THE DYNAMICS OF DEFLATION
IN A GROWING ECONOMY

Rui P. N. Santos*

*Independent researcher

Abstract

In this paper I study the behavior of prices in a growing economy in which the money supply is held constant. I show that with increasing levels of output, it is a natural outcome that prices of economic goods will decrease over time, which it is what we define as deflation. In this context, I study in particular the behavior of real and nominal incomes (wages and profits) over time, the evolution of nominal and real GDP and the effects of deflation on debt contracts. Specifically, I assess the common claims that deflation decreases incomes, postpone spending and favors creditors at the expense of debtors. I have found that none of these claims is supported by theoretical analysis in the case that price deflation is the consequence of economic growth with constant money supply.

Keywords: Deflation, Growing Economy, Money Supply

1. INTRODUCTION

Deflation is taken frequently to be an evil. Most central banks have a mandate to keep a positive growth rate of prices and avoid by all means that the general price level in (usually measured by the Consumer Price Index) the economy goes through a downward path. This fear of deflation, or *Apoplithorismosphobia,* as termed by Mark Thornton (2002) is a generalized phenomenon and pervades also the media news. We rarely see, though, a distinction being made between the various kinds of deflation. Sometimes we read that there is "good deflation" and "bad deflation" – the first being the consequence of decreasing costs of production and the latter being due to a decrease in aggregate demand.

Yet, a distinction between the various types of deflation is crucial in assessing the deflation's impact. According to Bagus (2015, pp. 35-83) we can identify at least four main causes of price deflation: growth deflation; cash-building deflation; bank credit deflation (also known as debt-deflation) and fiat deflation (directly caused by the government through interference with prices or the money supply). This need to identify the concrete cause of price deflation is all the more necessary as the term "deflation" in its traditional sense became to denote a decrease in prices rather than a decrease in the money supply. As Mises (1945) cap. 6, sec. 3) already noted, writing about the meaning of the term inflation: "The semantic revolution which is one of the characteristic features of our day has obscured and confused this fact. *The term inflation is used with a new connotation.* What people today call inflation is not inflation, i.e., the increase in the quantity of money and money substitutes, but the general rise in commodity prices and wage rates which is the inevitable consequence of inflation. This semantic innovation is by no means harmless. First of all there is no longer any term available to signify what inflation used to signify." So, it makes all the

difference if deflation is caused by a decrease in money supply or by a decrease in the average cost of production.

In particular, falling prices due to an increase in productivity represent a normal process in the economy which signalizes a greater abundance of goods and services available. The contribution of this article is to set out in precise terms the dynamics of the economy when the money supply is kept constant and, at the same time, the economy is growing over time. We shall see that in this case, the natural outcome is a gradual decrease in prices over time and a constant level of nominal incomes (wages and profits). This last point is critical because there is a widespread understanding in the general press and media that wages and profits also decrease along with prices in a scenario of deflation. But this would only be the case if the money supply were decreasing. When that supply is constant the tendency is for nominal incomes to stay constant. Therefore, it follows that in this context, real wages and profits rise over time. I have also addressed the issue of whether the real burden of debt increases in this particular case of growth-deflation and found that it does not.

Concerning empirical studies a good summary is given by Atkeson and Kehoe (2004) in the following words: «The data suggest that deflation is not closely related to depression. A broad historical look finds many more periods of deflation with reasonable growth than with depression and many more periods of depression with inflation than with deflation. Overall, the data show virtually no link between deflation and depression.» That is, even allowing for the several kinds of deflation indicated above the data does not support a casual link between deflation and recessions.[1]

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[1] The same conclusion can be seen, for example, in Bordo and Reddish (2004), Capie and Wood (2004) or Friedman and Schwartz (1982, esp. Table 4.9). A good review on some of the classical economists' views on deflation is Humphrey (2004),

Now for the rest of this paper, in section 2, I present and solve the formal model of the economy which will be the framework upon which we are going to be able to draw qualitative results about the behavior of capital accumulation, output, prices and incomes. In sections 3, 4 and 5 I discuss some modifications that might be made to the base model and discuss some possible counter-arguments that could be directed against my main conclusion. In particular, in section 3, I address (and hopefully rebut) the alleged argument of deferral of consumption as a case against price deflation. In section 6, I discuss, at the firm level, the reason why price deflation is a natural and smooth outcome in an economy with growth and constant money supply. Finally, section 7 concludes.

