BRAZILIAN TERM STRUCTURE OF INTEREST RATE MODELING: A NELSON-SIEGEL APPROACH

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Abstract

Forecasting interest rates structures plays a fundamental role in the fixed income and bond markets. The development of dynamic modeling, especially after Nelson and Siegel (1987) work, parsimonious models based in a few parameter shed light over a new path for the market players. Despite the extensive literature on the term structure of interest rates modeling and the existence in the Brazilian market of various yield curves from different traded asset classes, the literature focused only in the fixed rate curve. In this work we expand the existing literature on modeling the term structure of Brazilian interest rates evaluating all the yield curves of Brazilian market using the methodology proposed by Nelson and Siegel. We use Non Linear Least Squares (NLLS) to estimate the model parameters for almost 10 years of monthly data and model these parameters with the traditional VAR/VEC model. The results show that it is possible to estimate the Nelson Siegel model for the Brazilian curves. It remains for future research the modeling of their variances as well as the possibility to develop a global Brazilian model using Kalman Filter using the Diebold. Li. and Yue (2006) approach.

Keywords: Term structure, interest rates, Dynamic factor model, Brazilian yields, Bond market
JEL classification: G1 E4 C5

1. INTRODUCTION

The relationship between interest rates or fixed income securities with maturities is called term structure of interest rate (TSIR) and has a central role in the economy, serving as a reference for pricing of financial assets flows. However the structure of the TSIR is not directly observable and must be estimated from prices of fixed income securities or derivatives. From this set of discrete data it builds up a continuous function that fits the observed data.

In the literature several authors developed functions to model the term structure of interest rates. starting with Vasicek (1977) and passing through Nelson and Siegel (1987). The model proposed by Nelson and Siegel, and extended by Svensson (1994), suggests parametric curves that are flexible enough to describe almost any observed term structure format. and is consistent with the interpretation of the factors in Litterman and Scheinkman (1991).

This paper aims to evaluate the estimated monthly parameters for the Nelson Siegel model of the yield curves traded in Brazil using market data from February 2004 until October 2013, as well as to model the behavior of these parameters through this period. In Brazil due to historic importance of inflation, besides the fixed rate assets, the market also trades other assets classes that pay real interest rates linked to inflation indexes, such as IPCA and IGPM, linked to floating rates like TR and linked to the variation of exchange rate of Brazilian real to the U.S. dollar.

A frequent criticism to these models is that they are very good to explain normal times patterns. With that in mind our data selection was centered in the 2007/8 subprime crisis. Starting in August 2007 we observe a sharp increase in money market interest rates. Risk aversion and loan rates increase sharply and credit lines where cut especially after the crisis deepening with financial institutions failures in September 2008, Duchin, Ozbas and Sensoy (2010) and Almeida et al. (2009) use similar methodology in their data selection and crisis analysis. Therefore our data includes this whole period starting 40 months before its onset and ending 60 months after its deepening.

Our results show the behavior for Nelson Siegel model parameters during this period and we find evidence that it is possible to estimate models for several curves, except for the curve linked to the exchange rate and the curve linked to the IGPM that do not have the same order of integration for the parameters. Despite this limitation it was possible to estimate a global model without including the exchange rate linked curve, but including the IGPM linked for Brazilian curves. To the best of our knowledge there is no other paper that has modeled the Nelson Siegel parameters for the market curves in Brazil including the financial crisis period.

This paper is organized as follows: in section 2 we present the review of the literature on modeling of the interest rate structure, in section 3 there is a descriptive analysis of the data. Section 4 and section 5 we have the results for the Nelson Siegel models and for the VAR/VEC models, respectively. Finally, in section 6 are the conclusions.

2. LITERATURE REVIEW

In finance the term structure of interest rates is a curve showing the different yields, or interest rates, for securities with different maturities (6 months. 1 year. 2 years. etc.). The term structure is estimated from the observed prices of fixed income securities and derivatives.

The investment for a period of time t gives a yield or y(t). This function y is called Yield Curve and may or may not be an increasing function of t. In addition to the various possibilities of the Yield Curve format it is only possible to know their values with certainty for some specific maturity dates, while for the other different maturity values are calculated by interpolation. The Yield Curves are used by financial market participants to understand the conditions of the markets and seek investment opportunities.

In the literature attempts were made to model the term structure using parametric and non-parametric models. In the latter, the pioneering work is due to McCulloch (1971) with the proposal of the model the term structure of interest rates using splines. Similar non-parametric approaches appears in the work of Vasicek and Fong (1982) and Fama and Bliss (1987).

In the parametric approach Vasicek (1977), in a pioneering work, proposed a mathematical model for the evolution of interest rates based on a factor, the market risk. The model describes the variation in interest rates, dr, as:

$$dr_t = a(b - r_t)dt + \sigma dW_t. \quad (1)$$

where, a - is the mean reversion speed, b - the long-term average, σ - the volatility of interest rates and W - a Wiener process.

Within the class of parametric models. Nelson and Siegel (1987) proposed a model based on exponential components to determine the Yield Curve flexible enough to represent all the possibilities of the Yield Curve formats.

The model is based to forward r rates for the period m follow the following format:

$$r(m) = \beta_0 + \beta_1 exp\left(\frac{-m}{\tau}\right) + \beta_2 \left(\frac{m}{\tau}\right) exp\left(\frac{-m}{\tau}\right). \quad (2)$$

where, the betas and lambda are the parameters to be estimated.

As the yield is the average of forward rates, we have:

$$y(m) = \int_0^m r(x)dx. \quad (3)$$

Therefore, the model for the spot interest rate for the period m is given by:

$$y(m) = \beta_0 + \beta_1 \left[\frac{1-exp\left(\frac{-m}{\tau}\right)}{\left(\frac{m}{\tau}\right)}\right] + \beta_2 \left[\frac{1-exp\left(\frac{-m}{\tau}\right)}{\left(\frac{m}{\tau}\right)} - exp\left(\frac{-m}{\tau}\right)\right]. \quad (4)$$

The same model can be written according to Diebold e Li (2006) with a review of the parameter τ = 1/λ. so we can use λ =0 if needed:

$$y(m) = \beta_0 + \beta_1 \left[\frac{1 - exp(-m\lambda)}{(m\lambda)}\right] + \beta_2 \left[\frac{1 - exp(-m\lambda)}{(m\lambda)} - exp(-m\lambda)\right]. \quad (5)$$

where, λ is the decay rate to zero of the parameters β_0, β_1 and β_2.

Notice that β_0 represents the long-term level of y. and β_1 and β_2 are respectively its inclination and its curvature. That can be easily verified because when m tends to zero y(m) tends to $\beta_0 + \beta_1$ and when m tends to infinity y(m) tends to β_0.

The great advantage of this model is to summarize various economic variables that affect the behavior of the Yield Curve in a few factors that affect independently the behavior of the Yield Curve, according to the way the model is defined.

In finance, several studies have been done using factor analysis to the study of the yield curve. among them are Litterman and Scheinkman (1991) and Diebold and Li (2006). These studies show that over 95% of the variance of the Yield Curve can be explained by only three factors. These factors are level, inclination and curvature.

The level reflects the agents' expectation and the impact of parallel changes in the Yield Curve. Changes caused by this factor are such that rates of return associated with different maturity dates also vary. The inclination factor reflects the relationship between short interest rates and long term and the curvature factor determines the shape of the yield curve.

For the estimation of the parameters and its structure two approaches are possible. The first is to estimate the parameters and then to analyze these parameters with the tools available for time series. The second approach, discussed in Christensen, Diebold and Rudebusch (2011), is based on a joint estimation of the parameters and its temporal structure by the Kalman filter. In addition to the study of a specific yield curve, Aruoba, Diebold and Scotti (2007), discuss the possibility of jointly modeling several curves through the Kalman filter.

Recent researches with Brazilian data include authors such as Carvalho (2008) and Mendonca and Moura (2009). They study the application of the models above in the Brazilian term structure (yield curve), but their focus is given only to the fixed interest curve. There is another paper presented by Vilarino (2011) with a simple estimation using Kalman filter.

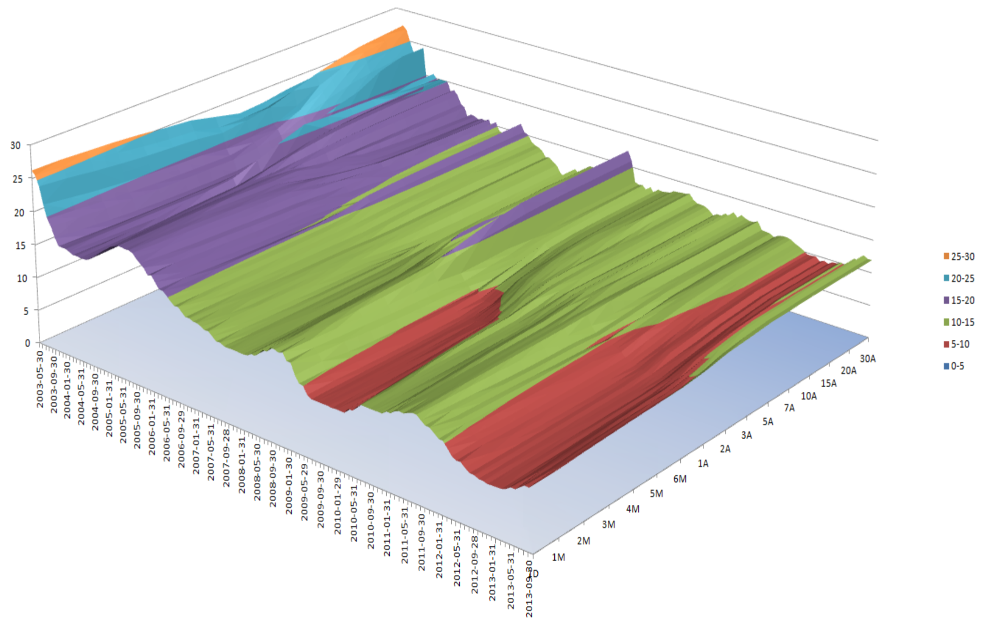
3. DATA

We use monthly data from Brazilian curves with values for the Fixed Rate yield curve, IPCA linked curve, IGPM linked curve, TR linked curve and exchange linked curve from February 2004 to October 2013. We select these dates to check the robustness of our results to the impact of the financial crisis of 2007/8. We include 40 months before the crisis start in mid 2007, when we see the first signals of the financial crisis by the Libor spread increase and 60 months after its impact in a global fashion after the Lehman Brother bankruptcy.

