VALUING STRATEGIC INVESTMENTS UNDER STOCHASTIC INTEREST RATES: A REAL OPTION APPROACH

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Abstract

One of the most challenging issues in management is the valuation of strategic investments. In particular, when undertaking projects such as an expansion or the launch of a new brand, or an investment in R&D and intellectual capital, which are characterized by a long-term horizon, a firm has also to face the risk due to the interest rate. In this work, we propose to value investments subject to interest rate risk using a real options approach (Schulmerich, 2010). This task requires the typical technicalities of option pricing, which often rely on complex and time-consuming techniques to value investment projects. For instance, Schulmerich (2010) is, to the best of our knowledge, the first work where the interest rate risk is considered for real option analysis. Nevertheless, the valuation of investment projects is done by employing binomial trees, which are computationally very expensive. In the current paper, a different modeling framework (in continuous-time) for real option pricing is proposed which allows one to account for interest rate risk and, at the same time, to reduce computational complexity. In particular, the net present value of the cash inflows is specified by a geometric Brownian motion and the interest rate is modeled by using a process of Vasicek type, which is calibrated to real market data. Such an approach yields an explicit formula for valuing various kinds of investment strategies, such as the option to defer and the option to expand. Therefore, the one proposed is the first model in the field of real options that accounts for the interest rate risk and, at the same time, offers an easy to implement formula which makes the model itself very suitable for practitioners. An empirical analysis is presented which illustrates the proposed approach from the practical point-of-view and highlights the impact of stochastic interest rates in investment valuation.

Keywords: Real Options, Investment projects, Stochastic Interest Rates

1. INTRODUCTION

One of the most popular techniques for valuing strategic investments such as an expansion or the launch of a new brand, or an investment in R&D and intellectual capital, is the use of a real options approach (Santos et al., 2014; Kalhagen & Elnegaard, 2002; Manley & Niquidet, 2010; Bernardo et al., 2012; Trang et al,. 2002; Chen et al., 2009; Erbas & Memis, 2012; Gamba & Fusari, 2009; Gibson & Schwartz, 1990; Huchzermeier & Loch, 2001; Liu et al., 2014; Lund, 2005; Paddock et al., 1988; Quigg, 1993; Taş & Ersen, 2012; Williams, 1991).

The present paper is devoted to assessing the impact of stochastic interest rates in real option pricing. The importance of considering non-constant interest rates in investment decisions is due to the fact that real options usually have a relatively long maturity and thus are strongly affected by the fluctuations of interest rates.

The relevance of this topic has been recently pointed out in Schulmerich (2010), where the pricing of real options under stochastic interest rates is considered. In his book, Schulmerich proposes to value real options using binomial trees. In our paper instead, in order to avoid numerical approximations, we assume that the interest rates follow the popular Vasicek model (Vasicek, 1977) and we manage to price real options using an exact closed-form solution. In particular, such an analytical formula is that obtained in Rabinovitch (1989), where it is applied to the pricing of financial options. In the present work, we apply the formula to the case of real options and to keep the analysis as close as possible to reality we calibrate the parameters of the Vasicek model to the Euribor/Eurirs indexes.

We point out that in Ballestra et al. (2017) we used the Rabinovitch approach in order to analyze the effect of stochastic interest rates on the valuation of investment projects. In particular, for the interest rate data and the option parameters (correlation between revenues and interest rates, project maturity, volatility) considered in that work, such an effect was found to be not significant. In the present paper, by performing a comprehensive investigation where the option parameters are varied, by considering a different time interval for the interest data and by taking into account a specific case-study, we show instead that interest rates can considerably affect real option valuation in some circumstances (see Sections 4 and 5).

In particular, we focus our attention on the practical application problem described in Santos et al. (2014). The results obtained in such a case study reveal that the effect of the stochastic interest rates strongly depends on the correlation between the interest rates themselves and the net present value of the future cash inflows. In particular, if the correlation parameter is large and positive, the value of the investment obtained in the case of stochastic interest rates is significantly smaller than that obtained under constant interest rates, whereas if the correlation parameter is large and negative, the value of the investment obtained in the case of stochastic interest rates is significantly bigger than that obtained under constant interest rates.

