A COMPARATIVE STUDY OF STOCK PRICE FORECASTING USING NONLINEAR MODELS

Diteboho Xaba*, Ntebogang Dinah Moroke*, Johnson Arkaah*, Charlemagne Pooe*

* North West University, Mafikeng Campus, South Africa

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Abstract

This study compared the in-sample forecasting accuracy of three forecasting nonlinear models namely: the Smooth Transition Regression (STR) model, the Threshold Autoregressive (TAR) model and the Markov-switching Autoregressive (MS-AR) model. Nonlinearity tests were used to confirm the validity of the assumptions of the study. The study used model selection criteria, SBC to select the optimal lag order and for the selection of appropriate models. The Mean Square Error (MSE), Mean Absolute Error (MAE) and Root Mean Square Error (RMSE) served as the error measures in evaluating the forecasting ability of the models. The MS-AR models proved to perform well with lower error measures as compared to LSTR and TAR models in most cases.

Keywords: Stock Price, Nonlinear Time Series Models, Error Metrics

1. INTRODUCTION

In recent times nonlinear time series has received great attention as opposed to linear time series models in modelling economic and financial data. This is due to the realization that linear models fail to describe the dynamics of financial time series. According to Maponga (2013) linear time series involves simple models that describe the behaviour of time series in terms of past values, may be used to describe the dynamics of an individual time series. Nonlinear time series are generated by nonlinear dynamic equations. These nonlinear dynamic equations show attributes that cannot be modeled by linear time series models. These attributes are time-changing variance, asymmetric cycles, higher-moment structures, thresholds and breaks data.

A variety of nonlinear models have been considered as alternative to standard linear models. For instance, the parametric nonlinear models such as the autoregressive conditional heteroscedastic (ARCH) of Engle (1982) and the generalized autoregressive conditional heteroscedastic (GARCH) of Bollerslev (1986) are some of the alternative linear models. However, the nonlinear models that receive great attention are the regime switching models (Franses and Dijk, 2000). These numbers of nonlinear models have been suggested in the literature to capture the suggested nonlinearities in economic and financial data. Commonly used among these models are the Threshold Autoregressive (TAR) of Tong (1978), Smooth Transition Regressive (STR) of Teräsvirta and Anderson (1992) and Markov-Switching Autoregressive (MS-AR) of Hamilton (1989).

These three models differ from conventional linear econometric models by their assumption of existence of different regimes, within which the time series may exhibit different behaviour. The study sought to explore the possibility of developing empirical models capable of describing and forecasting each of the five South African major banks' closing stock prices. In addition, the study investigates the question that although three different nonlinear univariate time series modeling and forecasting techniques are used for each of the five time series data used in the study; one particular method may outperform others. The performance of the model will be based on the margin of forecast error generated by each model. It is assumed that the data used satisfy the nonlinear properties in order to allow an efficient performance of the three suggested models.

The findings could empower stock market investors to make informed and accurate investment decisions. Again this may also boost the confidence of stakeholders in the financial industry to do more business with less risk exposure. Other beneficiaries of the study may be shareholders, regulators and other financial institutions as well as researchers in the academia.

The rest of the paper is organized as follows: in Section 2 study discuss the literature; in Section 3 study describe our methodology and data employed; the main results of the empirical analysis are presented in Section 4; finally, in Section 5 Study provide concluding remarks.

2. LITERATURE REVIEW

There is much interest in modeling and forecasting the nonlinearity in a variety of macroeconomic and financial series, such as stock market, exchange rate and Gross Domestic Products (GDP). A number of nonlinear time series models have been suggested in literature, for instance the bilinear models of Granger and Andersen (1978), the TAR, STR and the MS-AR models.

Moolman (2004) used the idea of MS-AR model as a tool to provide evidence that the South Africa stock market returns depends on the state of the business cycle. McMillan (2005) employed the STAR model to examine nonlinear behavior in the international stock market. Pérez-Rodriguez *et al.* (2005) concluded that the artificial neural network (ANN) and the STAR models in the Spanish market outperform the ARMA and the random-walk models. Cheung and Lam (2010) have compared profitability in the US stock market using the self-exciting threshold autoregressive (SETAR) and linear models. Ismail and Isa (2006) and Yarmohammadi et al. (2012) used MS-AR model to perform model comparison and it was found that MS-AR is a best-fitted model for modeling fluctuations of exchange rates.

Wasim and Band (2011) employed MS-AR examine the existence of bull and bear in the Indian stock market. Amiri (2012) have compared the forecasting performance of linear and nonlinear univariate time series models for GDP growth. The evaluation of the forecasting performance of their set of non-linear models using real time data is that the nonlinear models are able to capture the underlying processes of GDP rate time series as opposed to linear models. Cruz and Mapa (2013) also contributed to the literature by developing an early warning system (EWS) for predicting the occurrence of high inflation in the Philippines Markov switching model. The aim of the study was to develop models that could help quantify the possibility of the future occurrence of high inflation.

3. METHODOLOGY

This section discusses the data and methods used in the study

3.1. Sampling Technique, Data Description and Source

There are 31 banks registered with the South African Reserve Bank (SARB). Twenty-one (21) of these banks are listed on the JSE. The study used the purposive sampling technique, due to limited time and responses obtained from all the twenty-one (21) banks listed on the JSE when a request was made to help provide data for the study. Of the 21 banks listed on the JSE, only five (5) responded by providing data for this study. The banks that responded were ABSA Bank (ABSA), Capitec Bank (CAPB), First National Bank (FIRB), Nedbank (NEDB) and Standard Bank (STDB). These banks were considered to be the sampling frame for the study. This scenario fits in with the purposive sampling since the intention had been to find readily available banks willing to provide data for the realization of the aims and objectives of the study. Coincidentally, these five banks constitute the five largest banks listed on the JSE.

For the purpose of addressing the research objectives, the study uses weekly historical data starting from the first week of January 2010 to the last week of December 2012, a total of 563 observations. Using the purposive sampling technique, five (5) banks from a population of twenty-one (21) banks were used. Based on this sample, a formal request was made to the JSE for the weekly closing stock prices of the selected banks, a request that the JSE promptly responded to.

3.2. Preliminary Data Analysis

In statistics the norm is to perform preliminary data analysis in order to get the key features of the data and summarise the results. Before the main analysis of data, the study seeks to address important issues such as the normality of the actual data as suggested by Kline (2005) and Schumacker and Lomax (2004). Other descriptive statistics such as the mean, median and standard deviations of the variables are discussed. Furthermore, the skewness-kurtosis measures are estimated to check whether actual data is normal distributed, following the work of Joreskog (2000) and Cziraky *et al.* (2002).