2. THE MODEL

The conceptual framework which I use to describe a growing closed economy is the Solow growth model with an increasing returns to scale production function know in the literature as AK (see Acemoglu (2009), pp. 55-56). I am not interested in studying here the sources of economic growth but merely how monetary and real prices behave in a growing economy, so that the above formulation is satisfactory for this purpose and there's no need to enter in optimizing models. Like Tobin (1955), although in a different way, I then proceed to incorporate money in the basic Solow framework.[2] The way I structure the agents in the economy and their behavior follows Reisman (1998). It will be seen that my model is an exact replica of Reisman's model which he presents only with numerical examples. In this sense, I just build a mathematical formal model that generates exactly the same numerical results than Reisman's. This mathematical model, therefore, should be complemented with the reading of Reisman (1998, esp. pp. 623-29, 709-714, 728-735).[3] Following Reisman, I divide the agents in the economy in three components: Firms, Workers, and Capitalists (which we might call investors or savers). I will later show, in section 4, that this does not imply that in the model workers don´t save or capitalist don´t "work". It is just a functional division (and not a personal division) that facilitates the tractability of the model.

Capitalists invest part of their money directly in firms, being this money the capital equity of these firms, which in turn is spent to buy factors of production – labor, L, and physical capital, K. The other part of that money is spent by capitalists in consumption, NC (net consumption). At the beginning of each time-period firms pay wages, W, invest in physical capital and distribute dividends coming from last year's profit. Those wages and dividends are used to buy the output, Y, of firms that consists of consumption goods, C. Firms sell the output, C as well as K, that was produced during the previous period, receiving, thus, the total amount of money in the economy, M. In terms of firms' cash-flows we have the following situation, in which the variables D_C and D_K denotes money spent in consumption and capital goods, respectively:

Table 1. Cash flows

D_K	D_K
W	D_C
NC	

The sum in each column of the table equals M. In each period, capitalists can increase their savings by choosing to consume less and invest part of the dividends. This investment can be made in the form of increased spending in capital goods or in wages. As in Reisman, I assume that this investment is made exclusively in capital goods so that labor is constant (I normalize Labor, L, to one). Following the traditional assumption of the neo-classical growth model we assume that a given fraction of total spending, s, is spent in capital goods, so that, as usual, the growth equation for physical capital is given by

$$K_{t+1} = K_t(1-\delta) + sY_t \tag{1}$$

Where δ represents the rate of depreciation of K and Y_t is total output, as given by

$$Y_t = AK_t \tag{2}$$

Substituting equation (2) in (1) and dividing by K_t we get:

$$\frac{K_{t+1}}{K_t} = (1-\delta) + sA \tag{3}$$

From the above equation, in order to obtain a positive rate of growth in capital and output the following condition for parameters' values must be assumed:

$$sA - \delta > 0 \tag{4}$$

Reisman assumes initially $\delta = 1$, $s = 0.5$, $A = 2$. As this implies a stationary economy with no economic growth, later he assumes $s = 0.6$. He further assumes $M = 1000$.

Now we study the evolution of the price level, P_t.

