

MODELLING EA BANKS DEFAULT RATES WITH JOINTLY SPANNED AND UNSPANNED INTEREST RATES AND UNSPANNED BEI RATES

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Abstract

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The paper quantifies the influence of interest rates and inflation rates on default rates of banks. By expanding the work of Duffee (1998), with the unspanned risks as in the work of Joslin, Priebsch, and Singleton (2014), we estimate a multifactor model with unspanned interest rates and inflation rates to test the performance of unspanned variables in the default rate term structure of banks. The model is trained in samples of positive interest rates and evaluated in samples of negative interest rates. We check the robustness of the model by comparing the results with the performance of alternative model specifications. The model reveals that unspanned variables have worse performance than alternative models specifications. The negative effect of interest rates on default rates over longer maturities may lead the EA banks to decrease the loan supply to the real economy. As a consequence EA banks will have a lower net interest margin as the return of assets is lower. This may increase the future probability of default. Thus, the solution for EA banks is on the reach to yield behavior as described by Bruno and Shin (2015). This means that EA banks have to modify the allocation of assets more in favour of riskier and longer maturity securities to obtain higher profitability.

Keywords: Default Rates, Interest Rates, Inflation Rates, Term Structure, Kalman Filter, Out of Sample, Model Specification

Authors' individual contribution: The author is responsible for all the contributions to the paper according to CRediT (Contributor Roles Taxonomy) standards.

1. INTRODUCTION

The resilience of the European banking system is crucial to absorb shocks arising from the financial and economic environment and to reduce spillovers from the financial sector to the real economy as the consequences of the Great Recession on the real economy. The failure of Lehman Brothers and the breakdown of the Libor-OIS basis as a consequence of an increased counterparty risk perception in the European interbank market made spillover effects in the real economy. As a consequence, euro area (EA) banks had liquidity shortages that boosted higher default rates as shown by Cucinelli (2013). In this scenario, the Zero Lower Bound interest rates in Europe, as a result of the unconventional monetary

policy implemented by the European Central Bank (ECB), put huge strains for the Euro area banking system. Two are the main source of risk for the overall stability of the European banking system:

- Persistently low rates erode the profitability of banks, lowering net interest margin (NIM) (Claessens, Coleman, & Donnelly, 2018; Borio & Gambacorta, 2017) lead EA banks to rebalance their portfolio towards riskier assets (Bottero et al., 2019).
- Unprecedented low European interest rates lead European inflation rates to remain low, below ECB desired level.

As a consequence, firms have difficulties to pay back their debt because of higher debt values in real terms (firms are affected by the debt deflation

phenomenon described by David (2008) as the previous findings of Fisher (1933). This leads EA banks to increase losses from borrower's default putting huge strains for the resilience of the European banking system. Furthermore, the unprecedented spillover effects of the Greek crisis on the Sovereign debt of EA countries on EA banks led them to decrease the capital ratios. This boosted EA banks default rates. The spread between banks corporate bonds and risk-free counterparts is one of the indicator factors for the development of the crisis. This quantity contains useful information for both policymakers and investors for the development of a crisis. This information is embedded in the term structure of the spread between banks corporate bonds and risk-free rates and in the dynamics of the related risk factors. Modelling the term structure of interest rates has always been of interest of a wide array of economic agents such as:

- risk managers of financial and non-financial institutions for pricing financial instruments and hedging financial and non-financial risks;
- policymakers and central bankers for forecasting the future value of interest rates and evaluating the impact of macro variables on them so as to implement more effective economic policies;
- financial regulators to simulate a macro scenario for setting new capital requirements.

These economic agents will use this term structure model to:

1. Identify the relations that occur between EA banks corporate bonds credit risk premia and macroeconomics to handle financial risks more efficiently in light of the recent macroeconomic environment.

2. Better evaluate the effect of macroeconomic variables in financial regulatory requirements (such as the CVA risk or the liquidity covered ratio) so as to implement more effective capital and asset allocation strategies.

3. Implement monetary policies more banking oriented like the LTROs (long term refinancing operations) to incentive EA banks to lend more financial resources to the real economy.

Therefore, investigating the impact of inflation rates on EA bank default rates in light of Zero Lower Bound interest rates environment, through a term structure model may help financial and economic agents to preserve the stability of the European banking system by understanding the interconnection among monetary policy, inflation rates and banking system in the euro area. The structure of the paper is as follows: Section 2 reviews the existing literature, Section 3 analyses the data and the methodology used for answering the research question, Section 4 presents the results in light of the ongoing macroeconomic scenario focusing the attention on the importance of the results for the resilience of the EA banks system, Section 5 concludes.

2. LITERATURE REVIEW

The relationship between default rates and interest rates is exhaustively investigated by existing financial literature (Duffee 1998, 1999; Longstaff & Schwarz, 1995; Collin-Dufresne, Goldstein, & Martin,