3.3. Assessment of Data for Linearity

In order to apply the various methods needed to address the research aims and objectives of the current study, the data must first be tested for linearity and stationarity. Since nonlinearity in time series may occur in several ways, there exists no single test that dominates others in detecting nonlinearity. To test for nonlinearity in the data sets, the RESET (Regression Specification Error Test) and BDS (Brock- Dechert-Scheinkman) tests are used and the Cumulative Sum (CUSUM) test is used to investigate stability.

3.3.1. The RESET Test

According to Ramsey (1969) the RESET test is a specification test for linear regression analysis. In the context of the study, the commonly used linear regression model is the univariate autoregressive model of order p, denoted by AR (p):

$$X_t = \beta_0 + \sum_{j=1}^{p} \beta_j X_{t-p} + \varepsilon_t \qquad (3.1)$$

where, $\beta_0, \beta_1, \beta_2, \dots, \beta_p$ are parameters and ε_t is independent and identically distributed random variable with mean 0 and variance σ_ε^2. The AR order, p, is selected to minimize the error, ε_t. This is practically accomplished by selecting a value for p that minimizes an information criterion, such as the SBC (Franses & Dijk, 2000). If $X_t = (1, X_{t-1}, X_{t-2}, \dots X_{t-p})'$, equation (3.1) becomes

$$X_t = X_{t-1}'\beta + \varepsilon_t \qquad (3.2)$$

The RESET test involves, first, obtaining the OLS estimate, β, in equation (3.2), the residual $\hat{\varepsilon}_t = X_t - \hat{X}_t$, and the sum squared residuals:

$$SSR_0 = \sum_{t=p+1}^{n} \hat{\varepsilon}_t^2 \tag{3.3}$$

- The second step involves estimating the regression

$$\hat{\varepsilon}_t = X'_{t-1}\lambda_1 + M'_{t-1}\lambda_2 + e_t \tag{3.4}$$

where, $M'_{t-1} = (\hat{X}_t^2\ X_t^3\ \dots\ X_t^{s+1})$ for some $s \geq 1$, e_t is an independent and identically distributed random variable with mean 0 and variance σ_e^2. From the estimated residuals $\hat{e}_t = \hat{\varepsilon}_t - \hat{\hat{\varepsilon}}_t$, the sum of squared residuals is computed as:

$$SSR_1 = \sum_{t=p+1}^{n} \hat{e}_t^2 \tag{3.5}$$

- If the underlying AR (p) is adequate, the RESET test asserts that λ_1 and λ_2 are zero. Thus, the hypotheses to test are:

$H_0 : \lambda_1 = \lambda_2 = 0$ (Specification is indeed linear)

vs. $H_0 : \lambda_j \neq 0$ for at least one j (Specification is nonlinear)

The test statistic is the usual F-statistic of the equation given by:

$$F^* = \frac{(SSR_0 - SSR_1)/r}{SSR_1/(n-p-r)} \sim F_\alpha(r, n-p-r) \tag{3.6}$$

where, $r = s + p + 1$. At the α level, the null hypothesis of linearity is rejected in favour of the alternative hypothesis if

$$F^* > F_\alpha(r, n-p-r) \text{ or } \operatorname{prob}(F^*) < \alpha. \tag{3.7}$$

This means that the F test statistic is greater than the F critical value, and the study rejects the null hypothesis that the true specification is linear (which implies that the true specification is non-linear).

3.3.2. The Brock-Deckert-Schienkman (BDS) Test

If equation (3.1) is correctly specified, then under the null hypothesis of linearity, the residuals should be serially independent. This forms the basic idea behind various tests of nonlinearity. In practice, diagnostic tests of serial independence typically are based on certain aspects of the data such as the serial correlations or ARCH-type dependence while other tests explore dependence by testing the identical-and-independence-distributed (iid) condition of the residual term, which is sufficient for serial independence (Kuan, 2008; Kuan, 2009). One such test is the so-called Brock-Deckert-Schienkman (BDS) test - a form of portmanteau test. Portmanteau tests are residual-based tests in which the null hypotheses are well-stated but do not necessary have well-stated alternative hypotheses.

The BDS test can be applied to the estimated residuals from any time series process provided the time series process can be transformed into a form with iid errors. The BDS test, which focuses on the residual obtained after a linear structure has been removed from a process, tests the null hypothesis of linearity against a variety of alternative hypotheses. Under the null hypothesis of the BDS test, if the residuals are iid or follow a white noise process, then its m-lagged (also referred to as embedding dimension) correlation integral (also referred to as correlation function) is equal to the correlation integral of the (m-1)-lagged residuals. BDS test statistic is given by (Brock *et al.,* 1996) as:

$$BDS_{m,n} = \frac{\sqrt{n}\left[C_{m,n}(\epsilon) - C_{1,n}(\epsilon)^m\right]}{\sigma_m(\epsilon)} \tag{3.8}$$

where, $C_{m,n}(\epsilon)$ is the correlation integral, $\sigma_m(\epsilon)$ is the asymptotic standard deviation of the numerator, and ϵ is the maximum difference between pairs of observations used in calculating the correlation integral. Brock *et al.* (1996) showed that, under the null hypothesis of the residuals being iid or following a white noise process,

$$BDS_{m,n} \sim N(0,1) \tag{3.9}$$

The null hypothesis of iid residuals (whiteness or linearity) is rejected if the test statistic exceeds the critical value at the α-level of significance or if the p-value of $BDS_{m,n}$ is lower than α. Rejection of the null hypothesis is indicative of nonlinear dependence in time series data.

3.2.3. CUSUM Test

Stability is another aspect of nonlinearity in data. CUSUM examines data stability by testing for possible structural change in the data. On the one hand, if the model is stable, then β and the variance of the residuals do not change over time. In that case, the coefficients, $\hat{\beta} = (1\ \beta_1\ \beta_2\ \dots\ \beta_p)'$, in equation (3.2) can be obtained from the matrix (Brown *et al.,* 1975):

$$\hat{\beta} = (X'_{t-1}\ X_{t-1})^{-1} X'_{t-1} X_t \tag{3.10}$$

where, X_t is the dependent variable in equation (3.1) and $X_{t-1} = (1\ X_{t-1}\ X_{t-2}\ \dots\ X_{t-p})'$ and $\varepsilon_t \sim iid(0, \sigma_\varepsilon^2)$. On the other hand, if the model is unstable, then β and the variance of the residuals possibly change over time. In that case, then β is replaced by b_t, say, and so:

$$\hat{\beta} = (X'_{t-1}\ X_{t-1})^{-1} X'_{t-1} X_t \tag{3.11}$$

where, $X_{t-1} = (1\ X_{t-1}\ X_{t-2}\ \dots\ X_{t-p})'$ and $\varepsilon_t \sim iid(0, \sigma_{t,\varepsilon}^2)$. If the AR(p) is stable, the parameters remain constant over time, suggesting the absence of any structural change in the data. Thus, the hypotheses to test are:

$$\begin{aligned} &H_0 : \beta_1 = \beta_2 = \dots = \beta_p = \beta \quad \text{(say)} \\ \text{or}\quad &H_0 : \sigma_{1,\varepsilon}^2 = \sigma_{2,\varepsilon}^2 = \dots = \sigma_{n,\varepsilon}^2 = \sigma_\varepsilon^2 \quad \text{(say)} \end{aligned} \tag{3.12}$$