At a given period a definite amount of money, M, is spent in the output produced in the previous period, in the form of consumption or investment, so that

$$P_t Y_{t-1} = M_t = D_K + D_C. \tag{5}$$

So,

$$P_t = \frac{M}{Y_{t-1}} \tag{6}$$

Solving equation (3) for K_t and substituting equation (2) in (6) we obtain:

$$P_t = \frac{M}{AK_0(1+sA-\delta)^{t-1}}, \ t \geq 1 \tag{7}$$

According to equations (4) and (7) output grows over time and therefore the price level decreases correspondingly. The logic behind this result follows pretty forward by just assuming flexible prices, a constant amount of money and growing output. In fact it is similar to the classical equation of exchange as expounded for instance in Fisher (1922, p. 48) by assuming a constant rate of velocity of money. In this model the rate of deflation is given by

$$\frac{P_{t+1}}{P_t} - 1 = \frac{\dfrac{M}{Y_t}}{\dfrac{M}{Y_{t-1}}} - 1 \qquad (8)$$

Observing that

$$Y_t = AK_t = AK_0\left(1 + sA - \delta\right)^t, \ t \geq 1 \qquad (9)$$

and using equation (6) in expression (8), the rate of variation of the price level is:

$$\frac{P_{t+1}}{P_t} - 1 = \frac{1}{1 + sA - \delta} - 1 \qquad (10)$$

which is clearly negative, given assumption (4), so that we have a decrease over time of the price level, that is, a constant rate of price deflation.

As gross nominal output (GDP), in an income perspective, is equal to wages plus profits plus depreciation and by equation (6) this nominal output is equal to M, we must only prove that in this scenario of growing real output, nominal depreciation of physical capital is constant for a given set of parameters satisfying condition (4). Then I will find the steady-state levels of nominal wages and nominal profits and, provided these have positive values, it follows that, with decreasing P_t, a progressive rise in real wages and profits over time must ensue.

So, I now demonstrate that nominal depreciation of physical capital is constant over time. Note that at a given time-period depreciation equals $\delta P_{t-1} K_{t-1}$. By equations (3) and (7),

$$\delta P_{t-1} K_{t-1} = \delta \left[\frac{M}{AK_0\left(1 + sA - \delta\right)^{t-2}} K_0\left(1 + sA - \delta\right)^{t-1}\right] \qquad (11)$$

Simplifying that expression:

$$\delta P_{t-1} K_{t-1} = \delta \frac{1 + sA - \delta}{A} M \qquad (12)$$

It is convenient now to make the following notation:

$$x = \frac{1 + sA - \delta}{A} \qquad (13)$$

So that equation (12) is rewritten as:

$$Depr = \delta P_{t-1} K_{t-1} = \delta x M \qquad (14)$$

So nominal depreciation is indeed constant and the result is that GDP is constant over time implying also that a constant level of nominal wages and profits mean a progressive rise of their real values.

I know show what are precisely the values of nominal wages and profits and the consequent value of GDP.

Accounting profit, π, equals sales minus costs of production. As we assume that production in *t-1* is sold in *t*,

$$\pi_t = D_{K_t} + D_{C_t} - \left(W_{t-1} - \delta x M\right) \qquad (15)$$

I show in the *appendix* that the steady-state levels of nominal wages and profits equal:

$$W = M\left(1 - \gamma - s\right) \qquad (16)$$

$$\pi = M\left(\gamma + s - \delta x\right) \qquad (17)$$

In these equations above γ is a measure of intertemporal preference in consumption. Please see the appendix below. I also show in the appendix that, if we don't assume hoarding of money, GDP just equals M.

I now address the question of whether there is a real impact on *creditors or debtors* due to this scenario of deflation.

We must first notice that, as in the case of positive inflation, the nominal interest rate can be adjusted to take account of the expected rate of deflation, as is illustrated by Fisher's formula (1907, chs. V and XIV), $\left(1 + i\right) = (1 + r)(1 + \dot{P})$, where *i* is the nominal interest rate, *r* is the real interest rate and $\dot{P}$ is the expected rate of inflation. But, with a constant level of money, even a rate of deflation greater than expected does not harm the debtor as long as he earns some form of income, wages or profits. This is because, as I will presently show, an increase in the rate of deflation is synonymous with an equal increase in the rate of real incomes (profits and wages).