2001; Neal, Rolph, Dupoyet, & Morris, 2000). However, the impact of interest rates on default risk premium and the impact of inflation rates on the default risk of banks have not been yet analysed. Differently from the structural approach on credit risk and the macroeconomics (Gourio, 2012, 2013; Chen, 2010; Kang & Pflueger, 2015; Bhamra, Fisher, & Kuehn, 2011; Bhamra, Dorion, Jeanneret, & Weber, 2018; David, 2008), the approach followed by financial corporations (Bellini, 2017) and rating based approach of credit risk (Carty & Fons, 1994; Jarrow, Lando, & Turnbull, 1997), this paper follows a reduced form of approach to identify the impact of macroeconomic variables on default rates of banks. By extending the analysis made by Lucheroni and Pacati (2004) and by Mari and Renò (2005) as the development of the previous study of Duffie and Singleton (1999), this paper provides an empirical analysis on EA banks default rates and macroeconomics, highlighting the effect of inflation rates and interest rates on EA banks default rates. This work will merge two streams of literature: the one on MTSM (macro-term structure modelling) (Joslin, Singleton, & Zhu, 2011; Joslin et al., 2014; Dewachter & Iania, 2011; Dewachter, Iania, Lemke, & Lyrio, 2019), and the other on the affine term structure of credit risk modelling (Lando, 1998, 2004) to identify how the macroeconomic factors extracted from traded instruments (such as interest rate swaps or inflation swaps) affect the credit risk of EA banks over time. Analysing the determinants of bank credit risk over time is of great importance for the stability of the EA banking system. It is well known that credit risk is a determinant factor for bank profitability. As a consequence, the level of some macroeconomic variables has great importance for the economic performance and the portfolio allocation of EA banks. When the European interest rates decrease and the European inflation rates remain low, EA Banks, performing the asset transformation process have a negative impact on their profitability by investing in shorter maturities assets and less risky borrowers, as a consequence of a reduction on EA banks, the net interest margin (Cruz-Garcia, Fernández de Guevara, & Maudos, 2019; Eisenschmidt & Smets, 2019; Borio, Gambacorta, & Hoffman, 2017). As a result, the Zero Lower Bound European interest rates lead EA banks, likes US banks (Aramonte, Lee, & Stebonovs, 2019) to change the composition of the portfolio of the assets in favour of riskier securities, as described by Demiralp, Eisenschmidt, and Vlassopoulos (2019). Furthermore, the Zero Lower Bound European interest rates and the low European inflation rates lead EA banks to change the portfolio of liabilities in favour of issuing more debt conditionally to the financial regulation boundaries (liquidity coverage ratio and leverage ratio among others) (Lucas, Schaumburg, & Schwaab, 2019).

3. DATA AND METHODOLOGY

3.1. Interest rates, inflation rates and default rates description

We consider a panel of European OIS (overnight indexed swaps) rates from Datastream over the period from November 3, 2008 till November 3, 2016. For each day in our sample period, we

consider OIS rates for time to maturities 1-10 years, and 15, 20, 25 and 30 years. We fill in the missing integral maturities (e.g., 11, 12, ... years) by linear interpolation, as shown, for example, by Hagan and West (2006). We bootstrap short term interest rates following the procedure written by Bernhart (2013).

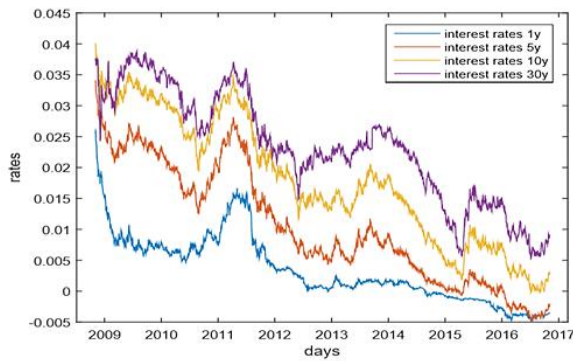
The (spot instantaneous) interest rates for each m are computed from the standard bootstrapping model:

$$s_m = \frac{1 - P(t, t + m)}{\sum_{i=1}^m P(t, t + 1)} \quad (1)$$

By inverting the OIS rates and solving them with respect to the discount factors, we find that the risk-free yields are:

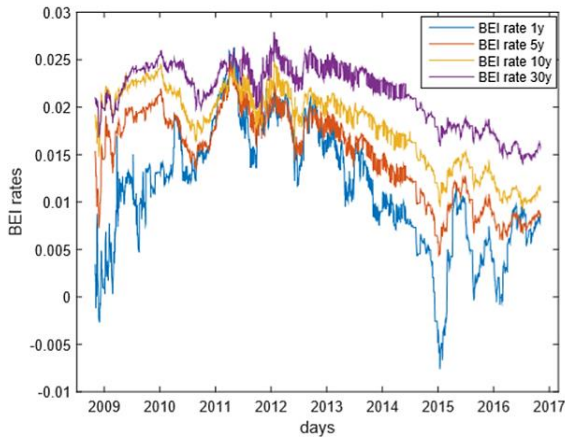
$$y(t, t + m) = -\frac{1}{n} \ln P(t, t + m) \quad (2)$$

Figure 1. Interest rates at 1y, 5y, 10y and 30y maturities



We consider a panel of European ZCIS (zero coupon inflation swaps) rates from Datastream over the period from November 3, 2008 till November 3, 2016. For each day in our sample period, we consider ZCIS rates for time to maturities 1-10 years, and 15, 20, 25 and 30 years. We fill in the missing integral maturities using linear interpolation. ZCIS rates are readily zero-coupon rates and no bootstrap is needed.

Figure 2. BEI (breakeven inflation) rates at 1y, 5y, 10y and 30y maturities



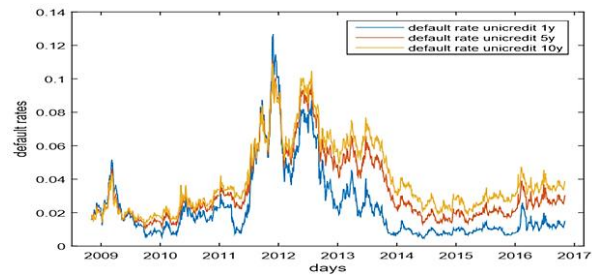
We consider a panel of standard CDS (credit default swaps) spreads from Datastream over the period from November 3, 2008 till November 3, 2016, written on the default of UniCredit bank.

Why Unicredit? Unicredit bank is the major banking group in Italy whose activity is widely spread around 32 countries. The leading markets in which Unicredit operates are Austria, Italy, Germany and East European countries and it is quoted in leading stock market indexes such as FTSE MIB (Italy) and DAX (Germany). As a consequence, it seems crucial to consider Unicredit bank performance as a proxy of the resilience of the EA banking system.