Moreover, if the correlation parameter is close to zero the stochastic interest rates have only a very small influence on the value of the investment. In addition, we also find that the stochastic interest rates have a more pronounced effect on the project valuation when the volatility and the time left to maturity are large.

Finally, an empirical study is presented in which both the investment cost and the net present value of the future cash inflows depend on the investment date. Such an analysis reveals that the stochastic interest rates affect the value of the project, but not the optimal time to invest.

The paper is organized as follows: in Section 2 some of the most common examples of real options are described; in Section 3 the basic facts about option pricing under both the Black-Scholes and the Rabinovitch models are recalled; in Section 4 an empirical analysis of the effect of stochastic interest rates is performed; in Section 5 a practical application study is presented; Section 6 discusses the main limitations of the study conducted in the present paper; finally, in Section 7 some conclusions are drawn.

2. REAL OPTION PRICING UNDER STOCHASTIC INTEREST RATES

Real option pricing is a very useful and popular tool to value investment decisions that are strongly characterized by uncertainty and managerial flexibility.

Here, we illustrate some examples of investment projects in various fields. These projects are characterized by different times to maturity, that range from one to thirty years, different values of the net present value of the future cash inflows, different investment costs and different volatilities of the future cash inflows, that range from 8% to 60%. Despite some heterogeneities, all these projects are usually valued using the real option approach. In the following, we provide a brief description of these projects while in Sections 4 and 5 we analyze in detail the first two projects of the following list:

- Santos et al. (2014) consider a firm that at the current time t_0 has the opportunity to invest at a time T in a mini-hydro plant. The project requires an investment equal to C_T (to be made at the time T) and generates random future cash inflows whose net present value is currently equal to S_{t_0}. The option of deferring the project is justified by the high uncertainty of electricity prices in the open market. Indeed, upon the current economic crisis, governments in Europe believe that the support given to electricity generation from renewable resources (as is the case of a hydro plant) is no longer a priority. This implies that the remuneration, or cash inflow, of the hydro plant, does not remain constant over time but it is instead affected by market conditions. It follows that the project is characterized by a certain amount of volatility which is due to the volatility of the prices of electricity. Santos et al. 2014, by means of Monte Carlo simulations, estimate project volatility of 40%.
- Taş and Ersen (2012) face the issue of assessing an investment in a solar energy plant. Due to uncertain economic conditions in the renewable energy market, the net present value future cash inflows generated by the solar energy plant S_{t_0} and the investment costs C_T will change substantially in the future years and will depend on the time T at which the project will be launched. Then, Taş and Ersen (2012) using a real options approach to determine the optimum time to undertake the investment given volatility of the future cash inflows equal to 15,79%.
- Kalhagen and Elnegaard (2002) and Charalampopoulos et al. (2011) examine the case of an incumbent's investment decision to upgrade its telecommunication services from an asymmetric digital subscriber line (ADSL) to a very-high-speed digital subscriber line (VDSL). The ADSL is a technology that enables data transmission over conventional copper telephone lines, while the VDSL is a faster technology that relies on fiber optic cabling. The case study concerns a dominant incumbent telecommunication company that offers ADSL services in a suburban area and that has the opportunity to invest at a time T in a VDSL upgrade at a cost C_T. This investment outlay will generate future cash inflows having a net present value S_{t_0}. The volatility of future cash inflows is estimated to be relatively high and equal to 60%.

- Manley and Niquidet (2010) consider a firm that has the opportunity to harvest a forest in New Zealand. In particular, since it is possible to collect timber only at the optimum rotation age, there is a relatively long elapse of time before the forest can be harvested. It follows that the firm has to consider the option to harvest in a certain future year versus do not harvest at all. The volatility of the future price of timber is estimated to be equal to 8%.
- Bernardo et al. (2012) consider the example of a computer firm that at the current time has the opportunity to immediately invest an amount of money in a hardware project which generates a negative expected net present value. Nevertheless, this investment also gives the firm an option to invest in an additional hardware project at a future time T. Such a second investment has a cost C_T, net present value S_{t_0} and relatively high volatility equal to 52%.
- Trang et al. (2002) describe an R&D project in the pharmaceutical industry. In particular, they consider the case of Nihon Schering, which once their drug Y (the name used for convenience) gets the approval from the government, is allowed to undertake further research at a future time T for developing other drugs similar to Y. Trang et al. (2002) assume that the total cash inflow resulting from the growth opportunity is equal to a certain fraction of the total cash inflow due to drug Y, while the cost of the investment to be made at the time T is equal to C_T. The volatility of the project is assumed to be equal to 35%.