First, as shown in the appendix (see equation (A.11)), for a given set of parameters, the nominal rate of profit is constant and positive. Also, assuming a perfect foresight equilibrium, the interest rate is equal to the profit rate and it follows from the Fisher equation that:

$$(1 + r) = \frac{\left(1 + i\right)}{1 + \dot{P}}.$$

So, a negative rate of inflation (deflation) makes the real interest rate rise as the nominal interest rate is positive and constant (for a given set of parameters); the higher the deflation rate is the higher is the real interest rate. That is to say that the real burden of debt increases proportionally to the rate of deflation.

But, then we could see that real incomes (profits and wages), out of which agents should pay the debt, grow exactly in the same inverse proportion as the rate of deflation. If we divide equation (17) by *P* as given by equation (7), we see that real profits grow at exactly the inverse rate of the variation in prices, $sA - \delta$. So, an increase in deflation is matched by an increase in the real profit rate. Additionally, if we divide equation (16) by *P* as given by equation (7), we see that real wages also increase in the same magnitude as the real interest.

In summary, any variation in the deflation rate makes automatically for an identical variation in incomes, so that, for a given nominal interest rate, the correspondingly variation in real interest is matched by a similar variation in income, so that debtors (supposing they have income, which they should have, anyway) do not see their situation worsened.

3. MONEY BALANCES

I now assume that individuals and firms do not spend all their income in a given period, so that a fraction ϕ of total money is maintained in the form of cash-balances by these agents. The price level is now, accordingly, lower and is given by the relation between total spending and sales:

$$P_t = \frac{M(1-\phi)}{Y_{t-1}} \tag{18}$$

Wages are given by:

$$W = M(1-\phi)(1-\gamma-s) \tag{19}$$

It also follows that depreciation is just a slight modification of equation (14):

$$Depr = \delta P_{-1}K_{t-1} = \delta(1-\phi)xM \tag{20}$$

Subtracting equations (20) and (19) from total sales, $M(1-\phi)$, profit is now given by:

$$\pi = M(1-\phi)(\gamma+s-\delta x) \tag{21}$$

Adding up the last three equations above we obtain the expression for nominal GDP which is equal to $M(1-\phi)$. So, we can see that nothing substantially changes when money balances are taken into consideration. Specifically, by equation (18), real wages and profits stay the same when compared to section 2. Total output, given by equation (9) is of course also unchanged. So, the level of cash balances only affects nominal values – the larger these balances are the lesser are nominal incomes and prices.

A recurrent argument against price deflation says that hoarding of money would increase by a significant amount because the mere holding of money would mean a positive rate of interest. By the fisherian equation a rate of deflation of 1% would mean a real rate of interest of 1% on holding money. At the same time, the argument goes, consumption and investment will be indefinitely postponed for the future because of expected declines in prices (which is the same thing as a positive real interest rate on money).

Now, it is likely that these would happen at first, but once expectations are adjusted people will see themselves with larger money holdings than desired because prices are going down and consumption is always the ultimate end of the individuals. We can see that today almost risk-free investments are available and it is not because of

that that people stop consuming. A positive interest rate available for investing our money just means that a large purchasing power will be available for future consumption. This is exactly what happens with falling prices. The only difference in a scenario of deflation is that there will be available another risk-free asset other than government bonds or fixed interest deposits– that asset is money.