For each day in our sample period, we consider CDS rates for time to maturities 1-5 years, 7 and 10 years (standard CDS durations). We fill in the missing integral maturities using linear interpolation. We bootstrap the forward risk-neutral default intensities $y^d(t, t + m)$, following the procedure written by Castellacci (2008). For all maturities $m = 1, 2, \dots, M$, the (forward risk-neutral instantaneous spot) default rate:¹

$$y^d(t, t + m) = -\frac{1}{m} \ln Q(t, t + m) \quad (3)$$

Figure 3. Default rates at 1y, 5y and 10y maturities



The (instantaneous spot) zero-recovery risky rate is:

$$y^*(t, t + m) = -\frac{1}{m} \ln[Q(t, t + m)P(t, t + m)] = y^d(t, t + m) + y(t, t + m) \quad (4)$$

Default rate $y^d(t, t + m)$ is, therefore, the spread we have to add to the risk-free rate $y(t, t + m)$ to obtain the zero-recovery risky rate.

How many factors do we need to describe the zero-recovery risky rate and instantaneous default rates? How do interest rates and inflation rates affect instantaneous default rates?

By performing the principal component analysis (PCA) on data we find the following:

Table 1. Variability of the term structure explained by each PCA factor of risky yield (on the left) and on default rates (on the right)

Factors	Unicredit risky yield	Unicredit default rates
1	0.950	0.967
2	0.030	0.025
3	0.008	0.004
4	0.004	0.001
5	0.002	0.000
Tot var.	0.994	0.998

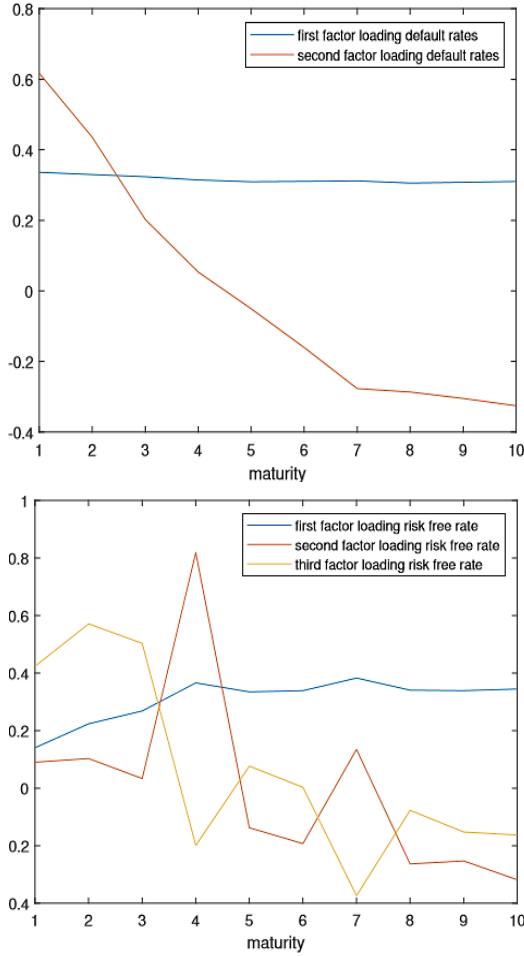
¹ $Q(t, T)$ be such that the price of a contingent claim paying 1 unit of cash at time T in case of non-default of the reference entity at time T is $Q(t, T)p(t, T)$. Therefore $Q(t, T)$ is the forward risk-neutral survival probability.

Therefore, the zero-recovery risky yield is described by five factors, where:

- two factors describe the evolution of the default rates;
- three factors are needed to describe the variability of risk-free rates.

Factor weights (factor loadings) of Unicredit default rates and Unicredit risky yield are presented below:

Figure 4. Factor loadings of Unicredit default rates (up) and risk-free rates (down)



As a consequence, inflation rates play no role in the evolution of the variability of zero-recovery bond yield over time.

3.2. The model

Following Lando (1998) and Duffie and Singleton (2003), we model a credit default event as the first jump of a Poisson process with stochastic intensity $\lambda(t)$. Under this assumption, at time t the survival probability at time T is following:

$$E_t \left[\exp^{-\int_t^T \lambda(u) du} \right] \quad (5)$$

Alternatively to more recent articles (Monfort & Renne, 2013), we model the default intensity as the sum of two Gaussian factors:

$$\lambda(t) = \lambda_1(t) + \lambda_2(t) \quad (6)$$

By setting $\Lambda(t) = (\lambda_1(t), \lambda_2(t))$, we assume the following risk-neutral dynamics:

$$d\Lambda(t) = A(\Gamma_\lambda - \Lambda(t) + Cr(t))dt + \Sigma dW^Q(t) \quad (7)$$

where $r(t)$ is the (nominal) risk free short rate;

$$A = \begin{bmatrix} \alpha_1 & 0 \\ 0 & \alpha_2 \end{bmatrix}; \Gamma_\lambda = \begin{bmatrix} \gamma_\lambda \\ 0 \end{bmatrix}; C = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}; \Sigma = \begin{bmatrix} \alpha_1 & 0 \\ 0 & \alpha_2 \end{bmatrix}$$

and dW^Q is a vector of correlated Wiener processes of this form:

$$dW^Q = \begin{bmatrix} dW_1^Q(t) \\ dW_2^Q(t) \end{bmatrix}, \quad \text{corr} \left(dW_1^Q(t), dW_2^Q(t) \right) = \rho_\lambda dt$$

We assume normally distributed default rates. The Gaussian distribution:

- has higher mathematical tractability than alternative distributions (Schönbucher, 2003);
- is used in the literature of default rates term structure models (Amato & Luisi, 2006; Wu & Zhang, 2008; Le Corlois & Nakagawa, 2013; Russo, Giacometti, & Fabozzi, 2017);
- gives a positive probability of negative default rates (but negligible if long term mean is positive and mean reversion speed is high with respect to the volatility);
- allows introducing macroeconomic variables in essentially affine models (Duffie, 2002). The alternative affine price of risk specification for non-Gaussian dynamics (extended affine models) proposed by Cheredito, Filipović, and Kimmel (2007) does not give improvements for multi-factor term structure models.