The evolution of curves and their descriptive statistics are presented below. As can be noted all series present high serial correlation and it is expected that many of them behave according to a local level model. We will confirm this fact later in the analysis of the curve parameters.

3.1. Fixed rate

Figure 1. Fixed rate yield curves from 02/2004 to 10/2013



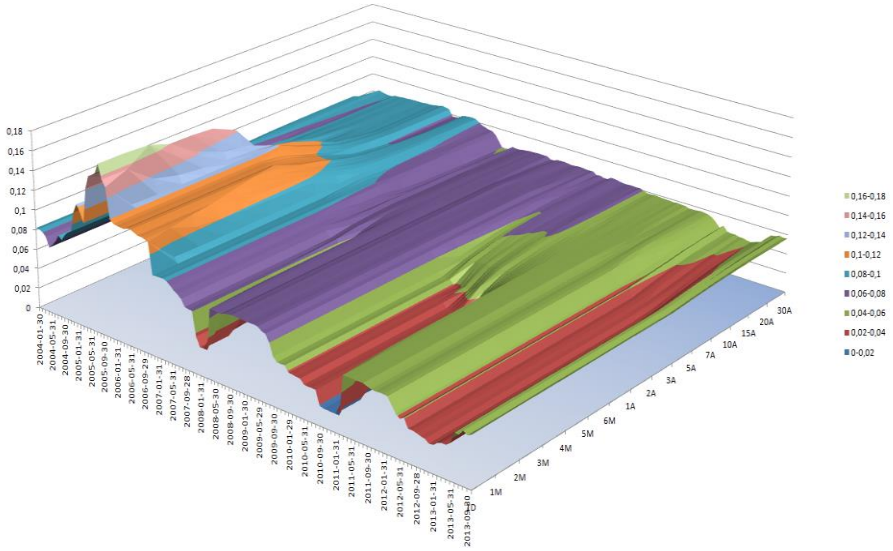
During the observed period the fixed rate yield curve has a downward trend for all maturity terms, and the difference between short term maturities and long term maturities are small. Table 1 shows, that the first and second order autocorrelations for all maturities are high (greater than 90%).

Table 1. Descriptive of Fixed rate Yield data from 02/2004 to 10/2013

					Fixed rate						
Term	Min	Max	Mean	Std. Dev.	ρ(1)	ρ(2)	ρ(3)	ρ(4)	ρ(6)	ρ(12)	ρ(24)
1D	6.9000%	19.7700%	12.24419%	3.56979%	0.993	0.980	0.961	0.933	0.867	0.645	0.513
1M	6.9750%	19.7700%	12.23787%	3.54693%	0.995	0.982	0.961	0.934	0.867	0.643	0.515
2M	7.0000%	19.8079%	12.23448%	3.53841%	0.995	0.981	0.960	0.933	0.866	0.641	0.511
3M	7.0316%	19.8100%	12.24515%	3.52450%	0.994	0.980	0.959	0.931	0.863	0.637	0.508
4M	7.0434%	19.7907%	12.26659%	3.50022%	0.993	0.979	0.957	0.929	0.860	0.634	0.505
5M	7.0549%	19.6900%	12.28591%	3.48134%	0.992	0.977	0.955	0.926	0.857	0.630	0.500
6M	7.0718%	19.6371%	12.30731%	3.46306%	0.991	0.975	0.952	0.924	0.854	0.626	0.495
1A	7.1380%	19.1840%	12.46183%	3.30435%	0.984	0.964	0.939	0.909	0.838	0.608	0.470
2A	7.7008%	20.1909%	12.79798%	2.98495%	0.969	0.941	0.912	0.885	0.816	0.607	0.458
3A	8.1470%	21.4073%	12.96536%	2.83120%	0.957	0.921	0.891	0.870	0.806	0.622	0.479
5A	8.6413%	21.1326%	13.09879%	2.69009%	0.956	0.919	0.890	0.865	0.798	0.629	0.486
7A	8.9302%	21.0151%	13.15651%	2.64032%	0.956	0.918	0.888	0.862	0.790	0.625	0.490
10A	9.2526%	20.9271%	13.21077%	2.59533%	0.954	0.915	0.883	0.855	0.777	0.605	0.485
15A	9.2600%	20.4206%	13.21077%	2.59576%	0.953	0.916	0.883	0.854	0.769	0.590	0.475
20A	9.2600%	20.6057%	13.20645%	2.59958%	0.952	0.918	0.883	0.854	0.766	0.581	0.470
30A	9.2600%	20.6057%	13.19778%	2.59146%	0.950	0.918	0.882	0.851	0.760	0.571	0.468

3.2. IGPM Linked Curve

Figure 2. IGPM linked curve from 02/2004 to 10/2013



IGPM is a Brazilian inflation index, and the IGPM linked curve is obtained with swap prices. The IGPM linked curve has behavior different than the Fixed Rate curve, with the long term maturity rates more stable than the short rates. This behavior is characterized by the autocorrelations being higher for long term maturities than for short term maturities, as shown in the Table 2.

Table 2. Descriptive of IGPM Linked from 02/2004 to 10/2013

IGPM											
Term	Min	Max	Mean	Std. Dev.	ρ(1)	ρ(2)	ρ(3)	ρ(4)	ρ(6)	ρ(12)	ρ(24)
1D	1.79%	17.08%	6.840%	3.562%	0.968	0.927	0.884	0.842	0.751	0.526	0.381
1M	1.79%	17.08%	6.859%	3.443%	0.967	0.921	0.880	0.842	0.754	0.533	0.397
2M	1.79%	16.93%	6.869%	3.348%	0.972	0.929	0.890	0.851	0.770	0.545	0.414
3M	1.81%	15.92%	6.873%	3.241%	0.977	0.940	0.902	0.866	0.789	0.561	0.435
4M	1.82%	15.70%	6.872%	3.136%	0.979	0.946	0.913	0.883	0.805	0.577	0.455
5M	1.93%	15.29%	6.866%	3.032%	0.980	0.950	0.925	0.898	0.819	0.591	0.472
6M	2.62%	14.11%	6.854%	2.924%	0.982	0.958	0.936	0.911	0.835	0.606	0.489
1A	2.64%	12.15%	6.792%	2.512%	0.988	0.969	0.948	0.926	0.866	0.660	0.545
2A	2.64%	11.27%	6.784%	2.171%	0.990	0.975	0.956	0.936	0.888	0.725	0.614
3A	2.64%	10.36%	6.718%	1.958%	0.990	0.976	0.958	0.940	0.897	0.773	0.702
5A	2.78%	9.32%	6.625%	1.723%	0.990	0.976	0.958	0.939	0.894	0.786	0.757
7A	3.16%	8.90%	6.621%	1.568%	0.990	0.975	0.956	0.936	0.893	0.794	0.776
10A	3.48%	8.64%	6.618%	1.445%	0.990	0.973	0.953	0.932	0.890	0.796	0.791
15A	3.76%	8.64%	6.598%	1.354%	0.990	0.975	0.955	0.936	0.897	0.811	0.771
20A	3.91%	8.63%	6.586%	1.313%	0.990	0.975	0.956	0.937	0.900	0.818	0.760
30A	4.05%	8.67%	6.562%	1.273%	0.987	0.973	0.953	0.935	0.898	0.813	0.719

3.3. IPCA Linked Curve

IPCA is also a Brazilian inflation index and it is the government official index, so the IPCA linked curve is obtained with the government bonds linked to it. The IPCA linked curve, as the IGPM linked curve, is an inflation linked curve and, during the observed period, has a similar behavior, with the long term maturity rates more stable than the short rates. Table 3 shows the same autocorrelation pattern, higher for long term maturities than for short term maturities.

Figure 3. IPCA Linked Curve from 02/2004 to 10/2013

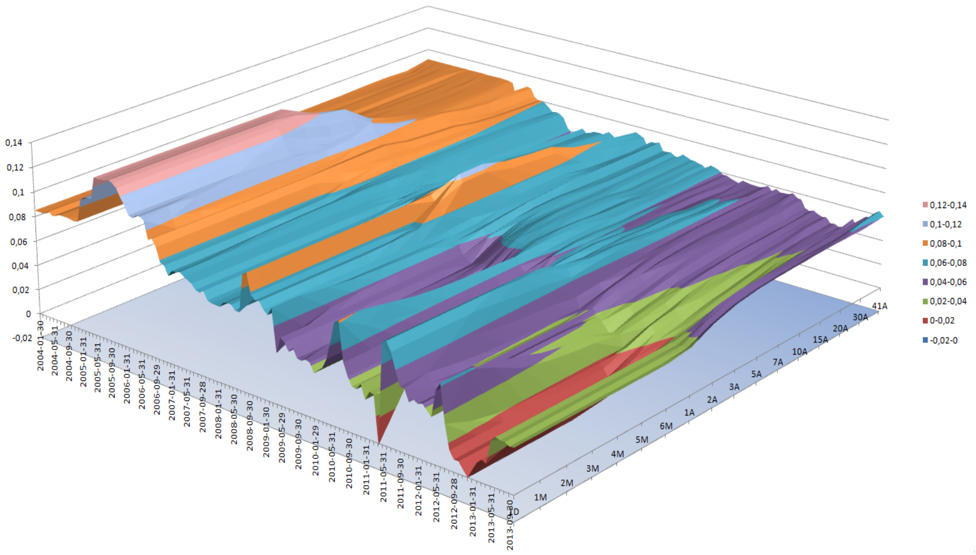
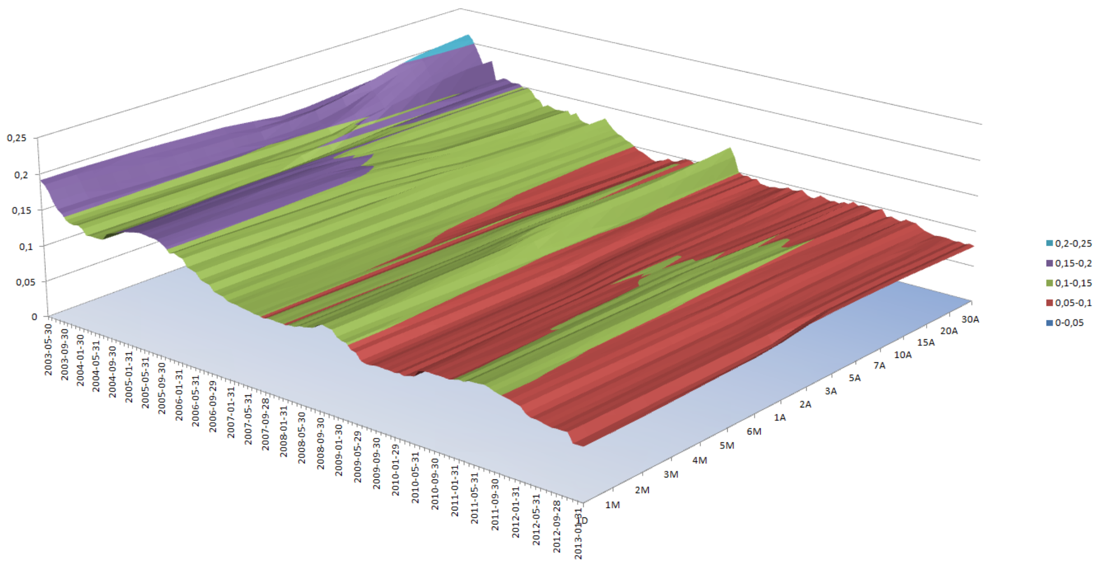