Then, the variance of the recursive residuals is computed as:

$$\begin{aligned} var\,(\varepsilon_t) = E(\varepsilon_t'\varepsilon_t) &= var\{\varepsilon_t - X_t'(b_{t-1} - \beta)\} \\ &= \sigma_\varepsilon^2 + X_t'\sigma_\varepsilon^2(X_{t-1}'X_{t-1})^{-1}X_t \\ &= \sigma_\varepsilon^2\{1 + X_t'(X_{t-1}'X_{t-1})^{-1}X_t\} \end{aligned} \quad (3.13)$$

Define the scaled recursive residuals, ω_t, as:

$$\begin{aligned} \omega_t &= \frac{\varepsilon_t}{\sqrt{\{1 + X_t'(X_{t-1}'X_{t-1})^{-1}\,X_t\}}} = \frac{\varepsilon_t}{\sqrt{var(\varepsilon_t)}} \\ &= \frac{\varepsilon_t}{s.e.(\varepsilon_t)} = \frac{\varepsilon_t}{\sigma_\omega} \end{aligned} \quad (3.14)$$

Then under constant parameters, $\omega_t \sim iid(0, \sigma_\omega^2)$. Then the CUSUM test statistic is given by:

$$W_t = \sum_{j=k+1}^{t} \omega_{j}, \qquad t = k+1, k+2, \dots, n \quad (3.15)$$

The cumulative sum of the square (CUSUMSQ) test statistic is given by:

$$S_t = \sum_{j=k+1}^{t} \omega_j^2 \Bigg/ \sum_{j=k+1}^{n} \omega_j^2, \quad t = k+1, k+2, \dots, n \quad (3.16)$$

These tests are performed by plotting W_t or S_t against time t. The confidence bounds are obtained by plotting the two lines that connect the points $[k, \pm a\sqrt{n-k}]$ and $[n, \pm 3a\sqrt{n-k}]$. At the 5% level (that is, 95% confidence interval) a = 0.948 while at the 1% level (that is, 99% confidence interval) a = 1.143. A test statistic meandering outside the confidence interval is indicative of a possible structural change, non-constancy in the parameters, and hence instability in the data, leading to the rejection of the null hypothesis of model stability.

Under the null hypothesis of model stability, Harvey and Collier (1977) developed a test with test statistic given by:

$$T^* = (n-1)^{1/2} S^{-1}.\bar{W} \quad (3.17)$$

where,

$$\bar{W} = \frac{1}{n-k}\sum_{j=k+1}^{n} \omega_j \text{ and } S^2 = \frac{1}{n-k-1}\sum_{j=k+1}^{n} (\omega_j - \bar{W})$$

The estimated T^* has a t-distribution with (n-k-1) degree of freedom. The null hypothesis of model stability is rejected if T^* is greater than a critical value at the α-level or if the p-value of T^* is less than α, often 0.05.

3.2.4 ARCH Test

Under the null hypothesis of linearity, the residuals of a properly specified AR(p) model should be independent. Denote the autocorrelations of the residuals by $\rho_1, \rho_2, \dots, \rho_m$, where $m = n/4$ (n=sample size), then the independence of the residuals, ε_t, can be tested based on the hypotheses (Engle, 1982):

$H_0 : \rho_1 = \rho_2 = \cdots = \rho_m = 0$ (Residuals are independent)

vs. $H_1 : \rho_j \neq 0$ for at least one j (Residuals are not independent)

The test statistic is the Q-statistic of squared residuals given by:

$$Q(m) = n(n+2)\sum_{k=1}^{m} \frac{\rho_k^2}{n-k} \sim \chi_\alpha^2(m-p) \quad (3.18)$$

At the α level, the null hypothesis of linearity is rejected in favour of the alternative hypothesis if

$$Q(m) > \chi_\alpha^2(m-p) \text{ or } prob[Q(m)] < \alpha \quad (3.19)$$

This same test is particularly useful in detecting conditional heteroskedasticity in X_t. A closely related test to the Q-statistic test is the Lagrange test of Engle (1982) for autoregressive conditional heteroskedasticity (ARCH) test based on the linear regression:

$$\hat{\varepsilon}_t^2 = \eta_0 + \sum_{i=0}^{m} \eta_i \varepsilon_{t-i}^2 + \upsilon_t \quad (3.20)$$

where, $\eta_0, \eta_1, \eta_2, \dots, \eta_m$ are parameters and υ_t is independent and identically distributed random variable with mean 0 and variance σ_ε^2. Testing for heteroskedasticity involves testing the hypotheses:

$H_0 : \eta_1 = \eta_2 = \cdots = \eta_m = 0$ (Homoskedasticity)

vs.

$H_0 : \eta_j \neq 0$ for at least one j (Heteroskedasticit)

The test statistic is the usual F-statistic:

$$F^* = \frac{R^2/m}{(1-R^2)/(n-m-1)} \sim F_\alpha(m, n-2m-1) \quad (3.21)$$

At the α level, the null hypothesis of linearity is rejected in favour of the alternative hypothesis if

$$F^* > F_\alpha(m, n-2m-1) \text{ or } prob(F^*) < \alpha \quad (3.22)$$

Asymptotically, $F^* \sim \chi_\alpha^2(m)$.

3.4. Modelling and Forecasting Methods

This section presents an overview of the three nonlinear time series modelling and forecasting methods which include the TAR model, STR model, and MS-AR model.