3.2.1. What is new in this model?

The default rate is stochastic, therefore, the credit risk is modelled by a so-called doubly stochastic intensity model as described by Lando (1998) and Duffie and Singleton (2003) extended for the macro variables. Alternatively to the recent default rates term structure models (Dewachter et al., 2019):

- Our credit risk term structure model considers the continuous time term structure modelling as presented by Duffie and Kan (1996) and not the discrete time modelling of interest rates (Le, Singleton, & Dai, 2010).
- Alternatively to other macro-finance term structure models, where both the risk-free part and the risky part of is extracted by the bond yield decomposition (Cochrane & Piazzesi, 2005), our model separates the risk-free part and the risky part of the bond yield by taking information from two different markets (CDS market from the risk premium and the money market for the risk-free part).
- We take macroeconomic variables from derivative instruments (OIS rates for interest rates and inflation swaps for inflation rates) at different maturities. We do not consider time series of macro variables from national statistical databases as done by others (see Amato and Luisi (2006) and Dewacher et al. (2019) as a reference for credit risk term structure modelling with macroeconomics variables).

- Alternatively to other macro-finance term structure models, we extract the embedded information of macro variables by modelling the term structure of such macro variables. We do not consider the time series dimension of the macro variables.

- We allow bond yields to be priced with an error. This makes the analysis of the term structure more realistic, as we consider the market microstructure noises (such as the bid-ask spread) that make bond yields difficult to observe.

Furthermore, according to Ang and Piazzesi (2003) and Pericoli and Taboga (2008), the dynamics of default rates is a mixture of unobserved factors and observed factors with a striking difference.

We don't consider factors from the PCA analysis as a reference to macroeconomic information to introduce in the term structure model. Alternatively, we use factors extracted from the estimated term structure of macroeconomic variables found in financial instruments regularly quoted in the financial markets.

3.2.2. Further assumptions

We make the following further assumptions:

Assumption 1 (inflation rate). We use the BEI rate (breakeven inflation) as a proxy of inflation expectations. Although the two quantities may not coincide, the choice of the BEI rate as a proxy of inflation expectations is widely used among market practitioners (Ciccarelli & Garcia, 2009).

$$\left\{ \begin{array}{l} \frac{\partial P^*}{\partial t} + \sum_{i=1}^2 \left(\alpha_i (\gamma_{\lambda_i} - \lambda_i + c_i(x_1 + x_2 + x_3)) \frac{\partial P^*}{\partial \lambda_i} + \frac{1}{2} \sigma_{\lambda_i}^2 \frac{\partial^2 P^*}{\partial \lambda_i^2} \right) + \sigma_{\lambda_1} \sigma_{\lambda_2} \rho_{\lambda} \frac{\partial P^*}{\partial \lambda_1 \partial \lambda_2} + \sum_{i=1}^3 \left(\kappa_i (\gamma_i - x_i(t)) \frac{\partial P^*}{\partial x_i} \right) + \\ \frac{1}{2} \sum_{i=1}^3 \left(\sigma_i^2 \frac{\partial^2 P^*}{\partial x_i^2} \right) + \frac{1}{2} \sum_{i=1}^3 \sum_{j=1}^3 \left(\sigma_i \sigma_j \rho_{ij} \frac{\partial^2 P^*}{\partial x_i \partial x_j} \right) = (\lambda_1 + \lambda_2 + x_1 + x_2 + x_3) P^* \\ P^*(T, x_1, x_2, x_3, \lambda_1, \lambda_2) = 1 \end{array} \right. \quad (9)$$

By standard arguments, the solution of this BVP is ($\tau = T - t$):

$$P^*(t, T) = \exp \left(A^d(\tau) - \sum_{i=1}^3 B_i^d(\tau) x_i(t) - \sum_{i=1}^2 C_i^d(\tau) \lambda_i(t) \right) \quad (10)$$

where $A^d(\tau)$, $B_i^d(\tau)$ and $C_i^d(\tau)$ are deterministic functions, solutions of a system of ODEs obtained in the usual way from our PDE:

$$\begin{aligned} \frac{\partial A^d(\tau)}{\partial \tau} &= -B_1^d(\tau) \kappa_1 \gamma_1 - C_1^d(\tau) \alpha_1 \gamma_1 + \\ &\frac{1}{2} \sum_{i=1}^2 \sigma_{\lambda_i}^2 C_i^d(\tau)^2 + \frac{1}{2} \sum_{i=1}^3 \sigma_i^2 B_i^d(\tau)^2 + \\ &\sum_{i,j=1}^3 \sigma_i \sigma_j \rho_{ij} B_i^d(\tau) B_j^d(\tau) + \sigma_{\lambda_1} \sigma_{\lambda_2} \rho_{\lambda} C_1^d(\tau) C_2^d(\tau) \end{aligned} \quad (11)$$

$$\frac{\partial B_i^d(\tau)}{\partial \tau} = -B_i^d(\tau) \kappa_i + \alpha_1 c_1 C_1^d(\tau) + \alpha_2 c_2 C_2^d(\tau) + 1 \quad (12)$$

$$\frac{\partial C_1^d(\tau)}{\partial \tau} = -C_1^d(\tau) \alpha_1 + 1 \quad (13)$$

$$\frac{\partial C_2^d(\tau)}{\partial \tau} = -C_2^d(\tau) \alpha_2 + 1 \quad (14)$$

The zero-recovery risky rate at time t for maturity T , therefore, is:

$$y^*(y, T) = -\frac{A^d(\tau)}{\tau} + \sum_{i=1}^3 \frac{B_i^d(\tau)}{\tau} x_i(t) + \sum_{i=1}^2 \frac{C_i^d(\tau)}{\tau} \lambda_i(t), \sum_{i=1}^2 \frac{C_i^d(\tau)}{\tau} \lambda_i(t) \quad (15)$$

Assumption 2 (interest rate and BEI rate). The short rate $r(t)$ is described by a three-factor Gaussian mean reverting models (G3). We assume the same functional form for BEI rates.