4. SAVINGS BY WORKERS

Wages are paid at the beginning of each time period out of a sum of Money $M(1-\phi)$ at the same time that dividends and capital goods are also paid to capitalists and firms. If workers decide to save part of those wages then what happens is that those savings are invested in firms through an increase of net investment in capital goods at the same time that total consumption decreases by the amount of those savings. Without further saving, in the next period total capital spending is greater by the amount of that previous saving and, with invariable money, nominal profits are lower, as wages stay equal and investment spending in physical capital is higher. So, this is just a particular case of an increase in savings accompanied by an increase in net investment as we saw in section 2. The only difference is that in the period in which new savings takes place we see a diminution in consumption out of wages and not out of profits. In the next period the situation is exactly the same as in section 2.

5. SAVINGS INVESTED IN WAGES

The model presented in this paper can also be adapted to the case where investment is made in workers' wages. As I am working with an exogenous model *a la* Solow all that would be necessary was to attribute an extra parameter to that variation in wages. But eventually we would find a steady-state situation with wages and profits constant and obtain the same general result – In a growing economy with constant money, that economy tends to an equilibrium with a constant level of nominal wages and profits, which is to say, a growing level of real wages and profits.

6. HOW WOULD FIRMS DECREASE PRICES?

In a scenario of perfect competition, which is what we are implicitly assuming here, prices tend, over time, to level with cost of production, allowance being made for normal profits. So a decrease in the cost of production tends naturally to induce a fall in prices so that excess profits become to dwindle. There is nothing dramatic in this and that's just the outcome of a growing output being sold against a constant amount of money. In the economy delineated above, average (unitary) cost equals:

$$\frac{P_K K + WL}{Y_t} \tag{22}$$

This means that average cost decreases over time (as Y increases and the numerator is constant): If prices decrease by the same rate, which is the case as seen by equation (6), then the profit margin in percentage terms stays the same. So the price

deflation works naturally as a constant restoration of profit's margins. See Selgin (2007, esp. pp. 5, 10).

7. CONCLUSION

In a scenario of a growing economy I have shown that with a constant level of money supply the price deflation that ensues does not decrease nominal wages and profits and, as a consequence, real growth in personal incomes is the natural outcome. In a standard growth model with a given exogenous amount of money, the accumulation of capital makes for a decrease in average costs of production, which, by means of competition induces individual firms to lower output prices.

I have also shown that this kind of deflation, which is frequently called growth-deflation (in contrast with debt or money deflation, for instance) rules out some common fears associated with deflation in general, namely an increase in the real burden of debts. This is because whether it is true that decreasing prices mean a real (in terms of purchasing power) increase to a given amount of nominal debt, on the other hand, and on the reasonable assumption that the debtor has some form of income, it is also clear that real incomes also rise. In particular, I have shown that a certain increase in the deflation rate is accompanied by an exact proportional increase in real profits and wages so that if, on one hand, the real value of debt increases, on the other hand the real value of incomes to pay that debt also increases.

I also address the other common fear associated with price deflation – that, supposedly, consumers and other agents postpone consumption and spending in general due to the expectation of general falling prices in the economy. This reduced spending, the argument goes, makes for decreasing incomes and even less spending and so on. I argue that in a scenario of price deflation what happens is that money turns out to be an interest bearing asset; that is, the mere possession of money increases its purchasing power given that prices decrease and the same amount of money buys more goods in the future. This is not substantially different from the situation where agents have at their disposal an asset with risk-free interest, such as most government bonds and insured time-deposits today. So, in a scenario of price deflation the only difference is that we have one more risk-free interest bearing asset – money. There is no additional reason for agents to save more money in this scenario as they could already have done so given the existence of other assets with virtually the same risk and higher expected returns.