In all the above cases, the investment valuation requires us to calculate the expected net present value of the future cash inflows, S_{t_0}, minus the investment cost C_T. Then, taking into account the possibility of not running the project once the maturity T is reached if the economic conditions are not favorable, we have to compute:

$$E\left[e^{-\int_{t_0}^{T} r_t dt} max\,(S_T - C_T, 0)\right] \tag{1}$$

The above-expected value can be evaluated using the explicit formula described in the next section.

3. REAL OPTION PRICING UNDER CONSTANT AND STOCHASTIC INTEREST RATES

In the Black-Scholes framework, see Black and Scholes (1973), it is assumed that the future cash inflows resulting from the investment to be valued follow the geometric Brownian motion:

$$dS_t = \alpha S_t dt + \sigma S_t dZ_{1,t} \tag{2}$$

where α and σ are the (constant) drift and the (constant) volatility of the future cash inflows, respectively, and $Z_{1,t}$ is a standard Wiener process (Øksendal, 2016) for a detailed explanation of stochastic processes and Wilmott (1998) and Hull (2014) for an overview of the Black-Scholes model. In addition, let r_f denote the spot interest rate, which is assumed to be constant and let C_t be the cost of the investment to be made at a future date T, so that $\tau = T - t_0$ represents the so-called time to maturity, t_0 being the current time. Then, under these assumptions, the expected value (1) can be computed using the popular Black-Scholes formula:

$$V_{BS}(\tau, S) = S_{t_0} N(d_1) - C_T e^{-r_f \tau} N(d_2) \tag{3}$$

Where

$$d_1 = \frac{\ln\left(\frac{S_{t_0}}{C_T}\right) + \left(r_f + \frac{1}{2}\sigma^2\right)\tau}{\sigma\sqrt{\tau}}, \qquad d_2 = d_1 - \sigma\sqrt{\tau} \tag{4}$$

Let us observe that in (3) $e^{-r_f \tau}$ is the price P_{BS} of a bond that pays one Euro at the future time T, i.e. under the assumption of constant interest rates, we have:

$$P_{BS} = e^{-r_f \tau} \tag{5}$$

Nevertheless, the dynamics of the interest rates in the last decade suggests that they experience significant random variations and volatility. Therefore, in order to perform an accurate investment valuation in a world dominated by turbulent financial markets, it is essential to take into account the stochastic dynamics of interest rates, which is done in the present paper. In particular, we assume that the spot interest rate follows a mean-reverting process of Vasicek type:

$$dr_t = q(m - r_t)dt + v dZ_{2,t} \tag{6}$$

Where q, m and v are the so-called speed of mean reversion, long-run mean and instantaneous volatility, respectively (q, m and v are assumed to be constant), and $Z_{2,t}$ is a standard Wiener process having a constant correlation ρ with $Z_{1,t}$.

Based on the geometric Brownian motion (2) and the Vasicek process (6), the value of a real option can be determined using the closed-form solution as obtained in Rabinovitch (1989):

$$V_{RAB}(\tau, S_{t_0}, r_{t_0}) = S_{t_0} N(D_1) - C_T P_{RAB}(\tau, r_{t_0}) N(D_2) \tag{7}$$

Where

$$\begin{aligned} D_1 &= \frac{\left(\ln\left(\frac{S_{t_0}}{C_T P_{RAB}(\tau, r_{t_0})}\right) + \frac{\vartheta}{2}\right)}{\sqrt{\vartheta}}, \\ D_2 &= D_1 - \sqrt{\vartheta}, \\ \vartheta &= \sigma^2\tau + \left(\tau - 2B + \frac{(1 - e^{-2q\tau})}{2q}\right)\left(\frac{v}{q}\right)^2 \\ &\quad - \frac{2\rho\sigma(\tau - B)v}{q}, \end{aligned} \tag{8}$$

and $P_{RAB}(\tau, r_{t_0})$ is the price at the time t_0 of a bond that pays one Euro at time T computed based on the stochastic process (6). The value of $P_{RAB}(\tau, r_{t_0})$ has been found by Vasicek (1977):