3.4.1 Threshold Autoregressive Model

The TAR model is basically an extension of the Autoregressive model, which allows for the parameters to change in the model according to the number of segments (breaks), m, deemed to exist within the data. If the time series, $\{X_t : t = 1, 2, 3, \dots, n\}$ changes structurally with m break points, then there are $w = m + 1$ segments or regimes with a TAR model representation given by (Tong, 1978):

$$X_t = \begin{cases} \alpha_{1,0} + \varphi_{1,1}X_{1,t-1} + \varphi_{1,2}X_{1,t-2} + \varphi_{1,3}X_{1,t-3} + \cdots + \varphi_{1,p_1}X_{1,t-p_1} + \varepsilon_{1,t} & t = 1, 2, \dots, n_1 \\ \alpha_{2,0} + \varphi_{2,1}X_{2,t-1} + \varphi_{2,2}X_{2,t-2} + \varphi_{2,3}X_{2,t-3} + \cdots + \varphi_{2,p_2}X_{2,t-p_2} + \varepsilon_{2,t}\,, & t = n_1 + 1, n_1 + 2, \dots, n_2 \\ \vdots & \\ \alpha_{w,0} + \varphi_{w,1}X_{w,t-1} + \varphi_{w,2}X_{w,t-2} + \varphi_{w,3}X_{w,t-3} + \cdots + \varphi_{w,p_w}X_{w,t-p_w} + \varepsilon_{w,t} & t = n_w + 1, n_{w+1}, \dots, n \end{cases} \quad (3.23)$$

where, $\varepsilon_{j,t}$ $(j = 1, 2, \dots, w)$ are iid error term and $n_1, n_2, \dots, n_w$ (where $n_1 < n_2, < \cdots, < n_w$) are respectively the sample sizes of segment 1, segment 2, …, and segment w. The TAR model in equation (3.23) allows different variances for all w segments (regimes). In order to stabilize the variance over different segments (regimes), restriction of the form is applied:

$$X_t = I_1\left[\alpha_{1,0} + \sum\nolimits_{i=1}^{p1}\varphi_{1,i}X_{1,t-i}\right] + I_2\left[\alpha_{2,0} + \sum\nolimits_{i=1}^{p2}\varphi_{2,i}X_{2,t-i}\right] + \cdots + I_w\left[\alpha_{w,0} + \sum\nolimits_{i=1}^{p_w}\varphi_{w,i}X_{w,t-i}\right] + e_t \quad (3.24)$$

Where, I_t is the indicator function such that $I_t = 1$ when it correspond to segment j and $I_t = 0$, if otherwise. Each of the m segments can easily be estimated using OLS while the TAR model in equation (3.24) can be estimated using Nonlinear Least Squares (NLS), however, boundaries for the segments need to be determined. One possible approach to determining boundaries for the segments is by possible locating structural breaks. The existence of at least structural break in a time series is indicative that the data is nonlinear.

To test for structural change due to the presence of one break point, the Chow test is widely used. However, for multiple break points the Bai-Perron test is usually applied. The Bai-Perron test assumes the following vector-form multiple-structural-break model with m breaks (w segments/ regimes):

$$X_t = \begin{cases} X'_{t-1}\beta + Z'_t\delta_1 + \varepsilon_{1,t}\;, & t = 1, 2, \dots, n_1 \\ X'_{t-1}\beta + Z'_t\delta_2 + \varepsilon_{2,t}\;, & t = n_1 + 1, n_1 + 2, \dots, n_2 \\ \vdots & \\ X'_{t-1}\beta + Z'_t\delta_{m+1} + \varepsilon_{m+1,t}\;, & t = n_m + 1, n_m + 2, \dots, n \end{cases} \quad (3.25)$$

where is $X_{j,t-1} = (1 \;\; X_{j,t-1} \; X_{j,t-2} \dots X_{j,t-p_j})'$ is the column vector of with j=1,2,..,m+1 at time t whose effects are invariant with time and Z_t is a column vector of the explanatory variables at time t whose effects vary over time, and $\varepsilon_{j,t}$ are the error terms. The break points, $n_1, n_2, \dots, n_m$, are treated as unknowns and are estimated together with the unknown coefficients, β and δ_j are coefficients, when n observations available. A structural change in a given time series means $\beta = 0$. Using the OLS principle, the Bai-Perron test involves sequentially estimating the regression coefficients of the m+1 data segments/regimes along with the break points in the sample of n observations. Bai and Perron (2003) discussed three types of test – a test of no break vs. a fixed number of breaks, a double maximum test, and a sequential test – notable among them is the sequential test. The sequential test involves the following steps:

- Using the full sample, a test of parameter constancy with unknown break is conducted. If the test rejects the null hypothesis of constancy, the breakpoint associated with this result is estimated and noted as the first breakpoint. A test statistic called the Fisher statistic associated with the first breakpoint is then obtained.
- If the Fisher statistic associated with the first breakpoint is greater than the critical value, this first breakpoint is then used to divide the sample into two samples. For each of the two sub-samples, a single unknown breakpoint test is conducted in each subsample. If the Fisher statistic is greater than the critical value for each of the two samples, the date corresponding to the higher value is chosen as the second breakpoint.
- Sequentially, this procedure is repeated until all of the subsamples do not reject the null hypothesis of constancy (that is, no further breakpoints are left).

3.3.2. Smooth Transition Regression Models

Smooth Transition Regression models are a set of nonlinear models that incorporates both the deterministic changes in parameters over time and the regime switching behaviour within the time series data (van Dijk, Teräsvirta & Franses, 2002).

The general STR model for a time series $\{X_t : t = 1, 2, 3, \dots, n\}$ is:

$$X_t = \left(\alpha_0 + \sum_{i=1}^{p} \alpha_i X_{t-i}\right) + \left(\beta_0 + \sum_{i=1}^{p} \beta_i X_{t-i}\right) G(S_{t-d}, \gamma, c) + \varepsilon_t \quad (3.26)$$

where, $G(S_{t-d}, \gamma, c)$ is the transition function with S_{t-d} as the transition variable which determines the switching point, d is the decay parameter, γ is the smoothing parameter that determines the smoothness of the transition variable, c is the threshold parameter, $\alpha_0, \alpha_1, \alpha_2, \dots, \alpha_p$ and $\beta_0, \beta_1, \beta_2, \dots, \beta_p$ are the parameters of the two autoregressive components of the model with optimal lag length p, and ε_t is an error term. The two most popular transition functions are the logistic smooth and exponential functions given, respectively, by:

Logistic Function:

$$G(S_{t-d}, \gamma, c) = \frac{1}{1 + \exp\{-\gamma(S_{t-d} - c)\}}, \quad \gamma > 0$$

Exponential Function:

$$G(S_{t-d}, \gamma, c) = \frac{1}{1 + \exp\{-\gamma(S_{t-d} - c)^2\}}, \quad \gamma > 0.$$

The optimal lag length, p, of the autoregressive components is selected using automatic selectors based on information criteria. Using the appropriate transition function and transition variable, the STR model can be estimated using nonlinear least squares (NLS). The estimated parameters are obtained by minimizing the sum of squared residuals:

$$RSS(\Psi) = \sum_{t=1}^{n} \varepsilon_t^2 \quad (3.27)$$

where, $\psi = (\alpha', \beta', \gamma, c)$ with $\alpha = (\alpha_0\ \alpha_1\ \alpha_2 \dots \alpha_p)'$ and $\beta = (\beta_0\ \beta_1\ \beta_2 \dots \beta_p)'$. Using nonlinear optimization algorithm, a two-dimensional grid search is conducted over γ and c, allowing the selection of the pair that gives the smallest estimator for the residual variance, $\sigma_\varepsilon^2(\gamma, c)$.