Assumption 3 (independence between interest rates and BEI rates). We assume that the Wiener's driving the interest rate dynamics are independent of the Wiener's driving the BEI dynamics.

Assumption 4 (independence between default rates and BEI rates). We let BEI rates to affect only the future P -expected default rates and not the survival probability. This choice is consistent with the market bootstrapping practice, where BEI rates play no role in the bootstrapping procedure from CDS spread.

3.2.3. Zero-recovery bond prices

According to Lando (1998), in this model the price at time t of a zero-recovery unit default table zero-coupon bond maturing in T is given by:

$$P^*(t, T) = E^Q_t \left[\exp^{-\int_t^T [\lambda(u) + r(y)] du} \right] \quad (8)$$

so that default intensity has the role of the short credit spread. The price of the bond is a function of λ_1 , λ_2 , and of the factors x_1 , x_2 and x_3 of the risk-free short rate that solves the following boundary value problem (BVP):

The credit spread of a ZCB zero-recovery is:

$$y^*(y, T) - y^r(y, T) = \left(\frac{A^r(\tau) - A^d(\tau)}{\tau} \right) + \left(\sum_{i=1}^3 \frac{B_i^d(\tau)}{\tau} - \sum_{i=1}^3 \frac{B_i^r(\tau)}{\tau} \right) x_i(t) + \sum_{i=1}^2 \frac{C_i^d(\tau)}{\tau} \lambda_i(t) \quad (16)$$

where functions $B_i^d(\tau)$ and $C_i^d(\tau)$ are factor loadings of the risk factors λ_1, λ_2 , and of the factors x_1, x_2 and x_3 on default rates and risky yield respectively.

3.2.4. Market price of risk and P -dynamics

The market price of risk is assumed to have the form:

$$\omega_{0i} + \omega_{1i} \lambda(t) + \omega_{bi} b(t) + \omega_{ri} r(t) \quad (17)$$

where $r(t)$ is the short rate dynamics and $b(t)$ is the instantaneous breakeven inflation (BEI) dynamics. Therefore, the natural drift is of the form:

$$\begin{aligned} \bar{\alpha}_i \left(\bar{\gamma}_{\lambda_i} - \lambda(t) + \bar{c}_i r(t) + \varphi_i b(t) \right) = \\ \alpha_i (\gamma_{\lambda} - \lambda(t) + c_i r(t)) + (\omega_{0i} + \omega_{1i} \lambda(t) + \omega_{bi} b(t) + \omega_{ri} r(t)) \sigma_{\lambda_i} \end{aligned} \quad (18)$$

with

$$\bar{\alpha}_i = \alpha_i - \omega_{1i} \sigma_{\lambda_i} \quad (19)$$

$$\bar{\gamma}_{\lambda_i} = \frac{\alpha_i \gamma_{\lambda} + \omega_{0i} \sigma_{\lambda_i}}{\bar{\alpha}_i} \quad (20)$$

$$\varphi_i = \frac{\omega_{bi}\sigma_{\lambda_i}}{\bar{\alpha}_i} \quad (21)$$

$$\bar{c}_i = \frac{a_i c_i + \omega_{ri}\sigma_{\lambda_i}}{\bar{\alpha}_i} \quad (22)$$

The dynamics of the default factors λ under the natural probability measure P can be written as:

$$d\lambda(t) = \bar{A}(\bar{\gamma}_\lambda - \lambda(t) + \bar{C}r(t) + \Phi b(t))dt + \sum dW^P(t) \quad (23)$$

where

$$\bar{A} = \begin{bmatrix} \bar{\alpha}_1 & 0 \\ 0 & \bar{\alpha}_2 \end{bmatrix}; \bar{\gamma}_\lambda = \begin{bmatrix} \bar{\gamma}_{\lambda_1} \\ \bar{\gamma}_{\lambda_2} \end{bmatrix}; \bar{C} = \begin{bmatrix} \bar{c}_1 \\ \bar{c}_2 \end{bmatrix}; \Phi = \begin{bmatrix} \varphi_1 \\ \varphi_2 \end{bmatrix}$$

and $dW^P(t)$ is a vector of correlated Wiener processes of the form:

$$dW^P = \begin{bmatrix} dW_1^P(t) \\ dW_2^P(t) \end{bmatrix}, \text{corr}(dW_1^P(t), dW_2^P(t)) = \rho_\lambda dt$$

For a time step $\Delta t = t + \Delta t - t$, the expected default rate λ is:

$$E^P[\lambda(t + \Delta t)|F_t] = \sum_{i=1}^2 (\bar{\gamma}_{\lambda_i} + \bar{c}_i(E^P[r(t)|F_{t-1}] + \varphi_i(E^P[b(t)|F_{t-1}]))(1 - \exp(-\bar{\alpha}_i \Delta t)) + \sum_{i=1}^2 \exp(-\bar{\alpha}_i \Delta t) \lambda_{t-1} \quad (24)$$