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Appendix A. Derivation of the formulas for nominal wages and profits

Noting that total sales, $D_k + D_c$, equal M and depreciation is constant over time I will now solve for the steady-state values of nominal profits and wages, starting with equation (15) of the main text, which can be written as:

$$\pi = M - W - \delta x M \tag{A.1}$$

I will now look for a more convenient expression for π. First, I show that profits equal consumption by the capitalists, NC, plus net Investment, I. Net investment, in nominal terms, is the increment in physical capital and labor between two periods.[4]

$$I_t = D_{K_t} + W_t - \left(\delta x M + W_{t-1}\right) \tag{A.2}$$

NC, in turn, equals total consumption minus wages, by definition. So that, for a period t,

$$NC + I = \left(D_C - W\right) + \left(D_K + W - (\delta x_{t-1} M + W)\right) \tag{A.3}$$

As the rhs of equations (A.1) and (A.3) are equal, it follows that:

$$\pi = NC + I. \tag{A.4}$$

Now, as mentioned in the text, in each period capitalists can choose to consume more or less by withdrawing or increasing the money capital of the firms. I assume that capitalists decide in each period to use a fraction, γ, of M in their own consumption, ($NC = \gamma M$) so that substituting equation (A.4) in (A.1) we obtain:

$$W = M\left(1 - \gamma - \delta x\right) - I \tag{A.5}$$

The parameter γ is thus a measure of intertemporal preference for consumption so that the greater it is, the greater is the preference for present consumption as opposite to future consumption. Now, in order to obtain an expression for W that only depends on the parameters of the model I first note that $D_k + D_c$ equals:

$$D_K + \left(NC + W\right) = M \tag{A.6}$$

Using equation (A.5) and the definitions of D_K and N_C, equation (A.6) can be written as:

$$sM + \gamma M + M\left(1 - \gamma - \delta x\right) - I = M \tag{A.7}$$

From this it follows that:

$$I = M\left(s - \delta x\right) \tag{A.8}$$

That is, after all, the very definition of net investment – gross investment minus total depreciation (as wages stay constant and their difference between two periods is zero). See equation (14). Now substituting (A.8) in (A.5) we obtain the final expression for money wages:

$$W = M\left(1 - \gamma - s\right) \tag{A.9}$$

Using Reisman's numbers (M=1000, γ=0.1, s=0.6, A=2) total wages equal 300 monetary units. Now, going back to equation (A.1) we get the expression for nominal profits:

$$\pi = M\left(\gamma + s - \delta x\right) \tag{A.10}$$

We can also determine the *rate* of nominal profit as the amount of profit in t divided by total investment in t-1:

$$\frac{\pi_t}{W_{t-1} + D_{K_{t-1}}} = \frac{M\left(\gamma_t + s_t - \delta x\right)}{W_{t-1} + D_{K_{t-1}}} = \frac{M\left(\gamma_t + s_t - \delta x\right)}{M - \gamma_{t-1}M} = \frac{\gamma_t + s_t - \delta x}{1 - \gamma_{t-1}}.^{[5]} \tag{A.11}$$

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[4] Following Reisman I will assume from now on that new investment is used to increase physical capital only, wages remaining constant. Note, also, that x refers to parameters' values as of period t-1. See equations (12) and (13) of the main text.

[5] Notice, again, that x refers to the values of the parameters as of period t-1. See equations (12) and (13).

Using δ=1, as in Reisman's, we get x=0.6 and π=100, so that GDP equals W+π+$depr.$ = 300+100+600 = 1000. In a period with positive net investment this means that capitalists consume less and invest more, which corresponds to a change in the value of γ from 0.2 to 0.1 and in the value of s from 0.5 to 0.6, so that the nominal rate of profit in this period of transition equals, according to (A.11):

$$\frac{0.1+0.6-1\times\left(\dfrac{1+0.5\times2-1}{2}\right)}{1-0.2} = \frac{0.2}{0.8} = 25\%.$$

Now, formally, using (A.9), (A.10) and (14) of the main text, we see that GDP is just equal to the quantity of money:

$$W + \pi + \delta xM = M.$$

This is because as we are not assuming changes in money balances or even hoarding, all money must be spent either in consumption (W and NC) or investment in physical capital. This implies that as s is the fraction of money spent in investment, Υ must not exceed 1-s. Also, for a given s, an increase in Υ must imply a decrease in W.