$$P_{RAB}(\tau, r_{t_0}) = A e^{-r_{t_0} B} \tag{9}$$

Where

$$\begin{aligned} B &= \frac{1 - e^{-q\tau}}{q}, \\ A &= e^{k(B - \tau) - \frac{v^2 B^2}{4q}}, \\ k &= m + \frac{v\lambda}{q} - \frac{v^2}{2q^2}, \end{aligned} \tag{10}$$

and λ is the market price of risk. For the sake of simplicity, in the present work we set $\lambda = 0$. It is worth noting that formula (7) differs from the Black-Scholes one as the term ϑ takes into account not only the volatility of the future cash inflows but also the volatility of the interest rate and its covariance with the future cash inflows (compare (4) and (8)).

4. AN EMPIRICAL ANALYSIS OF THE EFFECT OF STOCHASTIC INTEREST RATES

We consider the case study reported by Santos et al. (2014), which has already been briefly described in Section 2. On such a benchmark problem, we perform a detailed investigation that shows how the value of the investment varies when the interest rate is modeled according to the Vasicek process (6).

To perform an accurate and reliable evaluation of the effect of the stochastic interest rates, the parameters of the Vasicek model are calibrated to real market data. Specifically, we consider the Euribor/Eurirs indexes in a time period ranging from one month to thirty years, which yields:

$$r_{t_0} = 0.0013, q = 0.41, m = 0.0094, v = 0.0123 \quad (11)$$

Let us compare the project valuation obtained using the Black-Scholes formula (3) with that obtained using the Rabinovitch formula (7). To this aim, by imposing that relations (5) and (9) yield the same bond value, the interest rate to be used in the Black-Scholes formula is obtained as:

$$r_f = -\frac{\log\left(P_{RAB}(\tau, r_{t_0})\right)}{\tau},$$

where $P_{RAB}(\tau, r_{t_0})$ is the bond price calculated as in (9).

Santos et al. (2014) consider the following real option parameters:

$$S_{t_0} = 881371,\ C_T = 830000, \tau = \frac{1825}{252}, \sigma = 0.4 \quad (12)$$

Figure 1. Effect of the correlation coefficient on the real option value

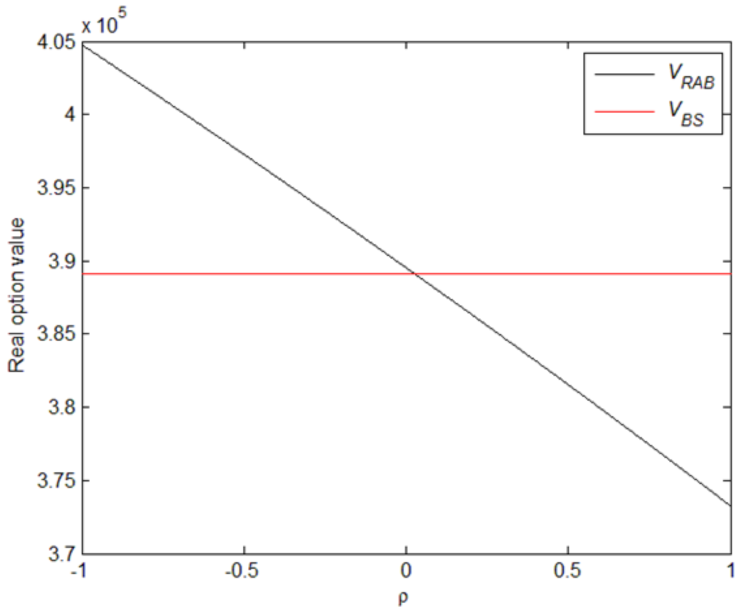
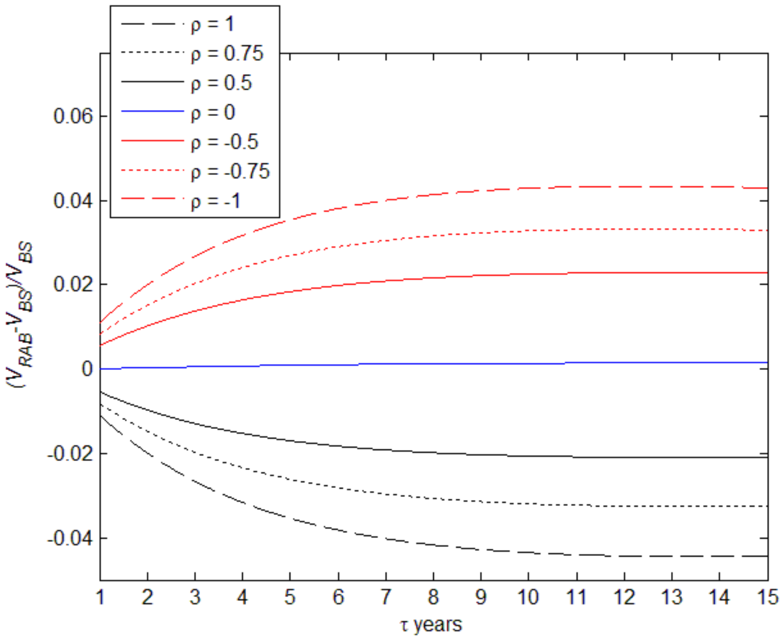