Table 3. IPCA Linked Curve from 02/2004 to 10/2013

IPCA											
Term	Min	Max	Mean	Std. Dev.	ρ(1)	ρ(2)	ρ(3)	ρ(4)	ρ(6)	ρ(12)	ρ(24)
1D	-0.13%	12.98%	6.543%	3.170%	0.931	0.877	0.838	0.797	0.731	0.654	0.569
1M	-0.01%	12.98%	6.570%	3.109%	0.960	0.908	0.865	0.824	0.754	0.664	0.578
2M	-0.01%	12.98%	6.581%	3.086%	0.970	0.924	0.880	0.837	0.764	0.663	0.581
3M	-0.01%	12.98%	6.583%	3.074%	0.975	0.935	0.893	0.851	0.776	0.660	0.586
4M	-0.01%	12.98%	6.595%	3.060%	0.978	0.942	0.903	0.862	0.788	0.658	0.590
5M	0.20%	12.98%	6.612%	3.043%	0.980	0.946	0.909	0.872	0.798	0.657	0.593
6M	0.46%	12.98%	6.635%	3.024%	0.981	0.949	0.915	0.881	0.810	0.657	0.597
1A	1.10%	12.81%	6.801%	2.857%	0.985	0.963	0.936	0.908	0.843	0.671	0.618
2A	1.66%	12.02%	7.058%	2.515%	0.983	0.959	0.932	0.903	0.839	0.669	0.620
3A	2.20%	11.64%	7.151%	2.260%	0.981	0.954	0.926	0.899	0.840	0.668	0.635
5A	2.74%	10.31%	7.043%	1.895%	0.980	0.953	0.926	0.901	0.851	0.718	0.690
7A	3.12%	9.98%	6.981%	1.743%	0.979	0.950	0.923	0.900	0.853	0.739	0.727
10A	3.52%	9.75%	6.951%	1.613%	0.977	0.946	0.917	0.894	0.847	0.745	0.740
15A	3.87%	9.32%	6.858%	1.522%	0.980	0.956	0.935	0.920	0.883	0.792	0.763
20A	3.98%	9.11%	6.825%	1.480%	0.981	0.959	0.941	0.929	0.896	0.810	0.756
30A	4.08%	9.09%	6.795%	1.467%	0.980	0.959	0.940	0.929	0.898	0.815	0.735
41A	4.11%	9.11%	6.818%	1.470%	0.979	0.961	0.941	0.930	0.899	0.807	0.701

3.4. TR Linked Curve

Figure 4. TR Linked Curve from 02/2004 to 10/2013



The TR is the index used to remunerate savings deposits and it is calculated by the Brazil Central Bank. The TR linked curve has a visually smooth behavior during the observed period, however the standard deviations are the same level as the previous curves. The descriptive data for this curve are presented in below.

Table 4. TR Linked Curve from 02/2004 to 10/2013

					TR						
Term	Min	Max	Mean	Std. Dev.	ρ(1)	ρ(2)	ρ(3)	ρ(4)	ρ(6)	ρ(12)	ρ(24)
1D	6.96%	16.10%	10.877%	2.525%	0.992	0.979	0.961	0.939	0.885	0.695	0.508
1M	6.96%	16.10%	10.876%	2.525%	0.992	0.979	0.961	0.939	0.885	0.695	0.508
2M	6.99%	16.11%	10.869%	2.513%	0.992	0.979	0.961	0.939	0.885	0.696	0.509
3M	7.02%	16.09%	10.866%	2.497%	0.992	0.979	0.962	0.940	0.886	0.697	0.512
4M	7.03%	16.08%	10.867%	2.477%	0.992	0.979	0.962	0.941	0.888	0.699	0.515
5M	7.04%	16.00%	10.864%	2.460%	0.991	0.979	0.962	0.942	0.889	0.700	0.518
6M	7.07%	15.95%	10.863%	2.445%	0.991	0.979	0.962	0.942	0.890	0.701	0.520
1A	7.13%	15.64%	10.857%	2.342%	0.989	0.977	0.961	0.943	0.893	0.705	0.531
2A	7.69%	15.25%	10.868%	2.165%	0.985	0.972	0.956	0.942	0.897	0.718	0.562
3A	8.11%	15.79%	10.832%	2.083%	0.981	0.966	0.951	0.940	0.900	0.738	0.597
5A	8.12%	15.80%	10.707%	2.009%	0.981	0.963	0.947	0.935	0.897	0.747	0.621
7A	8.12%	15.80%	10.626%	2.000%	0.980	0.961	0.946	0.933	0.895	0.756	0.645
10A	8.09%	15.80%	10.550%	2.007%	0.979	0.959	0.943	0.930	0.892	0.761	0.666
15A	7.94%	15.62%	10.452%	2.040%	0.975	0.954	0.936	0.922	0.880	0.747	0.657
20A	7.87%	15.63%	10.407%	2.061%	0.972	0.950	0.930	0.915	0.869	0.735	0.645
30A	7.79%	15.63%	10.363%	2.081%	0.967	0.946	0.924	0.908	0.858	0.721	0.629

3.5. Exchange rate linked Curve

The exchange rate linked curve is extremely volatile, as the Brazilian Real to US Dollar exchange rate. Due to the high volatility the graphics are separate in two periods:

a. Maturity terms from 1 to 30 days are displayed in Figure 5

b. Maturity terms above 30 days are displayed in Figure 6 below.

Figure 5. Exchange rate linked Curve below 30 days from 02/2004 to 10/2013

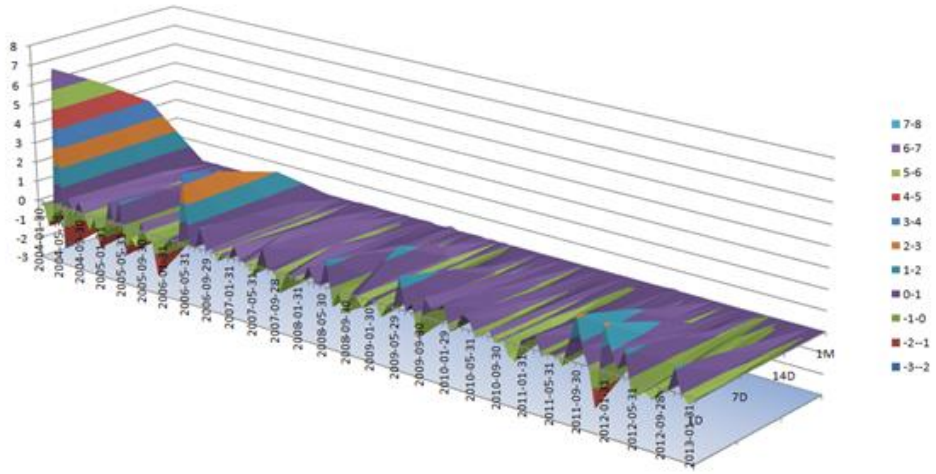
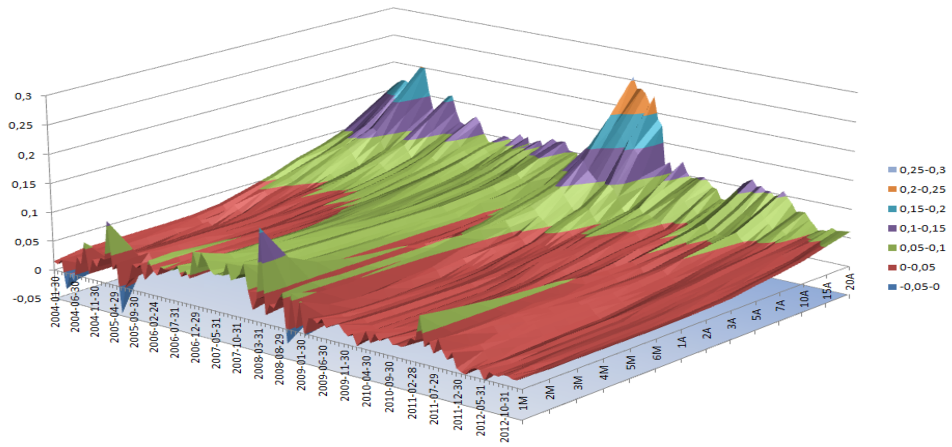


Figure 6. Exchange rate linked Curve above 30 days from 02/2004 to 10/2013



Comparing the lines in Table 5 we notice that the data and the serial correlations present a very irregular pattern, depending on which maturity is being analyzed. This behavior is expected due to the high volatility of exchange rates in Brazil, especially in the short term.