The variance of the default rate is:

$$\text{Var}^P[\lambda(t)|F_t] = \sum_{i=1}^2 (\bar{c}_i^2 \text{Var}^P[r(t)|F_{t-1}](1 - \exp(-2\bar{\alpha}_i \Delta t)) + \sum_{i=1}^2 (\varphi_i^2 \text{Var}^P[b(t)|F_{t-1}](1 - \exp(-2\bar{\alpha}_i \Delta t))) + \sum_{i=1}^2 \left(\frac{\sigma_{\lambda_i}^2}{2\bar{\alpha}_i} (1 - \exp(-2\bar{\alpha}_i \Delta t)) \right) + \left(\frac{\sigma_{\lambda_1} \sigma_{\lambda_2} \rho_\lambda}{\bar{\alpha}_1 + \bar{\alpha}_2} \right) (1 - \exp(-(\bar{\alpha}_1 + \bar{\alpha}_2) \Delta t)) \quad (25)$$

where $r(t)$ and $b(t)$ are the factors of the short rate and the BEI rate conditioned to time $t-1$. We consider the two rates as observable variables as they are previously filtered from ZCB (zero coupon bonds) yields and inflation swap respectively.

The factors of the expected default rates are composed of three parts: the observed interest rates, BEI rates, and a not observed part. All of the parts describe the dynamics of the expected default rate in a composite way. The same holds for the variance.

3.3. Estimation procedure

The Gaussian assumption of default rates and the affine relation between the zero recovery risky rates and the factors allow us to use the Kalman filter to estimate the model parameters. This choice leads to:

- Faster convergence towards the optimum than Sequential Monte Carlo filters (Doucet, de Freitas, & Gordon, 2001) and MCMC procedures

such as the Gibbs sampling (S. Geman & D. Geman, 1984) and the Metropolis-Hasting algorithm (Chib & Greenberg, 1995). All of these estimation procedures needed a couple of days to be completed. Thus when lots of data sets are needed to carry out the analysis, the estimation procedure could be dramatically long.

- Less asymptotic biases than the use of Kalman filter with non-Gaussian dynamics as in the analysis proposed by De Jong (2000), Chen and Scott (2003), Duan and Simonato (1999).

Considering a set of calendar times t_1, t_2, \dots, t_m and with constant time step $\Delta t = t_{k+1} - t_k$ for every $k = 1, 2, \dots, m-1$ and (2×1) vector of latent variables $X(t)$ (default rate components), two vectors (3×1) of observed variables $X^N(t)$ (risk free short rate components), $X^b(t)$ (BEI rate components) $\forall k$ a vector $\zeta(t_k) = (\zeta_1(t_k), \zeta_2(t_k), \dots, \zeta_p(t_k))'$ zero-recovery risky rates at fixed maturities $\tau_1, \tau_2, \dots, \tau_p$, we have our state space model of the form:

$$\text{(measurement equation)} \underbrace{z(t_k)}_{p \times 1} = \underbrace{A}_{p \times 1} + \underbrace{C}_{p \times 2} \underbrace{X(t_k)}_{2 \times 1} + \underbrace{B}_{p \times 3} \underbrace{X^N(t_k)}_{3 \times 1} + \underbrace{\eta(t_k)}_{p \times 1} \eta(t_k) \sim \text{IID}(0, R) \quad (26)$$

$$\text{(transition equation)} \underbrace{X(t_k)}_{2 \times 1} = \underbrace{\theta}_{2 \times 1} + \underbrace{F}_{2 \times 2} \underbrace{X(t_{k-1})}_{2 \times 1} + \underbrace{\varepsilon(t_k)}_{p \times 3} \varepsilon(t_k) \sim \text{IID}(0, Q) \quad (27)$$

where

$$\theta = \begin{bmatrix} \left(\bar{\gamma}_1 + \bar{C}_1 \sum_{i=1}^3 (x^N(t_k|Ft_{k-1}) + \varphi_1 \sum_{i=1}^3 (x^b(t_k|Ft_{k-1})) (1 - \exp(-\bar{\alpha}_i \Delta t))) \right) \\ \left(\bar{\gamma}_2 + \bar{C}_2 \sum_{i=1}^3 (x^N(t_k|Ft_{k-1}) + \varphi_2 \sum_{i=1}^3 (x^b(t_k|Ft_{k-1})) (1 - \exp(-\bar{\alpha}_i \Delta t))) \right) \end{bmatrix} \quad (28)$$

and

$$F = \begin{bmatrix} \exp(-\bar{\alpha}_1 \Delta t) & 0 \\ 0 & \exp(-\bar{\alpha}_2 \Delta t) \end{bmatrix} \quad (29)$$

$$\varepsilon(t_k) = \begin{bmatrix} \varepsilon_1(t_k) \\ \varepsilon_2(t_k) \end{bmatrix} \quad (30)$$

$$Q = \begin{bmatrix} \sigma_1 & \varrho_{12} \\ \varrho_{12} & \sigma_2 \end{bmatrix} \quad (31)$$

The variance of the latent variables and the variance of the measurement equation are:

$$P(t_k) = E \left[(X(t_k) - \hat{X}(t_k)) | Ft_k \right]^2 \quad (32)$$

$$S(t_k) = E \left[(z(t_k) - \hat{z}(t_k)) | Ft_k \right]^2 \quad (33)$$

Although heteroskedastic noises are considered in the estimation of both the interest rate process and the BEI rate process, the only pricing imperfection in this model comes from the imperfect observation of the bootstrapped default rates through maturities. We assume them to be fixed across maturities.

This approach agrees with the model proposed by Joslin, Le and Singleton (2013). In particular, the model presented here is the TS^f model where only the unobserved part, the default rate is filtered and the observable variables are introduced without measurement errors.

