confidence intervals suggest a very high stochasticity in the data. Captured in these figures are both the trend and the seasonal peaks. This is a confirmation that the two models have the ability to forecast the series.

The out-of-sample forecasts show that from May 2016 to April 2017, utility resource demand will increase gradually with some fluctuations just as also witnessed in Figures 3 and 4.

Table 4. One year forecasts

Date	Holt-Winters	SARIMA
MAY16	147.073	147.865
JUN16	141.978	147.616
JUL16	139.856	145.465
AUG16	140.782	145.104
SEP16	147.717	150.558
OCT16	149.103	154.878
NOV16	148.866	154.773
DEC16	148.984	158.389
JAN17	148.119	157.120
FEB17	148.433	160.805
MAR17	148.250	160.983
APR17	147.500	163.252

The forecasts reasonably follow the same pattern of the original data confirming that the two models are relevant as predictors of resource utilisation.

4.3. Evaluating forecast accuracy

Discussed in this section is the results of the measures used for evaluating the accuracy of forecasts from both models and the results are summarised in Table 5.

Table 5. Forecast accuracy measures

Model	AvMAE_Ratio	AvMAPE_Ratio	AvMSE_Ratio
Holt-Winters Model	1.72677	0.025823	4.28063
SARIMA Model	2.31255	0.026374	8.09056

From the analysis above, it is clear that the HWTES model provides minimum forecast error values compared to the Seasonal ARIMA model. Measures of forecast error metrics, MSE, MAE and the MAPE average ratios are very small for the HWTES model implying that this model could produce very small and insignificant errors when used for forecasting utility resource demand.

5. CONCLUSIONS

This paper compared the forecasting ability of HWTES and seasonal ARIMA models with respect to the electricity and water consumption, herein referred to as utility resource usage. Historical data used in this study was sourced from the South African Reserve Bank website, and covered the period Q3 2008 to April 2016. This was the only data available for this variable during the time of the study. Due to the nature of the data, seasonal differencing was imposed to induce stationarity prior to the primary analysis of data. This is also a preliminary requirement of the application of both the SARIMA and Holt-Winter’s frameworks. Upon inspection of the AIC and SBC, ARIMA (0,2,1)(0,2,1)₁₂ was found to be well suited for the data among three other candidate ARIMA models. This model together with the HWTES was used for forecasting purposes. Both models were found to be good forecasters since they mimic the data very well. However, the findings revealed the HWTES model to produce reliable forecasts than the seasonal ARIMA model. All the forecast measurement errors for the selected model were found to be less than those of the other model. Accurate forecasts of electricity and water demand is crucial for decision makers.

For better accuracy, the study can be enhanced by comparing other time series forecasting methods such as the Artificial Neural Networks, the Threshold Autoregressive and Smooth Transitional Autoregressive. In particular, the study recommends the application of SARIMA-GARCH enhanced models to similar studies in order to take care of the volatility shocks. This model may also capture the effects of heteroscedasticity in the data. Using the forecasts from either of the models, relevant authorities may be able to determine the consistent and reliable supply schedules during peak hours or demanding seasons. The availability of accurate short term forecasts might further make it possible for the authorities to implement effective load shifting between transmission substations as well as scheduling start-up times of peak stations. Further studies that model annual winter peaks in relation to

electricity demand using extreme value theory are also encouraged.

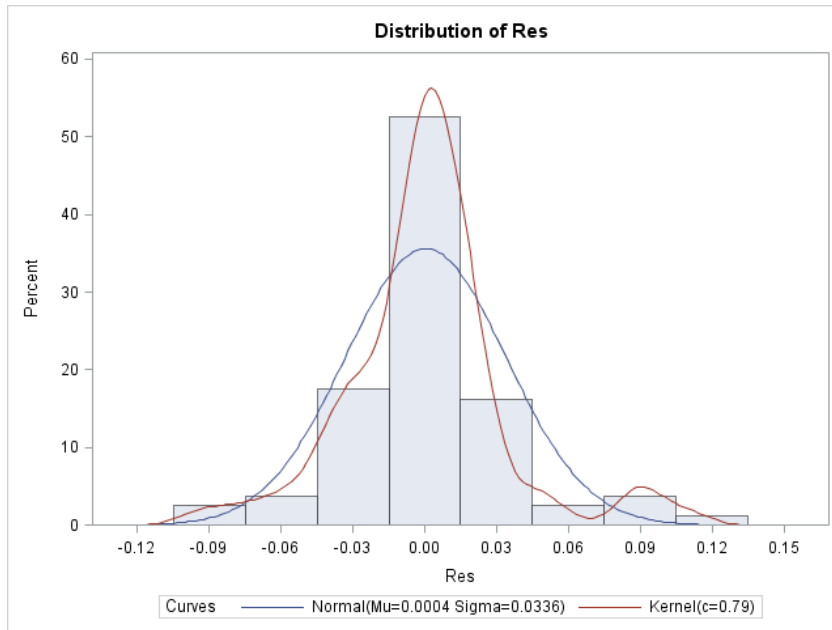
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APPENDIX:

Figure 6. Histogram of residuals



Goodness-of-Fit Tests for Normal Distribution

Test	Statistic		p-value	
Kolmogorov-Smirnov	D	0.06369224	Pr > D	>0.150
Cramer-von Mises	W-Sq	0.02594695	Pr > W-Sq	>0.250
Anderson-Darling	A-Sq	0.19446334	Pr > A-Sq	>0.250