Table 5. Descriptives - Exchange rate linked Curve from 02/2004 to 10/2013

Exchange rate linked Curve											
Term	Min	Max	Mean	Std. Dev.	ρ(1)	ρ(2)	ρ(3)	ρ(4)	ρ(6)	ρ(12)	ρ(24)
1D	-216.76%	710.52%	11.252%	116.705%	(0.206)	0.008	(0.147)	0.004	(0.013)	0.115	0.235
7D	-172.14%	563.53%	11.735%	82.501%	(0.233)	(0.031)	(0.095)	0.011	(0.096)	0.141	0.281
14D	-120.09%	392.04%	6.536%	48.418%	(0.231)	(0.054)	(0.026)	0.002	(0.141)	0.233	0.511
1M	-4.62%	15.95%	3.191%	2.841%	0.373	0.479	0.255	0.449	0.295	0.253	(0.051)
2M	-0.49%	9.51%	2.991%	2.065%	0.730	0.736	0.671	0.723	0.551	0.377	(0.092)
3M	0.06%	7.24%	2.979%	1.881%	0.834	0.823	0.764	0.772	0.642	0.422	(0.103)
4M	0.57%	7.09%	3.033%	1.785%	0.874	0.858	0.798	0.791	0.678	0.454	(0.095)
5M	0.82%	7.01%	3.070%	1.719%	0.898	0.877	0.819	0.799	0.698	0.475	(0.082)
6M	1.13%	6.94%	3.116%	1.681%	0.911	0.888	0.828	0.802	0.705	0.490	(0.061)
1A	1.31%	6.90%	3.387%	1.531%	0.929	0.889	0.836	0.795	0.705	0.546	0.048
2A	1.54%	6.90%	3.854%	1.371%	0.937	0.886	0.847	0.812	0.756	0.663	0.298
3A	1.83%	6.90%	4.284%	1.322%	0.931	0.885	0.856	0.820	0.793	0.727	0.489
5A	2.48%	7.93%	5.159%	1.385%	0.934	0.871	0.816	0.770	0.756	0.662	0.562
7A	2.95%	10.07%	5.986%	1.646%	0.942	0.868	0.797	0.746	0.723	0.537	0.429
10A	3.47%	13.48%	7.140%	2.225%	0.939	0.863	0.794	0.739	0.683	0.381	0.211
15A	4.66%	19.59%	9.426%	3.253%	0.951	0.879	0.800	0.735	0.641	0.224	0.021
20A	5.12%	25.61%	11.489%	4.461%	0.921	0.848	0.762	0.685	0.554	0.120	(0.127)

4. PARAMETER ESTIMATION

For the estimation of betas two approaches are possible. The first is to fix the τ parameter and estimate the beta parameters of the model (following linear Nelson Siegel model) and the second approach is to use non-linear methods and estimate the τ in together with the betas.

Note that we use the notation for τ and λ as follows: λ is the multiplicative factor of traditional Nelson Siegel and τ is the factor for the model by Diebold and Li (2006). For this work we estimate the parameters by the two approaches.

4.1. Identification Methodology

We will first estimate betas and tau and afterward we will verify their pattern and model these parameters along the time using VAR/VEC, where the model will be defined using Schwarz, Akaike and the Hannan-Quinn information criteria alongside with the Johansen test.

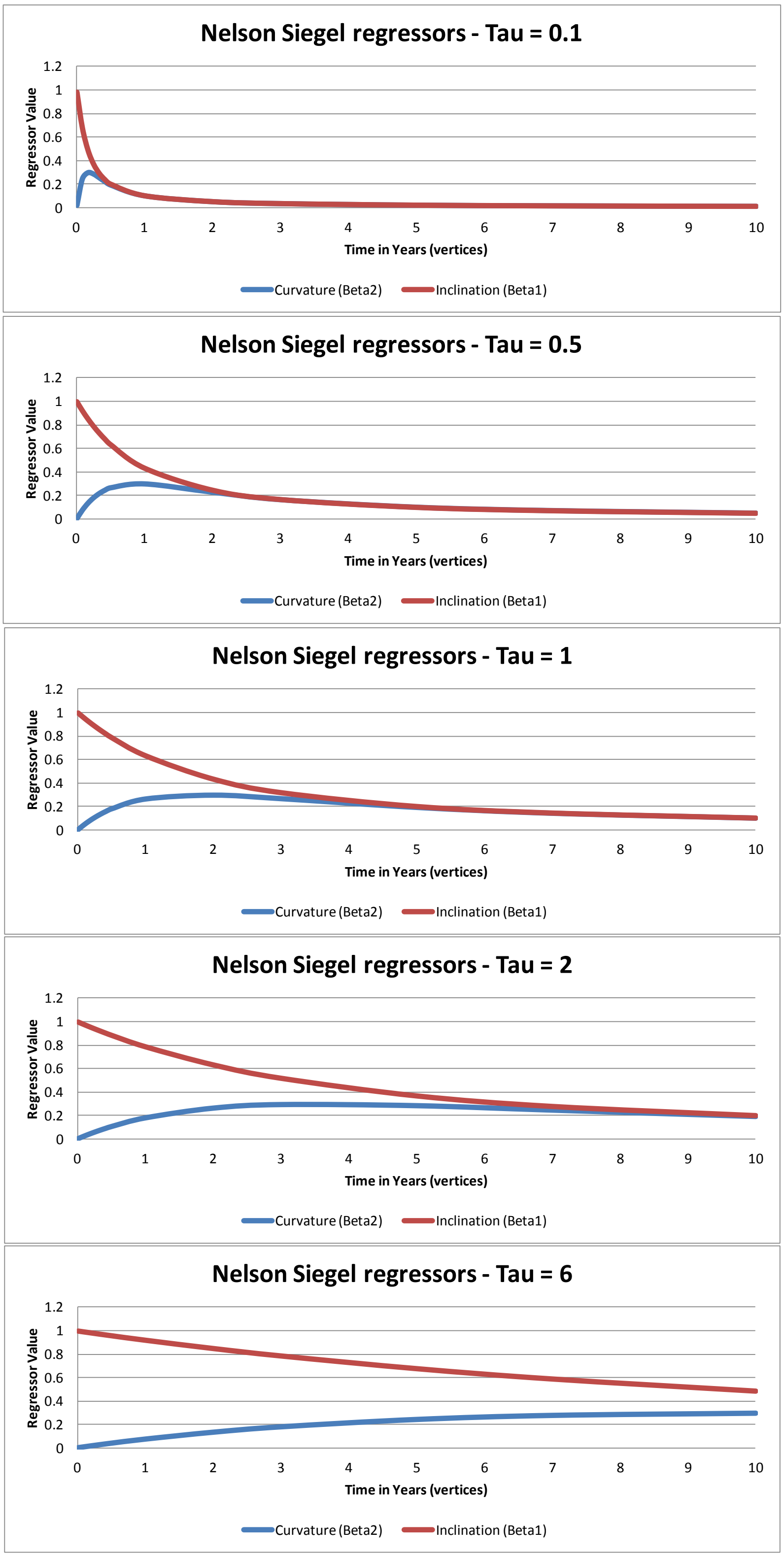
4.2. Theoretical Parameter Behavior Patterns

To understand the shapes of curves represented by the Nelson Siegel model, as in Eq.4, we display in the subsection the values that will be used as regressors for β_1 and β_2. Therefore we simulate for several values of τ and time maturities the values of $\left[\frac{1-exp(-m\lambda)}{(m\lambda)}\right]$ and $\left[\frac{1-exp(-m\lambda)}{(m\lambda)} - exp(-m\lambda)\right]$. As we can notice, τ has a great influence on the shape of the structure of yield curve of interest rates (yield curve), so by setting a value we have a specific shape curve with a maximum point of curvature, as can be seen in the graphs of Figure 7. It is noticeable that the maximum point of curvature of regressor grows monotonically in τ. In Table 6 these values are tabulated for detailed information. Based on these observations we will impose some restrictions in the regressions in the following section.

Table 6. τ - value implication

τ	*Inclination and Curvature Correlation*	*Curvature point of maximum*
0.005	0.98	1D
0.010	0.82	1D
0.025	0.41	1M
0.050	0.42	1M
0.100	0.48	2M
0.250	0.50	5M
0.300	0.48	6M
0.500	0.36	1A
0.750	0.18	1A
0.900	0.08	2A
1.000	0.03	2A
1.100	-0.03	2A
1.250	-0.10	2A
1.500	-0.19	3A
2.000	-0.34	3A
3.000	-0.52	5A
5.000	-0.71	10A
6.000	-0.77	10A
7.000	-0.81	15A
8.000	-0.85	15A

Figure 7. Regressors Pàttern for Load Factors according to the τ level and time



The interpretation for the beta parameters, as discussed in the literature review, follows the factor model approach: β_0 is the rate level, with an expected value around the rate mean for all maturities, β_1 is the slope of the curve, where a positive value indicates a downward term structure of interest rates (TSIR) curve, while negative indicates a growing TSIR, β_2 is the curvature, where a positive value indicates TSIR is concave and otherwise, if negative, points to convex TSIR.

4.3. All Parameters (Tau and Betas) estimation using NLLS

In this subsection we estimate the values of the Nelson Siegel model coefficients (β_0. β_1. β_2 and τ_1) using the nonlinear least squares model. In regressions where τ_1 values were above 10 the value was set at 1, which is neutral and with less multicollinearity level. Values above 10 or below 0.05 generate serious distortions in NLLS maximum likelihood estimation, with results without practical significance since the model becomes unstable due to excessive multicollinearity.

We estimated the values for βs and τ using NLLS. Table 7 shows the results for the parameters descriptive statistics, serial correlation as well as the ADF (augmented Dickey Fuller) test for unit root. We notice in particular from the results that for the fixed rate curve β_0 and β_1 are integrated order one, but β_2 and τ_1 are stationary. For IPCA and Exchange rate linked curves only β_0 is integrated order one, and β_1, β_2 and τ_1 are stationary. And for IGPM and TR only β2 is stationary and β_0, β_1 and τ_1 integrated order one.