The log likelihood function is:

$$\log L = -\frac{mp}{2} \log(2\pi) - \frac{1}{2} \sum_{k=1}^m \log \det S(t_k) - \frac{1}{2} \sum_{k=1}^m \eta(t_k) S^{-1}(t_k) \eta'(t_k) \quad (34)$$

4. RESULTS AND DISCUSSION

We evaluate the performance of the model both in sample and out of sample. The estimation period includes the period of the Great Recession (2008-2009) and the sovereign debt crisis (2011-2012). Those periods, as a consequence of financial market turmoil, show an unprecedented level of EA banks default rates and positive interest rates. Afterwards, European interest rates became at Zero Lower Bound and negative and default rates decreased. Therefore, it is crucial to estimate the model in the period of crisis and evaluate it in a period of Zero Lower Bound interest and negative rates.

Following Joslin et al. (2014), we perform the out of sample analysis by taking the parameter estimates as fixed on the level of the training set. We estimate interest rates models and BEI rates models over the training set and we project the models in the future. Afterwards, we introduce the forecasted factors of interest rates and BEI rates and the model parameters in the term structure of default rates model.

We estimate the model through an expanding window, updating the parameter estimates over three subsamples:

- the first one from 02/08/2014 to 16/04/2015, the period before the implementation of the quantitative easing (QE) policy by the ECB;
- the second from 17/04/2015 to 25/12/2015;
- the third from 26/12/2015 to 03/11/2016.

We test the parameter instability within the subsample above, by performing a Chow forecast test once every subsample is added to the training set. Furthermore, we compare the performance of the model (model 1, or m1) with a performance of two restricted models. In particular:

- We consider the price of risk of this form: $\omega_{0i} + \omega_{1i}\lambda(t)$

BEI rates and interest rates do not affect the price of risk of default rates, i.e. $\omega_{b1} = \omega_{r1} = \omega_{b2} = \omega_{r2} = 0$ (model 2, or m2).

- Independence of default rates and interest rates and BEI rates both in P and in Q , i.e. $\omega_{b1} = \omega_{r1} = \omega_{b2} = \omega_{r2} = 0$; $c_1 = c_2 = 0$, (model 3, or m3).

4.1. In sample analysis

The analysis in sample shows a statistically significant and negative relation between interest rates and default rates in the risk-neutral world for m2 and m1.

Interest rates and BEI rates do not affect significantly the default risk premium. This result holds for every sample analysed. The log likelihood ratio test shows that model 2 is preferable over the model 1 at 1%. Model 3 has worse goodness of fit in every sample analysed than model 2 and model 1.

Table 2. Log likelihood ratio test of model 1 (m1), model 2 (m2) and model 3 (m3)

Loglike ratio test = $-2(\log Lu - \log Lr) \sim \chi_n$		
Sample 03/11/2008 g - 01/08/2014		
	Loglike ratio	p-value
m1/m2	0.104**	0.0249
m2/m3	119.061***	0.0320
m2/m3	119.166***	0.0000
Sample 03/11/2008 - 16/04/2015		
	Loglike ratio	p-value
m1/m2	0.137**	0.0320
m2/m3	2220.771***	0.0000
m1/m3	2220.908***	0.0000
Sample 03/11/2008 - 25/12/2015		
	Loglike ratio	p-value
m1/m2	0.148**	0.0344
m2/m3	3560.694***	0.0000
m1/m3	3560.694***	0.0000
Sample 03/11/2008 - 03/11/2016		
	Loglike ratio	p-value
m1/m2	0.1796**	0.0410
m2/m3	3950.043***	0.0000
m1/m3	3950.222***	0.0000
$H_0 = Lu < Lr, H_1 = Lu > Lr$		

Notes: *significance at 10%; **significance at 5%; ***significance at 1%

Every model shows autocorrelated residuals. The Durbin Watson test shows model 1 has slightly less autocorrelated residuals than model 2 and model 3.

Table 3. Durbin Watson test over sample 03/11/2008 - 01/08/2014

Mat	DW(m3)	DW(m2)	DW(m1)
1	1.333	1.484	1.486
2	1.479	1.459	1.461
3	1.337	1.474	1.475
4	1.332	1.504	1.505
5	1.393	1.522	1.524
6	1.474	1.523	1.524
7	1.437	1.544	1.545
8	1.416	1.460	1.461
9	1.399	1.498	1.499
10	1.311	1.452	1.452
$DW = 2(1 - \rho(\epsilon))$			
$\rho(\epsilon) = 0.5(2 - DW)$			

In general, a Durbin Watson that is smaller than 1.5 indicates autocorrelated residuals. This result is shown in every sample analysed. Parameter estimates over all sample analysed show that unspanned variables are not statistically significant.

Furthermore, the Chow forecasting test shows a structural break in the period of decreasing interest rates for model 1, model 2 and model 3.

Table 4. Chow forecast test of model 1 (m1), model 2 (m2) and model 3(m3)

Chow forecast test = $\left(\frac{RSS(s1) - RSS(s2)}{RSS(s2)}\right) \frac{n(s2) - df}{n(s1) - n(s2)} \sim F(n(s1) - n(s2), n(s2) - df)$							
Samples/models		m3	p-value	m2	p-value	m1	p-value
03/11/2008 - 16/04/2015	01/08/2014 vs 03/11/2008 -	0.293***	0.000	0.241***	0.000	0.289***	0.000
03/11/2008 - 25/12/2015	01/08/2014 vs 03/11/2008 -	0.314***	0.000	0.299***	0.000	0.454***	0.000
03/11/2008 - 25/12/2015	16/04/2015 vs 03/11/2008 -	0.432***	0.002	0.458	0.976	0.839	0.957
03/11/2008 - 03/11/2016	01/08/2014 vs 03/11/2008 -	0.721***	0.000	1.034***	0.000	0.379***	0.000
$H_0 = para(s1) = para(s2), H_1 = para(s1) \neq para(s2)$							

Notes: *significance at 10%; **significance at 5%; ***significance at 1%

The results found in the estimation sample, show that the introduction of unspanned macro variables in the price of risk of EA banks default rates does not improve significantly the goodness of fit of the term structure model.