3.3.3. Markov Switching Autoregressive Models

The underlying principle of Markov Switching Models is to decompose nonlinear time series into a finite sequence of distinct stochastic processes, states or regimes, whereby the parameters are allowed to take on different values with regard to the state/regime prevailing at time t. Switches between states/regimes arise from the outcome of an unobservable regime variable, S_t, which is assumed to be evolve according to a Markov Chain. One particular type of MSM is the Markov Switching Autoregressive (MS-AR) model. Given the time series $\{X_t : t = 1, 2, 3, \dots, n\}$, the MS-AR model assumes the representation (Hamilton, 1989):

$$X_t - \mu(S_t) = \varphi_1[X_{t-1} - \mu(S_{t-1})] + \varphi_2[X_{t-2} - \mu(S_{t-2})] + \cdots + \varphi_p[X_{t-p} - \mu(S_{t-p})] + \varepsilon_t$$

which, when re-parameterised yields:

$$X_t = c + \varphi_1 X_{t-1} + \varphi_2 + \cdots + \varphi_p X_{t-p} + \varepsilon_t$$

or $$X_t = \sum_{i=1}^{p} \varphi_i X_{t-i} + \varepsilon_t \quad (3.28)$$

where, $\varphi_1, \varphi_2, \dots, \varphi_p$ represent the coefficients of the AR(p) process, $\varepsilon_t \sim \text{iid}(0, \sigma_\varepsilon^2)$ and $\mu\,(S_t)$ are constants that are dependent on the states/regimes S_t and represent μ_1 if the process is in state/ regime $1(S_t = 1)$, μ_2 if the process in state/regime $2(S_t = 2), \dots,$ and μ_R if the process is in state/regime $R(S_t = R)$, the last state/regime). The change from one state to another is governed by the R-state first-order Markov Chain with transition probabilities, expressed as:

$$p_{ij} = P(S_t = j\,|S_{t-1} = i), i, j = 1,2 \quad (3.29)$$

where, p_{ij} is the probability of moving from state i at time t-1 to state j at time t. Using the fact that:

$$p_{1i} + p_{2i} + \dots + p_{Ri} = 1 \quad (3.30)$$

the probability of state i being followed by state j (also known as the transition matrix) is given by:

$$P = \begin{pmatrix} p_{11} & p_{21} & \cdots & p_{R1} \\ p_{12} & p_{22} & \cdots & p_{R2} \\ . & & & \\ . & & & \\ p_{1R} & p_{2R} & \cdots & p_{RR} \end{pmatrix} \quad (3.31)$$

In the current study, two states or regimes assumed that R=2 and the underlying MS-AR (p) model is given by:

$$X_t = \begin{cases} c_1 + \sum_{i=1}^{p} \varphi_{1,i} X_{t-i} + \varepsilon_{1,t}, & \text{if } S_t = 1 \\ c_2 + \sum_{i=1}^{p} \varphi_{2,i} X_{t-i} + \varepsilon_{2,t}, & \text{if } S_t = 2 \end{cases} \quad (3.32)$$

The transition matrix is, thus, given by:

$$P = \begin{pmatrix} p_{11} & p_{21} \\ p_{12} & p_{22} \end{pmatrix} \quad (3.33)$$

so that $p_{11} + p_{12} = 1$ and $p_{21} + p_{22} = 1$. P represents the probability of change in regime. For this two-regime MS-AR model, there are four transition probabilities given by:

$$\begin{aligned} P(S_t = 1|S_{t-1} = 1) &= p_{11} \\ P(S_t = 2|S_{t-1} = 1) &= p_{12} = 1 - p_{11} \\ P(S_t = 2|S_{t-1} = 2) &= p_{22} \\ P(S_t = 1|S_{t-1} = 2) &= p_{21} = 1 - p_{22} \end{aligned} \quad (3.34)$$

The MS-AR allows one to make inferences about the value of the observed regime, S_t, through the observed behaviour of X_t. This inference takes the form of probabilities called 'filtered probabilities', which are estimated using a simple iterative algorithm that computes both the likelihood function recursively and $P(S_t = i|\Omega_t)$, the filtered probability conditional on the set of observations, $\Omega_t = (X_t, X_{t-1}, X_{t-2}, \dots, X_1, X_0)$ up to time t. If the whole data set is used, the probabilities obtained are called the 'smoothed probabilities' which is estimated conditional on all the n available observations, $\Omega_n = (X_t, X_{t-1}, X_{t-2}, \dots, X_1, X_0)$. An important result that can be derived from the transition matrix is the expected duration (or average duration) of regime *i* as well as the average duration of regime *i*. The expected duration of regime *i* is given by:

$$E[D(S_t = i)] = \bar{D}(S_t = i) = 1/(1-p_{ii}) = 1/p_{ij} \quad (3.35)$$

A small value of p_{ij} $(i \neq j)$ is an indication that the model tends to stay longer in state i while its reciprocal $1/p_{ij}$ describes the expected duration of the process to stay in state i.

3.5. Model Selection Criteria

Schwarz Bayesian Criterion (SBC) developed by Schwarz (1978) was derived from a Bayesian modification of the AIC criterion. The idea of SBC is to select the model that has a minimise value. SBC is a function of the number of observation *n*, the SSE, the number of independent variables $p \leq m + 1$ where p includes the intercept, as shown in equation (3.40).

$$SBC = n \ln\left(\frac{SSE}{n}\right) + p\ln(n) \quad (3.36)$$

The penalty term for SBC is similar to AIC, but uses a multiplier of ln *n* for p instead of a constant by incorporating the sample size *n*.