Figure 1 shows the real option price for different values of the correlation coefficient. We note that for values of ρ that are small in magnitude (approximately, $-0.1 < \rho < 0.1$) there is no significant difference between the Black-Scholes formula and the Rabinovitch formula. Instead, for values of ρ that are negative and large in magnitude (approximately, $\rho < -0.75$), the Black Scholes formula significantly underprices the real option, i.e. $V_{BS} < V_{RAB}$, whereas for values of ρ that are positive and large in magnitude (approximately, $\rho \geq 0.75$) the Black-Scholes formula significantly overprices the real option, i.e. $V_{BS} < V_{RAB.}$

This fact has the following economic explanation: if ρ is negative and large in magnitude then to the higher values of S_T (those that are bigger than C_T and hence concur to determine the option value, see (1)) correspond lower interest rates and therefore the present value of the investment tends to be higher.

Let us now vary the time to maturity. The results obtained are reported in Figure 2. Again, as experienced before, for values of ρ that are small in magnitude there is no significant difference between the Black-Scholes formula and the Rabinovitch formula. Instead, for values of ρ that are negative and large in magnitude we have $V_{BS} < V_{RAB}$ and the relative difference decreases as τ increases, whereas for values of ρ that are positive and large in magnitude we have $V_{BS} < V_{RAB}$ and the relative difference increases as τ increases. However, we can see that the differences between V_{BS} and V_{RAB} are appreciable only for times to maturity larger than approximately two years (we are assuming that the relative difference between V_{RAB} and V_{BS} is appreciable only if it is greater than 2% in magnitude).

Figure 2. The gap in the value of the real option between the Black-Scholes formula and the Rabinovitch formula as a function of the time to maturity



Let us conclude the sensitivity analysis by varying the volatility of future cash inflows. This is an important aspect as σ is a characteristic parameter of any project and it is not easy to determine.

As we can observe by comparison of Figure 3, Figure 4 and Figure 5, the qualitative shape of the curves obtained does not significantly depend on the time to maturity. On the contrary, the time to maturity can significantly affect the value of the relative difference between V_{BS} and V_{RAB}. In particular, if $\tau = 1$ and $|\rho| = 1$ the relative difference between V_{BS} and V_{RAB} has a maximum equal to approximately 0.0225 and a minimum equal to approximately -0.0225, whereas if $\tau = 15$ and $|\rho| = 1$ the relative difference between V_{BS} and V_{RAB} has a maximum equal to approximately 0.13 and a minimum equal to approximately -0.16.

However, the relative difference between V_{BS} and V_{RAB} are significant, i.e. greater than 2% in magnitude, only for values of, τ, ρ and σ which are large in magnitude, say $\tau \geq 2$, $\rho \leq -0.5$, or $\rho \geq 0.5$ and $0.05 \leq \sigma \leq 0.6$.