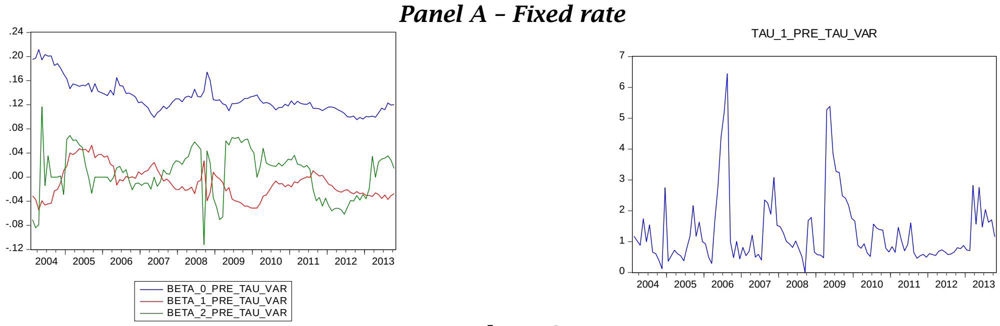
Table 7. Descriptive Summary and Unit Root for Estimated Nelson Siegel Parameters

						Fixed rate						
Param.	Min	Max	Mean	Std. dev.	ρ(1)	ρ(2)	ρ(3)	ρ(4)	ρ(6)	ρ(12)	ρ(24)	ADF	Sign. (%)
β0	0.095	0.212	0.1319	0.0258	0.95	0.92	0.88	0.85	0.74	0.56	0.50	-1.320	17.09%
β1	-0.054	0.053	-0.0099	0.0254	0.91	0.85	0.79	0.71	0.49	(0.01)	0.04	-1.895	**5.53%**
β2	-0.112	0.117	0.0057	0.0398	0.57	0.47	0.34	0.29	0.20	(0.05)	(0.17)	-5.727	**0.00%**
τ1	0.005	6.441	1.3217	1.1368	0.63	0.39	0.18	0.02	(0.03)	(0.13)	(0.08)	-3.141	**0.19%**
						IGPM						
β0	0.041	0.087	0.0649	0.0122	0.98	0.96	0.93	0.90	0.85	0.79	0.64	-2.080	54.73%
β1	-0.065	0.090	0.0026	0.0278	0.92	0.84	0.77	0.70	0.56	0.27	0.13	-2.700	23.66%
β2	-0.075	0.120	0.0124	0.0305	0.65	0.47	0.35	0.23	0.16	0.10	0.05	-4.901	**0.01%**
τ1	0.006	8.674	2.2898	2.2866	0.86	0.77	0.64	0.53	0.40	0.07	(0.23)	-2.965	14.68%
						TR						
β0	0.075	0.159	0.1028	0.0210	0.97	0.94	0.92	0.91	0.86	0.72	0.63	-2.784	20.63%
β1	-0.025	0.041	0.0056	0.0133	0.91	0.84	0.76	0.67	0.45	(0.09)	(0.05)	-2.233	19.58%
β2	-0.039	0.043	0.0094	0.0144	0.64	0.50	0.32	0.33	0.03	(0.36)	(0.01)	-5.539	**0.01%**
τ1	0.104	8.487	1.9163	1.4255	0.53	0.18	0.05	0.12	0.34	0.18	(0.12)	-2.440	13.31%
						IPCA						
β0	0.042	0.092	0.0669	0.0137	0.97	0.95	0.93	0.92	0.89	0.80	0.72	-2.722	22.97%
β1	-0.066	0.044	-0.0023	0.0218	0.83	0.72	0.64	0.55	0.46	0.41	0.33	-3.107	**2.88%**
β2	-0.100	0.102	0.0184	0.0368	0.79	0.69	0.61	0.49	0.36	0.17	0.01	-3.968	**1.23%**
τ1	0.103	8.316	1.7392	1.5226	0.43	0.25	0.07	(0.00)	(0.14)	0.07	0.09	-6.859	**0.00%**
						Exchange Rate						
β0	0.026	0.496	0.0871	0.0746	0.41	0.45	0.65	0.22	0.32	0.02	(0.06)	-2.810	19.47%
β1	-3.221	11.121	0.1519	1.7039	(0.14)	0.04	(0.08)	0.05	0.05	0.18	0.17	-11.650	**0.00%**
β2	-17.194	2.676	-0.5156	2.2222	(0.14)	0.05	(0.08)	0.03	0.01	0.15	0.15	-11.117	**0.00%**
τ1	0.001	8.305	0.6371	1.5334	0.04	0.19	0.22	0.03	0.09	(0.05)	(0.11)	-4.190	**0.63%**

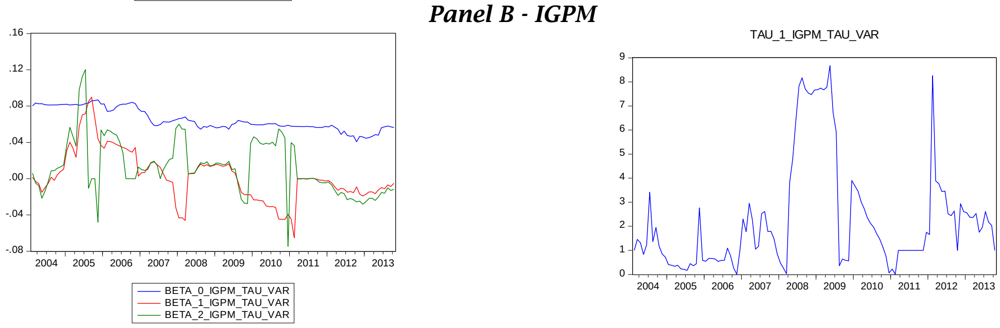
In Figure 8 we present the evolution of the parameters over the observed period. The τ for all curves does not show any pattern. The βs have a less erratic behavior, except for the Exchange Rate linked curve where the βs estimated over time resemble just noise. It is noticeable that β0s have a visible trend for the curves, while others βs the trend is less apparent.

Figure 8. Nelson Siegel parameters across the years for fixed rate curves

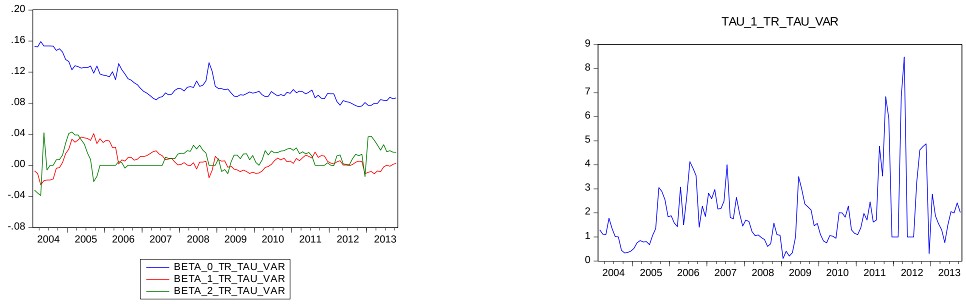
Panel A – Fixed rate



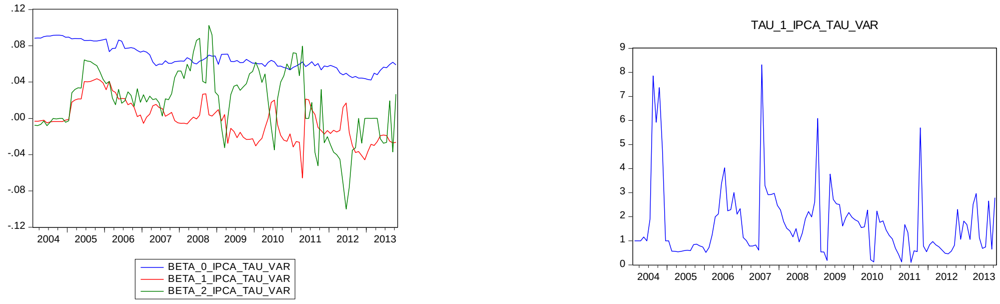
Panel B - IGPM



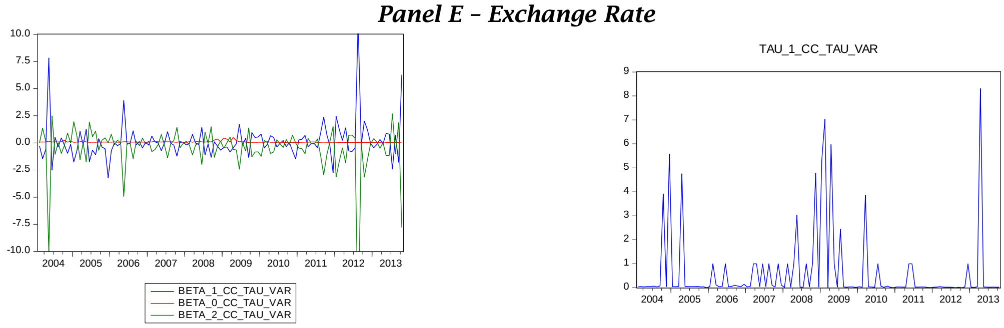
Panel C – TR



Panel D – IPCA



Panel E – Exchange Rate

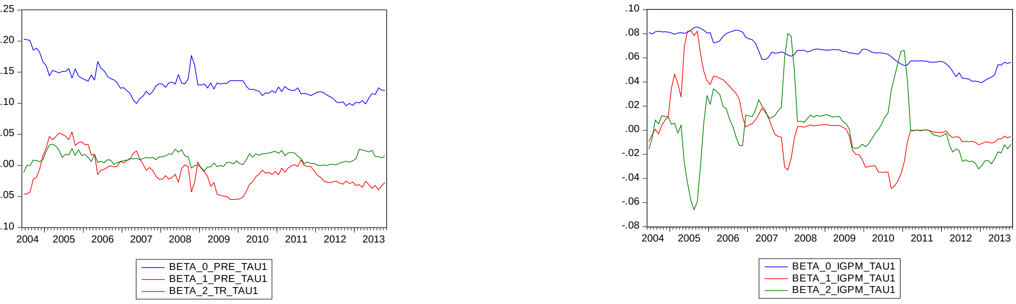


4.4. Beta Parameters estimation with fixed Tau using NLLS

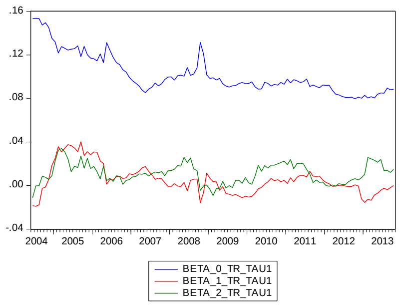
An alternative model to estimate Nelson and Siegel parameters is to fix τ to one and estimate only the βs with NLLS. Proceeding this way we reduce the error induced in the previous complete model that estimates four parameters using only 17 maturities. Our results are displayed in Figure 9 and Table 8. We remark that with 5% significance β_0, β_1 and β_2 are all integrated order one, except β_2 for the IGPM linked curve all betas for the Exchange rate linked. In the latter case, besides the fact that all betas are stationary we also notice that the parameters have very high volatility as in Table 8.