3.6. Comparison of Model Performance

On the basis of reliability, validity and wide use, the following performance (error) measuring metrics are recommended for evaluating models. In order to select the appropriate models for each of the five closing stock prices among the three nonlinear modelling techniques which include SETAR, STR and MS-AR, four error metrics, RMSE, MAE, MAPE, and RSMPE, are appealed to. Given the time series, X_t and estimated series, $\hat{X}_t$, the four error metrics are defined below:

$$RMSE = \sqrt{\frac{\sum_{t=1}^{n}(X_t - \hat{X}_t)^2}{n}} \quad (3.37)$$

$$MAE = \frac{1}{n}\sum_{t=1}^{n}|X_t - \hat{X}_t| \quad (3.38)$$

$$MAPE = \frac{1}{n}\sum_{t=1}^{n}\left|\frac{X_t - \hat{X}_t}{X_t}\right| * 100\% \quad (3.39)$$

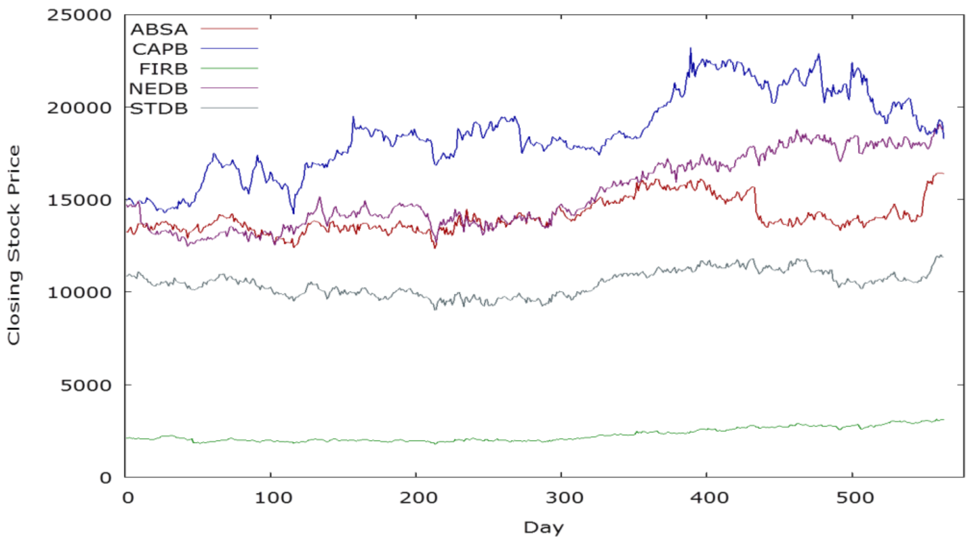
$$RMSPE = \sqrt{\frac{1}{n}\sum_{t=1}^{n}\left(\frac{X_t - \hat{X}_t}{X_t}\right)^2} \quad (3.40)$$

4. EMPIRICAL ANALYSIS

4.1. Preliminary Analysis

The study employed the stock prices of the South Africa collected daily for the period 2010-2012, a total of 563 observations and was obtained from http://www.jse.com. Study used the purposive sampling technique; five (5) banks from a population of twenty-one (21) banks were used. The banks that responded were ABSA Bank (ABSA), Capitec Bank (CAPB), First National Bank (FIRB), Nedbank (NEDB) and Standard Bank (STDB). Figure 1 depicts a picture of the closing stock price series.

Figure 1. Graphical Representation of the Five Closing Stock Prices



The results reveal that FIRB has the lowest stock prices and is estimated by an upward sloping trend. Stock prices of other banks are explained by irregular increasing patterns with ABSA and NEBD showing convergence at several stages. Given this movements by the stock prices, the data is not stationary at all levels. The series are further checked for nonlinearity by employing different tests.

Since nonlinearity in time series may occur in several ways, there exists no single test that dominates others in detecting nonlinearity. Therefore the study uses the Regression Specification Error Test (RESET) by Ramsey (1969) and Brock-Dechert-Scheinkman (BDS) by Brock et al. (1996) tests for this purpose. The null hypothesis of nonlinearity is rejected if the RESET and the BDS tests are greater than the critical values at conventional levelod significance, implying that the true specification is nonlinear. To determine the stability of the models, a Cumulative Sum (CUSUM) test by Brown et al. (1975) will be used. The null hypothesis is rejected if the CUSUM test exceeds the critical value. The results of the three tests are summarised in Table 1.

Table 1. Estimated AR Models with Nonlinearity Tests

		ABSA	CAPB	FIRB	NEDB	STDB
Parameter Estimate	α_0	263.614 (1.8731) [0.0616]	197.824 (2.001) [0.0459]	2.9902 (0.3089) [0.7575]	47.3057 (0.6324) [0.5274]	182.023 (2.013) [0.0446]
	α_1	0.862676 (20.5228) [0.0000]	0.9897 (187.60) [0.0000]	0.9995 (237.80) [0.0000]	0.8596 (20.51) [0.0000]	0.9828 (114.0) [0.0000]
	α_2	0.119039 (2.8155) [0.0050]			0.1379 (3.279) [0.0011]	
RESET Test for Specification	Test Statistic	4.00483 [0.0188]	3.4352 [0.0329]	3.6984 [0.0254]	4.9172 [0.0076]	8.7728 [0.0002]
CUSUM Test for Parameter Stability	Test Statistic (Harvey-Collier)	2.58915 [0.0099]	0.6004 [0.5485]	1.7090 [0.0880]	2.6447 [0.0084]	0.2375 [0.8123]
Test for ARCH Effects	LM	3.1967 [0.07379]	71.2252 [0.0000]	3.0925 [0.0787]	5.9051 [0.0151]	12.1992 [0.0022]
BDS	z-statistics	3.1967 [0.07379]	3.1967 [0.07379]	3.1967 [0.07379]	3.1967 [0.07379]	3.1967 [0.07379]

Figures in (•) are t-statistics while figures in [•] are p-values

Results from the RESET tests of the five variables suggested that the use of a linear regression modelling technique was inappropriate. In addition, the residuals from various autoregressive (AR) models fitted to the data were found to have ARCH structures, further supporting the use of nonlinear modelling methods. There is no evidence of structural change in the data according to the BDS tests. The preliminary results of the data proves that the data is suitable for the application of STR, TAR, MS-AR models.

4.2.Modelling and Forecasting models

This section presents the results of the three nonlinear time series models suggested.

4.3. Threshold Autoregressive Models for Closing Stock Price

Switches between one regime and another depend on a threshold variable and threshold value. This study followed the Hsu *et al.* (2010) structural break concept in selecting the thresholds. In particular, assuming that the numbers of thresholds are unknown, the Bai-Perron multiple breakpoint method was applied.