Figure 3. The gap in the value of the real option between the Black-Scholes formula and the Rabinovitch formula as a function of the volatility of the future cash inflows, $\tau = 1$

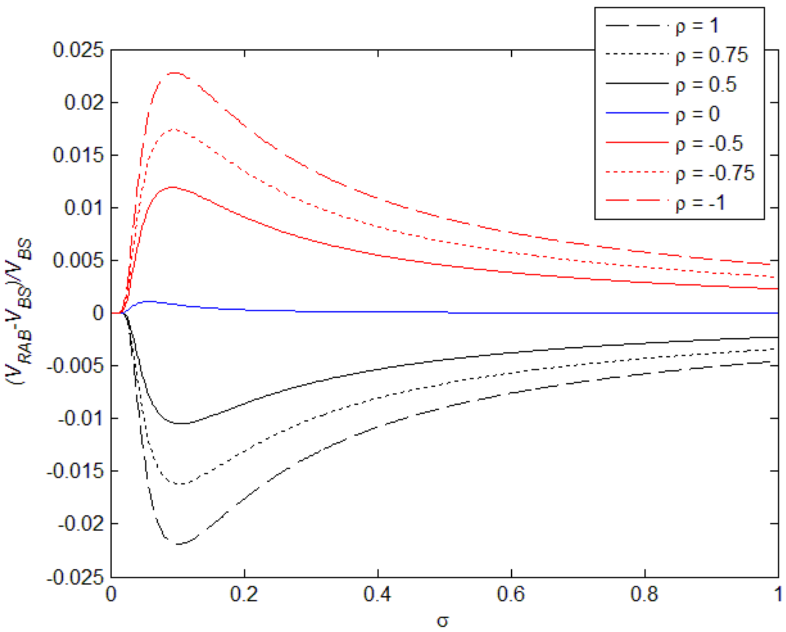


Figure 4. The gap in the value of the real option between the Black-Scholes formula and the Rabinovitch formula as a function of the volatility of the future cash inflows, $\tau = 1825/252$

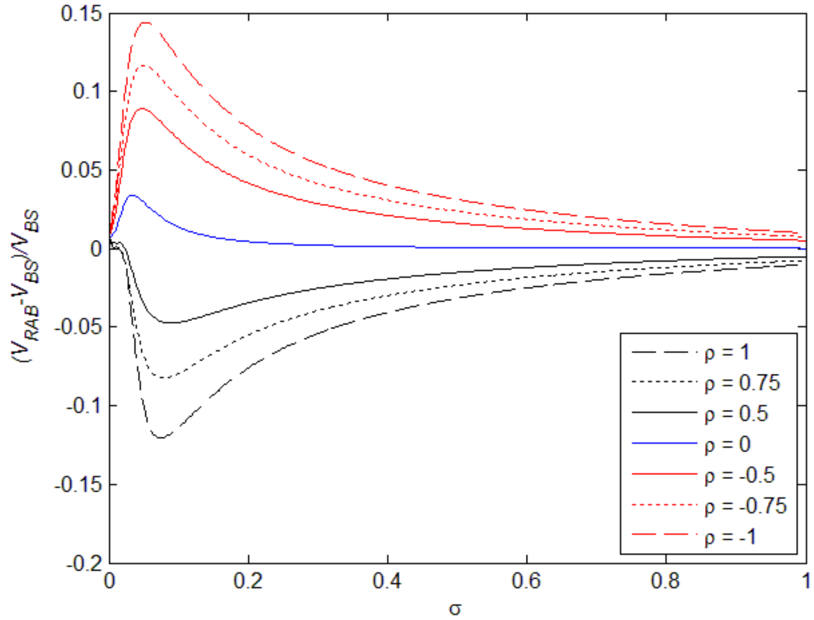
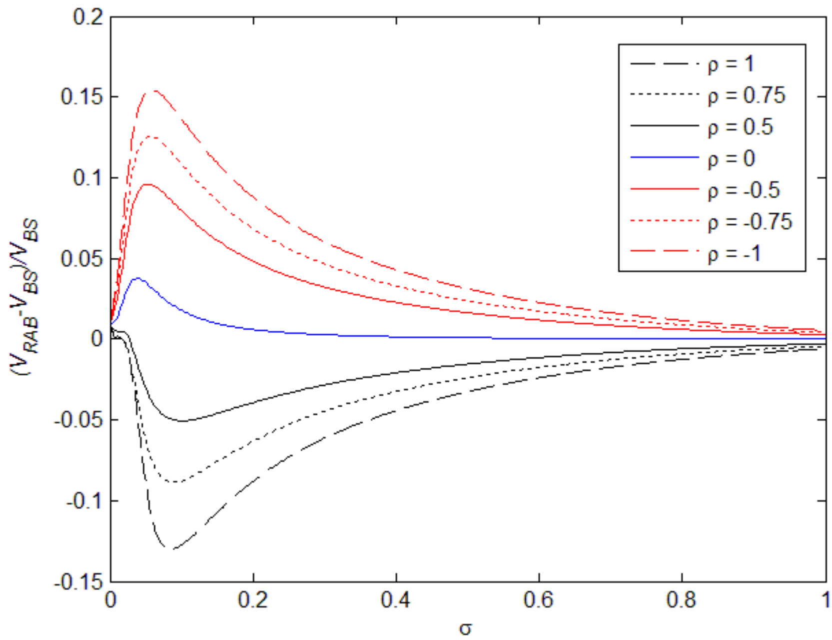