Figure 9. Nelson Siegel parameters across the years for fixed rate curves

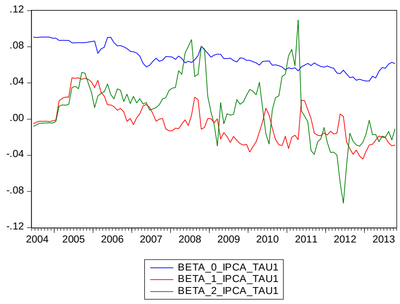
Panel A – Fixed rate ***Panel B – IGPM***



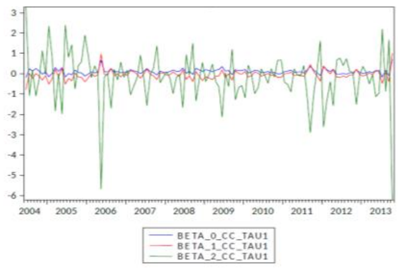
Panel C – TR



Panel D – IPCA



Panel E – Exchange Rate Linked



The βs for the exchange rate linked curve still have a completely erratic behavior, however the behavior of estimated βs for the other curves has become smoother.

Table 8. Descriptive Summary and Unit Root for Estimated Nelson Siegel Parameters (only Betas)

						Fixed rate							
Param.	Min	Max	Mean	Std. Dev.	ρ(1)	ρ(2)	ρ(3)	ρ(4)	ρ(6)	ρ(12)	ρ(24)	ADF	Sign. (%)
β0	0.096	0.213	0.1331	0.0255	0.94	0.91	0.87	0.84	0.74	0.53	0.46	-1.636	9.57%
β1	-0.056	0.054	-0.0116	0.0268	0.93	0.87	0.80	0.73	0.51	0.03	0.05	-2.205	20.57%
β2	-0.085	0.079	-0.0018	0.0288	0.74	0.62	0.47	0.42	0.19	(0.37)	(0.13)	-1.288	>10% *
						IGPM							
β0	0.040	0.086	0.0655	0.0125	0.99	0.96	0.94	0.91	0.86	0.78	0.72	-1.580	48.89%
β1	-0.048	0.082	0.0032	0.0267	0.96	0.90	0.84	0.78	0.64	0.33	0.13	-2.138	23.03%
β2	-0.066	0.080	0.0026	0.0250	0.90	0.70	0.48	0.30	0.08	0.09	(0.15)	-4.313	0.07%
						TR							
β0	0.080	0.159	0.1037	0.0203	0.97	0.95	0.92	0.91	0.86	0.72	0.65	-3.162	9.75%
β1	-0.025	0.040	0.0045	0.0136	0.92	0.85	0.77	0.69	0.46	(0.04)	0.02	-2.710	7.53%
β2	-0.041	0.034	0.0099	0.0120	0.71	0.65	0.46	0.48	0.19	(0.50)	(0.05)	-2.478	12.36%
						IPCA							
β0	0.042	0.091	0.0683	0.0136	0.97	0.94	0.91	0.90	0.85	0.77	0.72	-1.814	37.20%
β1	-0.044	0.046	-0.0040	0.0215	0.91	0.80	0.72	0.64	0.52	0.42	0.31	-2.196	20.89%
β2	-0.093	0.109	0.0120	0.0331	0.85	0.73	0.63	0.53	0.34	0.14	0.00	-3.442	5.09%
						Exchange Rate							
β0	-0.193	1.303	0.1156	0.1724	(0.15)	0.02	(0.07)	0.01	(0.05)	0.14	0.31	-5.884	0.00%
β1	-0.782	1.894	-0.0419	0.2948	(0.22)	(0.00)	(0.08)	0.04	(0.04)	0.16	0.27	-15.71	0.00%
β2	-11.378	3.272	-0.2641	16.379	(0.21)	(0.01)	(0.10)	0.01	(0.06)	0.15	0.31	-16.01	0.00%

** In this particular case we use DF-GLS test*

4.4. Estimates Summary

In the first approach, estimating τ and β parameters, the results for all curves present β_0 as random walk, β_1 is random walk only for fixed rate. TR and IGPM curves and β_2 is stationary for all curves. The τ is random walk to IGPM and TR and stationary in other cases. The curves of exchange linked have a completely different behavior when compared to the other curves and we will proceed modeling VAR/VEC for them. Our results suggest that Nelson Siegel model may not be an efficient approach to analyze exchange rate linked curves.

In the approach fixing τ = 1, all βs are random walks, except for the exchange rate linked curves, where all parameters have stationary behavior, and for the IGPM linked curves in which β_2 also presents a stationary pattern. In table 9 we indicate the percentage of regressions in which the F statistic for the null hypothesis of all coefficients equal zero with significance below 5%. Notice that the percentages are high for most estimations exception made to exchange rate curves.

Table 9. Joint Significance of Estimations*

Curve	***Including τ***	***Fixing τ = 1***
Fixed rate	99.10%	99.10%
IGPM	94.90%	92.30%
TR	95.70%	92.30%
IPCA	100.00%	98.30%
Exchange Rate	90.60%	65.80%

** % of NLLS regressions with F test below 5%*

5. VAR/VEC ESTIMATION FOR THE PARAMETERS TIME SERIES

With the results obtained in the previous section it is possible to estimate VAR/VEC models using the fixed τ for fixed rate, TR and IPCA linked curves. For the other curves the parameters do not share the same integration order which prevents the use of the VAR/VEC model.

5.1. Fixed rate Parameters

The information criteria for model order selection indicates as optimal the VAR in level with three lags (except for Schwarz Information Criteria, SIC, that indicates only one lag). We selected VEC with two lags, as the variables are integrated to first order. Johansen test indicates the use of model 4 with linear trend in the data and two equations of cointegration with trend and intercept (using the Akaike Information Criteria, AIC). Table 10 shows the EViews estimate of the model.

The equations for β_0 and β_2 show that there is a possible long run relationship them and their explanatory variables, as indicated by the negative coefficients for the cointegration equations.

Table 10. VAR/VEC Estimation for Fixed rate Betas

Vector Error Correction Estimates
Sample: 2004M06 2013M10
Included observations: 113 Standard errors in ( )

LR test for binding restrictions (rank = 2):			
Chi-square(1)	1.226124		
Probability	0.268162		
Cointegrating Eq:	CointEq1	CointEq2	
BETA_0_PRE_TAU1(-1)	1.000000	0.000000	
BETA_1_PRE_TAU1(-1)	0.000000	1.000000	
BETA_2_PRE_TAU1(-1)	1.613250	-2.158768	
	(0.36169)	(0.52538)	
@TREND(04M02)	0.000882	-0.000233	
	(0.00017)	(0.00025)	
C	-0.184527	0.024874	
Error Correction:	D(BETA_0_PRE_TAU1)	D(BETA_1_PRE_TAU1)	D(BETA_2_PRE_TAU1)
CointEq1	-0.063059	0.050092	-0.369048
	(0.03428)	(0.03574)	(0.07947)
CointEq2	-0.035366	0.000000	-0.163100
	(0.01003)	(0.00000)	(0.06075)
D(BETA_0_PRE_TAU1(-1))	-0.291331	0.529222	0.814989
	(0.39039)	(0.42805)	(0.55962)
D(BETA_0_PRE_TAU1(-2))	0.294818	0.016973	0.936788
	(0.39109)	(0.42882)	(0.56064)
D(BETA_1_PRE_TAU1(-1))	-0.113070	0.351625	0.436544
	(0.34587)	(0.37923)	(0.49580)
D(BETA_1_PRE_TAU1(-2))	0.317492	-0.004643	0.960419
	(0.34490)	(0.37817)	(0.49442)
D(BETA_2_PRE_TAU1(-1))	0.021743	0.015340	-0.191715
	(0.05712)	(0.06263)	(0.08188)
D(BETA_2_PRE_TAU1(-2))	-0.001364	0.028768	0.056059
	(0.05130)	(0.05624)	(0.07353)
C	-0.000758	0.000537	0.000671
	(0.00084)	(0.00093)	(0.00121)
R-squared	0.064963	0.153712	0.388384
Adj. R-squared	-0.006962	0.088613	0.341337
Sum sq. resids	0.007424	0.008925	0.015256
S.E. equation	0.008449	0.009264	0.012111
F-statistic	0.903199	2.361201	8.255171
Log likelihood	383.7807	373.3737	343.0864
Akaike AIC	-6.633287	-6.449091	-5.913034
Schwarz SC	-6.416061	-6.231866	-5.695808
Mean dependent	-0.000728	0.000207	-0.000611
S.D. dependent	0.008420	0.009704	0.014923
Determinant resid covariance (dof adj.)		5.01E-14	
Determinant resid covariance		3.91E-14	
Log likelihood		1263.123	
Akaike information criterion		-21.73668	
Schwarz criterion		-20.89192	

Table 11 shows that residuals from the VAR/VEC model exhibit serial correlation for the first lag, but for higher lags we cannot reject the hypothesis that this correlation is zero.

Table 11. VEC Residual Serial Correlation LM Tests

Null Hypothesis: no serial correlation at lag order h		
Sample: 2004M06 2013M10		
Included observations: 113		
Lags	LM-Stat	Prob
1	33.20481	0.0001
2	7.842286	0.5501
3	9.897002	0.3589
4	10.80077	0.2896
5	7.374919	0.5981
6	9.460233	0.3959
7	5.974240	0.7425
8	5.419124	0.7963
9	5.502222	0.7885
10	4.745792	0.8559
11	12.00459	0.2131
12	6.338428	0.7056
Probs from chi-square with 9 df.		

Figure 10 shows that the model residuals appear to be stationary however they may present some structure as shown above by the autocorrelation test. We remark also that at the turn of 2009 there is a more pronounced variability possibly attributed to the subprime crisis. To make sure that there is no structure in the variance we run the White test for joint heteroscedasticity in Table 12 and reject this hypothesis, although the individual waste res1 and res2 present a pattern in isolation and also crosswise. This result suggests that variance modeling may be a possible evolution in a future research.