This section focuses on estimating TAR models on the basis that each stock price is a linear AR within a regime of that particular variable. First, for each regime of a particular variable, an AR model was run by allowing maximum five (5) lags and their respective SBC. Using these optimal lag lengths, the AR models were estimated for each segment of each of the five variables. At the 5% level, the final estimated AR models were obtained by eliminating the insignificant lags. The final estimated TAR models are reported in the estimated TAR models for the closing stock prices, ABSA, CAPB, FIRB, NEDB and STDB are, respectively:

$$ABSA_t = I_1 * (2372.97 + 0.8258 * ABSA_{t-1}) + I_2 * (2386.70 + 0.8205 * ABSA_{t-1}) + I_3 * (3733.22 + 0.7329 * ABSA_{t-1}) + I_4 * (1840.19 + 0.8811 * ABSA_{t-1}) + I_5 * (1.0003 * ABSA_{t-1})$$

$$CAPB_t = I_1 * (1.0005 * CAPB_{t-1}) + I_2 * (1441.48 + 0.9202 * CAPB_{t-1}) + I_3 * (1.0008 * CAPB_{t-1}) + I_4 * (2983.46 + 0.7163 * CAPB_{t-1} + 0.3746 * CAPB_{t-2} - 0.2281 * CAPB_{t-3}) + I_5 * (0.9980 * CAPB_{t-1})$$

$$FIRB_t = I_1 * (0.9990 * FIRB_{t-1}) + I_2 * (355.556 + 0.8199 * FIRB_{t-1}) + I_3 * (1.0022 * FIRB_{t-1}) + I_4 * (1.00112 * FIRB_{t-1}) + I_5 * (1.00114 * FIRB_{t-1})$$

$NEDB_t = I_1 * (1414.94 + 0.8925 * NEDB_{t-1}) + I_2 * (3785.64 + 0.9518 * NEDB_{t-1} - 0.2153 * NEDB_{t-3}) + I_3 * (1726.92 + 0.8769 * NEDB_{t-1}) + I_4 * (0.7171 * NEDB_{t-1} + 0.2848 * NEDB_{t-3}) + I_5 * (3017.19 + 0.8329 * NEDB_{t-1})$

$STDB_t = I_1 * (1485.84 + 0.8590 * STDB_{t-1}) + I_2 * (1557.05 + 0.8438 * STDB_{t-1}) + I_3 * (1771.69 + 0.8170 * STDB_{t-1}) + I_4 * (1384.42 + 0.8772 * STDB_{t-1}) + I_5 * (1.0007 * STDB_{t-1})$

where, $I_k = \begin{cases} 1, & \text{regime k} \\ 0, & \text{other wise} \end{cases}$

4.4. Smooth Transition Regression Analysis

This section provides the results for the STR modelling technique. Also shown are the forecasts of the model for the five variables. As a starting point, AR models up to lag five (5) were estimated with each of the five variables in order to determine the appropriate lag order.

Once the suggested STR models have been specified, the nonlinear least squares (NLS) method was used to estimate them and the results are summarised in Table 2.

Table 2. Estimated LSTR Models

Dep. Var.	Variable	Estimate	t-Stat	p-Value	R-Square	Adj. R-Square
ABSA(t)	----- Linear Part ------				0.94704	0.9471
	CONST	1407.01652	4.4602	0.0000		
	ABSA(t-1)	0.89731	38.7516	0.0000		
	---- Nonlinear Part ----					
	ABSA(t-1)	0.01213	3.8226	0.0001		
	Gamma	14.19718	0.7681	0.4427		
	C1	14598.19084	138.9240	0.0000		
CAPB(t)	----- Linear Part ------				0.98467	0.9847
	CONST	244.32152	2.3887	0.0172		
	CAPB(t-1)	0.98762	177.9250	0.0000		
	---- Nonlinear Part ----					
	CONST	-244.32152	-2.3887	0.0172		
	Gamma	17.97761	1.2362	0.2169		
	C1	22476.78769	94.3540	0.0000		
FIRB(t)	----- Linear Part ------				0.99051	0.9905
	CONST	239.56090	3.8898	0.0001		
	FIRB(t-1)	0.87937	28.3434	0.0000		
	---- Nonlinear Part ----					
	CONST	-206.50774	-3.1367	0.0018		
	FIRB(t-1)	0.10941	3.3907	0.0007		
	Gamma	800.73372	0.1317	0.8953		
	C1	2132.44609	347.5154	0.0000		
NEDB(t)	----- Linear Part ------				0.98693	0.9870
	NEDB(t-1)	1.00023	1121.5372	0.0000		
	---- Nonlinear Part ----					
	CONST	813.8	2.2596	0.0242		
	NEDB(t-1)	-0.0	-2.2466	0.0251		
	Gamma	14.1	0.7563	0.4498		
	C1	15526.9	30.4711	0.0000		
STDB(t)	----- Linear Part ------				0.96003	0.9601
	CONST	268.61160	1.8589	0.0636		
	STDB(t-1)	0.99933	95.3909	0.0000		
	---- Nonlinear Part ----					
	CONST	-266.04	-2.2890	0.0225		
	Gamma	7.29	1.9281	0.0544		
	C1	9284.20	98.9096	0.0000		

As revealed by the results, all five variables have been found to be autoregressive processes since their lags are significant in both the linear and nonlinear parts. By observation the estimated models seem good judging from the high R^2 and R^2_{adj} values. Again, the transition values (C1), for ABSA, CAPB, NEDB, and STDB suggest that closing stock price of these banks switch between two regimes. In fact, a closing stock price less than transition values 14598.19 for ABSA, 22476.79 for CAPB, 2132.45 for FIRB, 15526.88 for NEDB, and 9284.20 for STDB are regarded as low stock yield periods for these banks. Closing stock price larger than these values implies even higher stock prices.

4.5. Markov-Switching AR Models for Stock Prices

First, in order to ascertain the possibility of using two-regime switching models for the variables, linearity likelihood ratio (LR) tests were conducted and the regime results reported in Table 3. The test rejects the null hypothesis of no regime switching in favour of the existence of two regimes since the p-value of the chi-square statistics for all the five variables are less than the 10%, 5% or 1% level. Therefore, the LR test results support a two-state regime for all the five variables. Similar results were reported by Ismail and Isa (2007), Psaradakis *et al.* (2009), Wasin and Bandi (2011) and Yarmohammadi *et al.* (2012).