Figure 5. The gap in the value of the real option between the Black-Scholes formula and the Rabinovitch formula as a function of the volatility of the future cash inflows, $\tau = 15$



5. A PRACTICAL APPLICATION

Quite often when considering investment projects both S_{t_0} and C_T depend on the investment date T. This case is described, for example, in Taş and Ersen (2012) and briefly recap in section 2, where an investment in a solar energy plant is considered.

In the following, we deal with the issue of determining the optimal investment time when both S_{t_0} and C_T depend on T. In particular, we assume that S_{t_0} and C_T vary as reported in the following Table 1 (data as in Taş & Ersen, 2012):

Table 1. Real option data as in Taş and Ersen (2012) (Part 1)

τ	C_T	S_{t_0}
1	195000000	180272599
2	190125000	172143911
3	185371875	163400165
4	180737578	153937253
5	176219139	143641621
6	171813660	132388890
7	167518319	121740379
8	163330361	111661905
9	159247102	102121366
10	155265924	93088602
11	151384276	84612249
12	147599699	76657309
13	143909677	69191047
14	140311936	62182841
15	136804137	55604041

Following Taş and Ersen (2012), the volatility of future cash inflows σ is set to 0.1579. Moreover, the interest rate parameters are chosen as in Section 3 (see (12)).

Figure 6 shows the option value as a function of maturity τ when ρ is equal to 0.5. As we can observe, the value of the investment reaches its maximum when τ is equal to approximately four years. However, the stochastic interest rates do not significantly affect the value τ at which the value of the project is maximum, but the maximum value of the project. Similar behavior is experienced also when ρ is equal to -0.5, see Figure 7.

Figure 6. Value of the option to defer investment in a solar energy plant, $\rho = 0.5$