Figure 2. Residual Analysis for Fixed rate VAR/VEC model

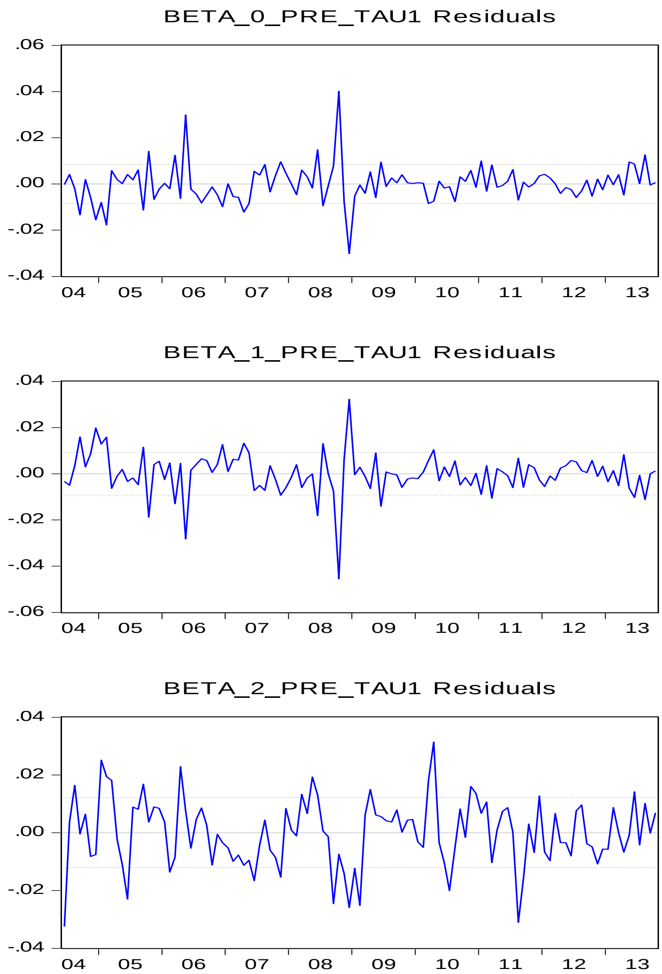


Table 12. VEC Residual Heteroskedasticity Tests for Fixed rate VAR/VEC model residuals

No Cross Terms (only levels and squares)				
Sample: 2004M06 2013M10				
Included observations: 113				
Joint test:				
Chi-sq	df	Prob.		
194.4923	96	0.0000		
Individual components:				
Dependent	R-squared	F(16.96)	Prob.	Chi-sq(16)
res1*res1	0.173477	1.259322	0.2396	19.60285
res2*res2	0.156346	1.111919	0.3554	17.66707
res3*res3	0.384553	3.749012	0.0000	43.45449
res2*res1	0.165537	1.190255	0.2898	18.70571
res3*res1	0.353903	3.286528	0.0001	39.99101
res3*res2	0.352528	3.266811	0.0002	39.83567

Table 13 shows Granger causality test which indicates no causality between variables, β_1 and β_2 in β_0 and β_0 and β_2 in β_1. However, there is Granger causality evidence of β_0 and β_1 in β_2 for the fixed rate curve parameters.

Table 13. Granger Causality Test for Fixed rate Betas

VEC Granger Causality/Block Exogeneity Wald Tests			
Sample: 2004M06 2013M10			
Included observations: 113			
Dependent variable: D(BETA_0_PRE_TAU1)			
Excluded	Chi-sq	df	Prob.
D(BETA_1_PRE_TAU1)	0.848728	2	0.6542
D(BETA_2_PRE_TAU1)	0.186558	2	0.9109
All	1.434818	4	0.8381
Dependent variable: D(BETA_1_PRE_TAU1)			
Excluded	Chi-sq	df	Prob.
D(BETA_0_PRE_TAU1)	1.668699	2	0.4342
D(BETA_2_PRE_TAU1)	0.262799	2	0.8769
All	2.693069	4	0.6104
Dependent variable: D(BETA_2_PRE_TAU1)			
Excluded	Chi-sq	df	Prob.
D(BETA_0_PRE_TAU1)	6.628894	2	0.0364
D(BETA_1_PRE_TAU1)	6.957127	2	0.0309
All	12.87522	4	0.0119

5.2. TR Linked Parameters

The information criterion for selection of model order indicates to the VAR in level two lags, except for the SIC and Hannan-Quinn (HQ) which indicate level one. We choose VEC with lag one, since the variables are integrated to first order. The Johansen test indicates the second model without a linear trend in the data and two equations of cointegration without trend and intercept (AIC criteria and SIC) with two cointegration vectors.

Table 14 exhibits the VAR/VEC model estimated for TR Linked parameters. In this case the parameters for the model for β_0 indicates a possible long run relationship between it and the independents variables.

As in the fixed rate curves VAR/VEC model, Table 15 shows that the residuals have serial correlation in the first lag.

Figure 11 illustrates that the residuals are stationary; however there is a large variance fluctuation during the 2008/9 crisis. As displayed in Table 16 although the White test rejects heteroscedasticity pattern for the overall model, we find structures in individual residues res1 and res2.

Table 16 shows no Granger causality between variables β_0, β_1 and β_2 for the TR Linked curve parameters.

Table 14. VAR/VEC Estimation for TR Linked Betas

Vector Error Correction Estimates			
Sample: 2004M06 2013M10			
Included observations: 113 Standard errors in ( )			
LR test for binding restrictions (rank = 2):			
Chi-square(3)	1.897319		
Probability	0.593990		
Cointegrating Eq:	CointEq1	CointEq2	
BETA_0_TR_TAU1(-1)	1.000000	0.000000	
BETA_1_TR_TAU1(-1)	0.000000	1.000000	
BETA_2_TR_TAU1(-1)	-1.272983	-1.843659	
	(0.65262)	(0.30625)	
C	-0.080135	0.019898	
	(0.00917)	(0.00430)	
Error Correction:	D(BETA_0_TR_TAU1)	D(BETA_1_TR_TAU1)	D(BETA_2_TR_TAU1)
CointEq1	-0.074663	0.105404	0.000000
	(0.01738)	(0.01836)	(0.00000)
CointEq2	0.000000	-0.133189	0.000000
	(0.00000)	(0.02004)	(0.00000)
D(BETA_0_TR_TAU1(-1))	-0.077814	0.136316	0.037843
	(0.18041)	(0.18467)	(0.21054)
D(BETA_1_TR_TAU1(-1))	0.063285	0.060365	-0.025461
	(0.15893)	(0.16268)	(0.18546)
D(BETA_2_TR_TAU1(-1))	-0.010373	-0.080173	-0.358746
	(0.06108)	(0.06253)	(0.07128)
R-squared	0.153600	0.214784	0.303474
Adj. R-squared	0.122252	0.185702	0.277677
Sum sq. resids	0.002377	0.002491	0.003237
S.E. equation	0.004691	0.004802	0.005475
F-statistic	4.899814	7.385445	11.76381
Log likelihood	448.1249	445.4860	430.6754
Akaike AIC	-7.842919	-7.796213	-7.534077
Schwarz SC	-7.722238	-7.675532	-7.413396
Mean dependent	-0.000603	0.000212	-8.40E-05
S.D. dependent	0.005007	0.005322	0.006442
Determinant resid covariance (dof adj.)		3.18E-15	
Determinant resid covariance		2.77E-15	
Log likelihood		1412.097	
Akaike information criterion		-24.58578	
Schwarz criterion		-24.03065	

Table 15. VEC Residual Serial Correlation LM Tests for TR Linked VAR/VEC model residuals

Null Hypothesis: no serial correlation at lag order h		
Sample: 2004M06 2013M10		
Included observations: 113		
Lags	LM-Stat	Prob
1	32.32256	0.0002
2	12.60283	0.1814
3	10.11706	0.3411
4	8.339280	0.5003
5	4.131220	0.9026
6	12.76646	0.1735
7	7.955239	0.5387
8	10.66128	0.2996
9	8.855414	0.4507
10	7.850719	0.5493
11	9.363453	0.4044
12	4.910498	0.8420
Probs from chi-square with 9 df.		

Figure 11. Residual Analysis for TR Linked VAR/VEC model

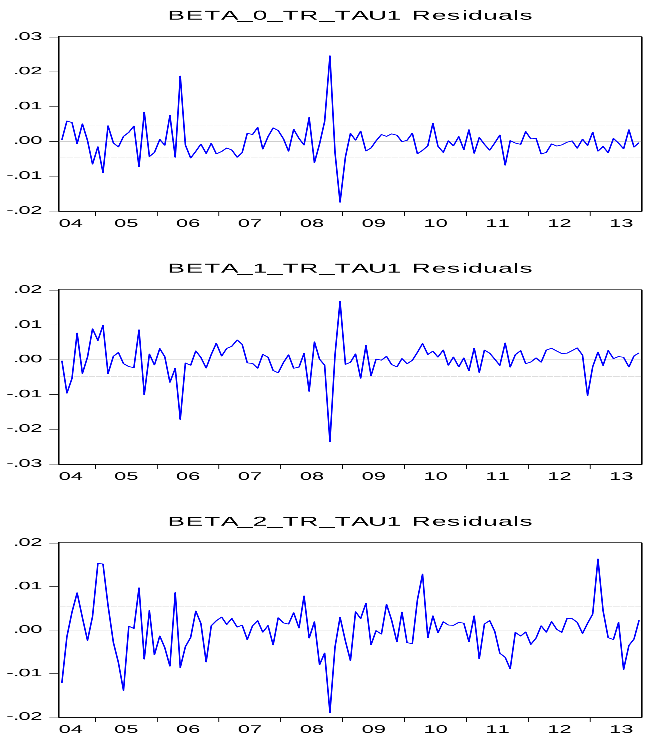


Table 16. VEC Residual Heteroskedasticity Tests for TR Linked VAR/VEC model residuals