Table 3. Linearity LR Test of Two-Regime Switch

Variable	Chi-Square Test Statistic	P-value
ABSA	53.794	0.0000
CAPB	100.1	0.0000
FIRB	21.788	0.0006
NEDB	11.296	0.0796
STDB	12.042	0.0610

In order to find the optimal lag length for the estimation of the univariate MS-AR (p) model, different AR models were estimated with up to five (5) and their SBC. From these results, an optimal lag length one (1) is deemed appropriate for each of the five two-regime MS-AR (p) models. Results for the estimated MS-AR (1) models are shown in Table 4.18. As observed from these results, with ABSA, CAPB and FIRB, the variances of Regime 2, $\sigma^2(s_t = 2)$, is greater than the variance of Regime 1, $\sigma^2(s_t = 1)$, suggesting that for these three closing stock prices, Regime 2 is more volatile than Regime 1. In other words, Regime 2 captures the behaviours in ABSA, CAPB and FIRB in an unstable manner while Regime 1 captures the behaviours of the three stock prices in a stable manner. The opposite happens in the case of NEDB and STDB since the variances of Regime 1, $\sigma^2(s_t = 1)$, is greater than the variance of Regime 2, $\sigma^2(s_t = 2)$. It is also observed that, for ABSA, FIRB, NEDB and STDB, the estimated regime-dependent intercepts (expected daily increments in closing stock prices) are higher in Regime 1 than in Regime 2 (that is, $\mu(s_t = 1) > \mu(s_t = 2)$ for ABSA, FIRB, NEDB and STDB) while the opposite holds in the case of CAPB. In other words, changes in ABSA, FIRB, NEDB and STDB closing stock prices increased in a stable state while opposite holds for NEDB.

Table 4. Two-Regime MS-AR Modelling Results

	ABSA	CAPB	FIRB	NEDB	STDB
$\mu(s_t = 1)$ $\mu(s_t = 2)$	13749.6 13642.6	17853.9 18761.8	2276.56 2194.16	15390.1 14488.6	10507.0 10457.0
$\varphi_1(s_t = 1)$ $\varphi_1(s_t = 2)$	0.996758 0.531652	1.00108 0.945820	0.998810 1.259510	0.994343 1.000960	0.973702 1.180030
$\sigma^2(s_t = 1)$ $\sigma^2(s_t = 2)$	178.457 241.037	201.356 331.188	34.4960 116.217	241.296 190.232	137.350 18.7274
p_{11} p_{12} p_{21} p_{22}	0.989355 0.061359 0.010645 0.938640	0.98621 0.041621 0.013793 0.958380	0.995928 0.999979 0.0040724 0.0000212	0.995980 0.004151 0.004019 0.995850	0.94730 0.69051 0.052702 0.309490
$E[D(s_t = 1)]$ $E[D(s_t = 2)]$	16.2975 93.9408	24.0263 72.5005	1.0000 245.5554	240.8884 248.8181	1.4482 18.9746

Furthermore, the probabilities of a closing stock price remaining in Regime 1, p_{11}, are smaller than the probability of a closing stock price staying in Regime 2, p_{22}, for all the five closing stock prices. In fact, the probabilities of a closing stock price staying in Regime 1 lie in the range of 0.947 to 0.996 with an expected duration, $E[D(s_t = 1)]$, of 1 to 241 days. Similarly, the probabilities of a stock price staying in Regime 2 lie in the range 0.000 to 0.958 with an expected duration, $E[D(s_t = 2)]$, of 19 to 249 days. In other words, closing stock prices can stay slightly longer in Regime 2 than in Regime 1.

Model performance

This section provides the results of the forecast perfomance of the three models. One Forecasted future values are of great importance for decision-making and policy formulation. The evaluation of nonlinear models is based on the properties of resulting residuals. Using the residuals, various tests for misspecification, including non-normality, parameter stability and autocorrelation checks were conducted. The diagnostic tests of the residuals of the three models did not violate the required assumptions and as a result rendered the models accurate and sufficient.

On the basis of reliability, validity and wide use, the performance (error) measuring metrics are recommended for evaluating the efficieny of models in forecasting. The study uses the four error metrics such as RMSE, MAE, MAPE, and RSMPE. The model that generate the least forecast error is chosen and suggested for further analysis. Table 5 provides the results for the four measures.

Table 5. Forecast Comparison among LSTR, TAR and MS-AR Models

Measure	Method	ABSA	Capitec	FRIB	NEDB	STDB
RMSE	LSTR	200.2572	270.9698	35.48659	219.5906	133.0790
	TAR	196.5424	266.1471	35.48629	210.4875	131.0235
	MS-AR	186.7458	217.5940	35.32322	213.6210	129.6859
MAE	LSTR	148.9902	189.5397	27.03180	167.8142	103.7846
	TAR	147.2353	186.6499	26.93976	162.2681	101.3549
	MS-AR	143.0377	160.6033	27.34744	165.5507	97.6963
MAPE	LSTR	0.010624	0.010107	0.011973	0.011023	0.009965
	TAR	0.010502	0.009945	0.011929	0.010653	0.009735
	MS-AR	0.010188	0.008587	0.012121	0.010863	0.009400
RMSPE	LSTR	0.251849	0.239601	0.283848	0.261327	0.236247
	TAR	0.248965	0.235339	0.282786	0.252104	0.230791
	MS-AR	0.241512	0.203568	0.287354	0.257530	0.222846

According to the results, the four error metrics select the MS-AR(1) model for ABSA, CAPB and STDB, and TAR model for NEDB accordingly. MAE, MAPE and RMSPE select the TAR model for FIRB, RMSE selects the MS-AR(1) model for FIRB. The results are in accordance with those by Dacco and Satchell (1999), whose study identified the FIRB as best modelled by the MS-AR(1).

4. CONCLUSION REMARKS

Study explored the perfomance of the TAR, STAR and the MS-AR models in modelling and forecasting daily stock prices series of five banks of South Africa. Five banks considered are the ABSA, Capitec , First Rand Bank, Nedbank , and Standard Bank for the period from 2010 to 2012. One of objective of the study was to provide evidence that the five variables used in the study were nonlinear in nature. Three test used proved that all series are nonlinear in nature and nonlinear models are more appropriate to model five varaibles. The study technique suggested the LSTR1 models for all five variables, while the TAR modelling technique involved a maximum lag of three in coming up with suitable TAR models for the five variables, and the MS-AR modelling technique allowed up to a maximum lag of one in determining the appropriate MS-AR models for the five variables. The study employed the four error metrics to select the best performing model. The results showed that while ABSA, CAPB, FIRB and STDB are best modelled by MS-AR(1), NEDB is best modelled by TAR. Generally, the results proved that the MS-AR modelling technique performed better in most cases compared to the LSTR and TAR models. From the discussions of the results, the following conclusions can be drawn:

- All five closing stock prices are nonlinear in nature.
- All five closing stock prices do not change structurally.
- The almost negligible error measures suggest that the various estimated predictive models for the five closing stock prices are robust, efficient and reliable for purposes of forecasting.

Although the three nonlinear models proved to be good, there is room for further improvement. More specifically, in the case of MS-AR results. It is recommended that the Neural Networks (NN) be used and results compared with the current results of MS-AR.

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