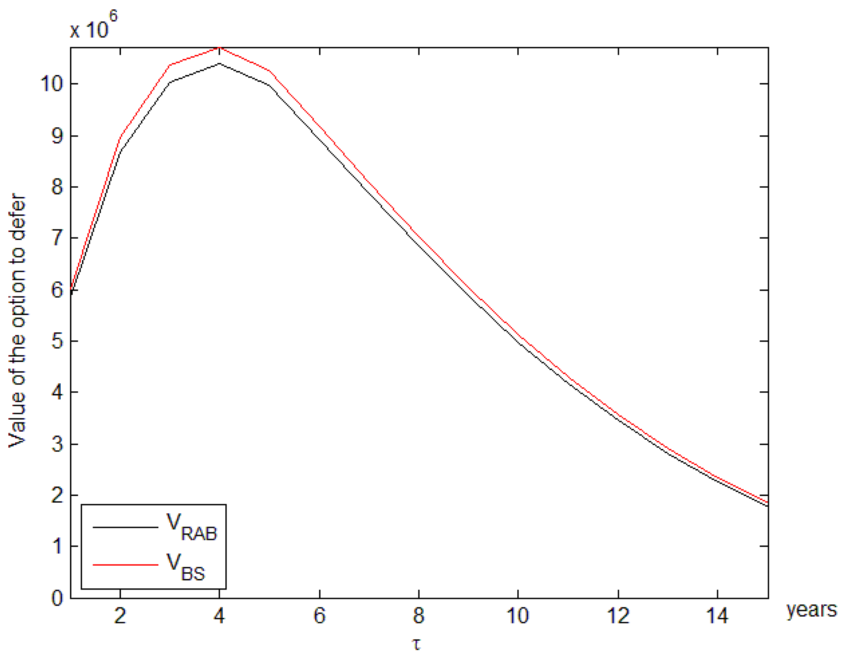
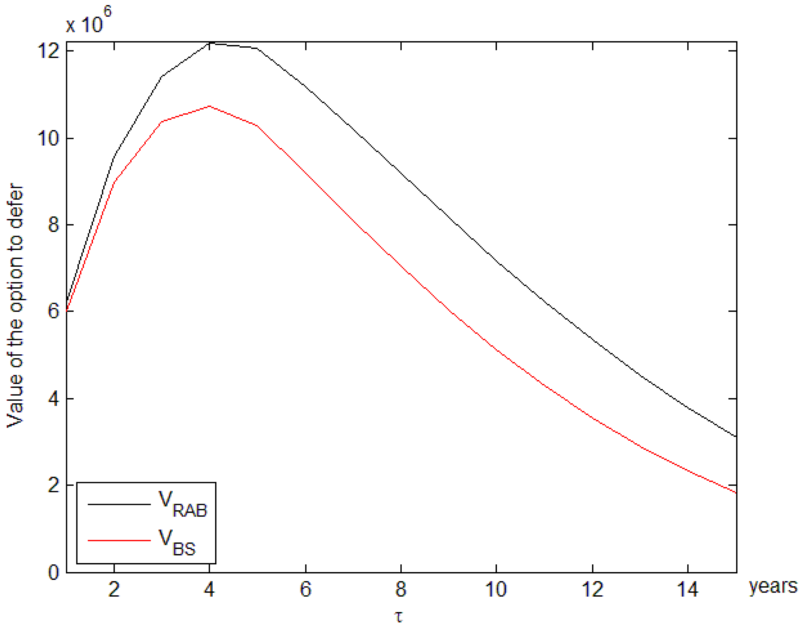


Figure 7. Value of the option to defer investment in a solar energy plant, $\rho = -0.5$



6. LIMITATIONS

First of all, the proposed approach is based on a quite simple stochastic interest rate model, the Vasicek's model, which has the disadvantage that interest rates are not prevented from becoming too small. Clearly, the possible occurrence of negative interest rate should be taken into account, nevertheless interest rates, albeit negative, cannot be too large in magnitude. More sophisticated short-term interest rate models are presented and discussed, for example, in Bringo and Mercurio (2007). Therefore, we shall acknowledge that the present work can be further developed by considering other kinds of stochastic interest rate models, such as the well-known CIR model, see Cox et al. (1985). It is worth pointing out, however, that most of such extensions require the use of numerical approximations to compute real option prices. Another limitation of the present study is the lack of data on a large scale to estimate the future cash flows that a project can generate. Specifically, this restricts the possibility of performing a systematic investigation of the effect of the interest rate volatility on strategic projects. A possible strategy to overcome this issue is to employ balance sheet data, so as to proxy the cash flows that projects undertaken by specific firms can generate.

7. CONCLUSION

Interest rates are a source of uncertainty which is important to take into account when valuing long term investments or investments that can be postponed in the future. In fact, the time variations of the interest rates can substantially affect the value of a real option to defer or expand.

In this paper interest rates are modeled using the popular Vasicek stochastic process, which allows us to price real options by means of a closed-form solution.

Such an analysis yields quite interesting results. In fact, the stochastic interest rates have a significant effect on the investment value when the correlation between the interest rates themselves and the net present values of the cash inflows, the time to maturity and the volatility of the future cash inflows are large (in magnitude). By contrast, for small values of the correlation parameter, the stochastic interest rates do not substantially affect the project valuation.

This has interesting practical implications. In fact, for companies that operate in the banking or insurance sector, where revenues can be strongly correlated with interest rates, we can expect differences between the Black-Scholes and Rabinovitch models. Therefore, in such a case, our suggestion is to use the most complex model to make the assessment. On the other hand, for those cases where interest rates are not expected to be highly correlated with future revenues, the evaluation with the traditional Black-Scholes real options model appears to be adequate.

Further, an empirical case study is considered where both the investment cost and the net present value of the future cash inflows depend on the time at which the project is undertaken. Such an investigation highlights that the presence of stochastic interest rates does not significantly modify the optimal time to invest.

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