No Cross Terms (only levels and squares)				
Sample: 2004M06 2013M10				
Included observations: 113				
Joint test:				
Chi-sq	df	Prob.		
90.74194	24	0.0000		
Individual components:				
Dependent	R-squared	F(4.108)	Prob.	Chi-sq(4)
res1*res1	0.037299	1.046086	0.3870	4.214765
res2*res2	0.037872	1.062794	0.3786	4.279537
res3*res3	0.502671	27.29002	0.0000	56.80182
res2*res1	0.033829	0.945377	0.4408	3.822729
res3*res1	0.051042	1.452264	0.2219	5.767759
res3*res2	0.122324	3.763065	0.0066	13.82263

Table 17. Granger Causality Test for TR Linked Betas

VEC Granger Causality/Block Exogeneity Wald Tests			
Sample: 2004M06 2013M10			
Included observations: 113			
Dependent variable: D(BETA_0_TR_TAU1)			
Excluded	Chi-sq	df	Prob.
D(BETA_1_TR_TAU1)	0.158564	1	0.6905
D(BETA_2_TR_TAU1)	0.028837	1	0.8652
All	0.216904	2	0.8972
Dependent variable: D(BETA_1_TR_TAU1)			
Excluded	Chi-sq	df	Prob.
D(BETA_0_TR_TAU1)	0.544851	1	0.4604
D(BETA_2_TR_TAU1)	1.644135	1	0.1998
All	3.380919	2	0.1844
Dependent variable: D(BETA_2_TR_TAU1)			
Excluded	Chi-sq	df	Prob.
D(BETA_0_TR_TAU1)	0.032307	1	0.8574
D(BETA_1_TR_TAU1)	0.018847	1	0.8908
All	0.306757	2	0.8578

5.3. IPCA Linked Parameters

The information criterion indicates a VAR model in level with two lags, with the exception of SIC and HQ which indicate only one lag. We choose a one lag model for the VEC, since the variables are integrated to first order. The Johansen test indicates model one without a linear trend in the data and a cointegration equation without trend and not intercept (AIC). Table 18 exhibits the VAR/VEC model estimated for IPCA Linked parameters. For this curve the model for β_1 is the one who presents evidence that it has a long term relationship with its explanatory variables.

Table 18. VAR/VEC Estimation for IPCA Linked Betas

Vector Error Correction Estimates			
Sample: 2004M06 2013M10			
Included observations: 113			
Standard errors in ( )			
LR test for binding restrictions (rank = 1):			
Chi-square(1)	0.218558		
Probability	0.640141		
Cointegrating Eq:	CointEq1		
BETA_0_IPCA_TAU1(-1)	1.000000		
BETA_1_IPCA_TAU1(-1)	4.085861		
	(1.40269)		
BETA_2_IPCA_TAU1(-1)	-3.442527		
	(0.88415)		
Error Correction:	D(BETA_0_IPCA_TAU1)	D(BETA_1_IPCA_TAU1)	D(BETA_2_IPCA_TAU1)
CointEq1	0.000000	-0.025683	0.029216
	(0.00000)	(0.00635)	(0.01413)
D(BETA_0_IPCA_TAU1(-1))	0.194139	0.063816	-0.037820
	(0.09909)	(0.27002)	(0.56481)
D(BETA_1_IPCA_TAU1(-1))	0.160778	0.210755	-0.415793
	(0.04085)	(0.11130)	(0.23282)
D(BETA_2_IPCA_TAU1(-1))	0.057907	0.094559	-0.161944
	(0.02242)	(0.06109)	(0.12778)
R-squared	0.121237	0.205854	0.089431
Adj. R-squared	0.097051	0.183997	0.064369
Sum sq. resids	0.001073	0.007967	0.034860
S.E. equation	0.003137	0.008549	0.017883
F-statistic	5.012685	9.418146	3.568447
Log likelihood	493.0661	379.7902	296.3951
Akaike AIC	-8.656037	-6.651154	-5.175135
Schwarz SC	-8.559493	-6.554610	-5.078590
Mean dependent	-0.000254	-0.000235	-6.53E-05
S.D. dependent	0.003302	0.009464	0.018488
Determinant resid covariance (dof adj.)		1.35E-13	
Determinant resid covariance		1.21E-13	
Log likelihood		1199.436	
Akaike information criterion		-20.96347	
Schwarz criterion		-20.60143	

Figure 12 shows that the residuals of the model may present some structure in their variance, however the possibility for serial correlation is discarded according to the test in table 19. Again, the White test in Table 20 led us to reject the joint heteroscedasticity, but with some pattern for res1 and the cross term res1 and res2, that may be object of future research.

Table 19. VEC Residual Serial Correlation LM Tests for IPCA Linked VAR/VEC model residuals

Null Hypothesis: no serial correlation at lag order h		
Sample: 2004M06 2013M10		
Included observations: 113		
Lags	LM-Stat	Prob
1	6.176016	0.7222
2	12.48072	0.1875
3	13.04542	0.1606
4	6.160258	0.7238
5	3.471929	0.9426
6	6.492640	0.6898
7	9.208368	0.4183
8	11.88845	0.2197
9	9.881253	0.3602
10	10.36918	0.3214
11	5.168670	0.8194
12	18.81727	0.0268
Probs from chi-square with 9 df.		

Figure 12. Residual Analysis for IPCA Linked VAR/VEC model

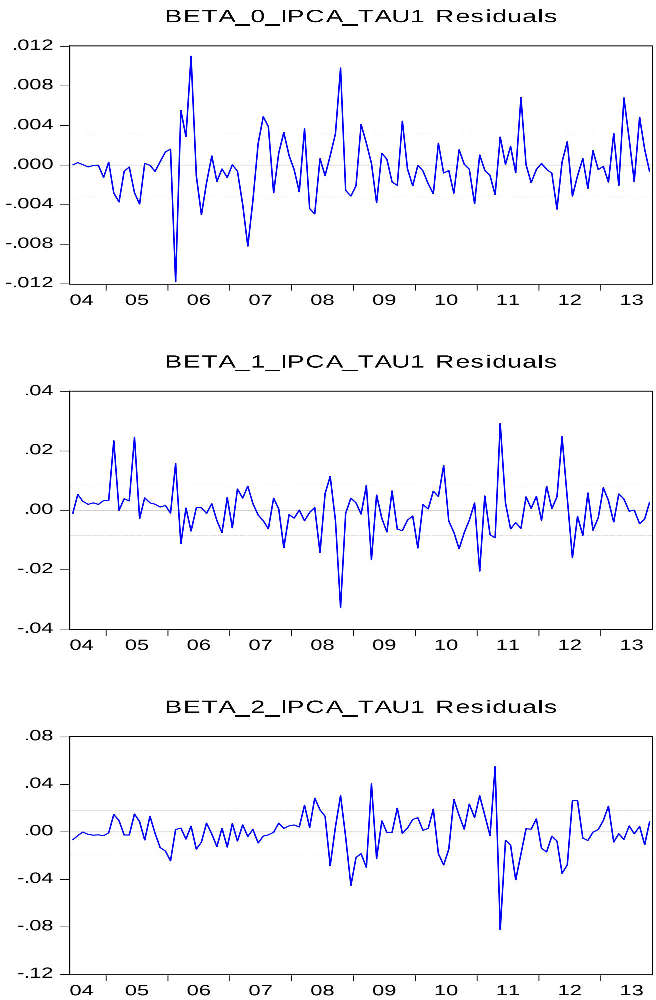


Table 20. VEC Residual Heteroskedasticity Tests for TR Linked VAR/VEC model residuals

Sample: 2004M06 2013M10					
Included observations: 113					
Joint test:					
Chi-sq	df	Prob.			
84.56649	48	0.0009			
Individual components:					
Dependent	R-squared	F(8.104)	Prob.	Chi-sq(8)	Prob.
res1*res1	0.026940	0.359910	0.9392	3.044170	0.9316
res2*res2	0.182129	2.894918	0.0059	20.58052	0.0083
res3*res3	0.474043	11.71684	0.0000	53.56685	0.0000
res2*res1	0.079863	1.128338	0.3506	9.024568	0.3402
res3*res1	0.205051	3.353249	0.0019	23.17076	0.0032
res3*res2	0.405559	8.869280	0.0000	45.82815	0.0000

Table 21 shows Granger causality test which indicates no causality between variables, β_1 and β_2 in β_0 and β_0 and β_2 in β_1. However, there is Granger causality evidence of β_1 and β_2 in β_0 for the IPCA curve parameters.

Table 21. Granger Causality Test for TR Linked Betas

VEC Granger Causality/Block Exogeneity Wald Tests			
Sample: 2004M06 2013M10			
Included observations: 113			
Dependent variable: D(BETA_0_IPCA_TAU1)			
Excluded	Chi-sq	df	Prob.
D(BETA_1_IPCA_TAU1)	15.49405	1	0.0001
D(BETA_2_IPCA_TAU1)	6.672132	1	0.0098
All	15.55760	2	0.0004
Dependent variable: D(BETA_1_IPCA_TAU1)			
Excluded	Chi-sq	df	Prob.
D(BETA_0_IPCA_TAU1)	0.055857	1	0.8132
D(BETA_2_IPCA_TAU1)	2.396101	1	0.1216
All	2.524260	2	0.2831
Dependent variable: D(BETA_2_IPCA_TAU1)			
Excluded	Chi-sq	df	Prob.
D(BETA_0_IPCA_TAU1)	0.004484	1	0.9466
D(BETA_1_IPCA_TAU1)	3.189549	1	0.0741
All	3.682444	2	0.1586

6. CONCLUSIONS

This paper aims to estimate the parameters of the Nelson Siegel model for Brazilian monthly yield curves for the period 2004-2013, encompassing the major asset classes traded in the market (Fixed Rate, IGPM, TR, IPCA and Exchange Rate linked). We estimated the parameters for all assets classes using NLLS in a monthly basis, generating time series for τ and the three βs (level, inclination and curvature). As the models with variable τ resulted in too volatile models, we decided to fix τ in the model and estimate only the βs. Besides, we noticed that the estimated values for the Exchange Rate Linked resulted in too many regressions with low F-tests and therefore we decided not to model this asset class. In the last part we modeled the β parameters for fixed rate, TR and IPCA linked curves with interesting results. Despite some limitations of the estimated models, they remain as a possible object for future research, modeling the residuals variance using GARCH or similar models and also with the use of the Kalman filter for the Brazilian curves.

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