On the other hand, there is a significant improvement of goodness of fit if interest rates are considered spanned in the term structure of default rates. This result is more evident after the implementation of the QE from the ECB.

4.2. Out of sample analysis

It is also important to evaluate the model outside the estimation procedure. It is well known in the literature that Bayesian estimation procedure “attach” historical data to model generated data through the optimization procedure. As a consequence, there may be poor forecasting accuracy outside the estimation sample. Serious problem that a researcher may have is the overfitting of the model outside the estimation sample. This may occur when the RMSE (root mean squared errors, also called root mean forecasting errors) of the evaluation sample are higher than the RMSE in the estimation sample.

Indicators of overfitting are the presence of autocorrelated residuals in the estimation sample. In this case, the estimation of the model is still unbiased but it has no longer the minimum variance of the errors. As a consequence, using the vector of model parameters of that estimation sample leads to poor forecasting accuracy of future data.

The poor forecasting accuracy is a problem widely addressed in the existing literature since early 2000 (Duffee, 2002).

To solve the forecasting accuracy problem of term structure models, macroeconomic variables are

introduced jointly with the yield curve factors (Ang & Piazzesi, 2003; Coroneo, Giannone, & Modugno, 2016).

To evaluate the forecasting ability of the models, we generate an out of sample iterative forecast of the factors:

$$E = [X^N(t_{k+h})|F_t] = \Gamma + F_N X^N(t_k) \tag{35}$$

$$E = [X(t_{k+h})|F_t] = \theta + FX(t_k) \tag{36}$$

where

- $X^N(t_k)$ are factors of risk-free yields and $X(t_k)$ are factors of default rates;
- h is equal to the forecast horizon; and
- F and F_N are estimated with the information available up to time t_k .

We then compute the out of sample forecasts of zero recovery risky yields given the projection of the factors:

$$E[z(t_{k+h})|F_t] = A + C[X(t_{k+h})|F_t] + BE[X^N(t_{k+h})|F_t] \tag{37}$$

Finally, we subtract the risk-free part of the zero recovery risky yield to obtain the forecasted default rates:

$$E[z(t_{k+h})|F_t] - E[z^N(t_{k+h})|F_t] = E[z^d(t_{k+h})|F_t] \tag{38}$$

Comparing the forecasted default rates with the forward risk-neutral default rates data over the test set, $n = t_1 - t_0$, we compute the root mean forecasting errors (RMFE). For every maturity $\tau = (\tau_1, \tau_2, \dots, \tau_p)$, the RMFE are:

$$RMFE(\tau_i, h, m) = \left(\sqrt{\frac{\sum_{t=t_0}^{t_1} (E[z^d(\tau_i)(t_{k+h})|F_t] - y^d(\tau_i)(t_{k+h}))^2}{n + 1}} \right) \tag{39}$$

Following Coroneo et al. (2016), we compute ratios between the RMFE of every model analysed.

Let $M = (m1, m2, m3)$ the models considered, we have:

$$RM(\tau_i, h, M) = \frac{RMFE(\tau_i, h, m_\alpha)}{RMFE(\tau_i, h, m_\beta)} \forall \alpha \neq \beta \tag{40}$$

Table 5. In sample and out of sample comparison of model 1 (m1), model 2 (m2) and model 3(m3)

Maturity	In sample RMSE			Out of sample RMFE		
	Estimation sample 03/11/2008 - 01/08/2014			Evaluation sample 02/08/2014 - 16/04/2015		
	m1	m2	m3	m1	m2	m3
1	0.0022	0.0022	0.0021	0.0032	0.0030	0.0027
2	0.0021	0.0021	0.0019	0.0046	0.0044	0.0050
3	0.0021	0.0021	0.0020	0.0055	0.0054	0.0064
4	0.0020	0.0021	0.0020	0.0058	0.0056	0.0067
5	0.0020	0.0020	0.0019	0.0063	0.0062	0.0071
6	0.0019	0.0019	0.0018	0.0063	0.0062	0.0069
7	0.0020	0.0021	0.0019	0.0060	0.0059	0.0063
8	0.0019	0.0020	0.0019	0.0068	0.0067	0.0067
9	0.0019	0.0019	0.0019	0.0070	0.0069	0.0065
10	0.0020	0.0020	0.0019	0.0070	0.0070	0.0061

Table 6. Out of sample analysis of model 1 (m1), model 2 (m2) and model 3 (m3)

The ratio m_α/m_β with $\alpha \neq \beta$ is $RM(\tau_i, h, m)$										
	Maturity									
	1	2	3	4	5	6	7	8	9	10
Evaluation period 02/08/2014 - 16/04/2015										
m1/m2	1.101	1.053	1.034	1.026	1.021	1.018	1.016	1.012	1.010	1.009
m2/m3	1.101	0.878	0.845	0.844	0.870	0.902	0.945	1.003	1.068	1.146
m1/m3	1.213	0.925	0.874	0.866	0.888	0.918	0.960	1.016	1.080	1.157
Evaluation period 17/04/2015 - 25/12/2015										
m1/m2	1.010	1.009	1.008	1.007	1.007	1.006	1.006	1.006	1.006	1.006
m2/m3	1.072	1.328	1.407	1.382	1.352	1.264	1.172	1.113	1.040	0.974
m1/m3	1.084	1.341	1.419	1.392	1.362	1.272	1.180	1.120	1.047	0.980
Evaluation period 26/12/2015 - 03/11/2016										
m1/m2	1.028	1.022	1.017	1.014	1.013	1.012	1.010	1.010	1.010	1.010
m2/m3	1.281	1.268	1.277	1.227	1.227	1.190	1.142	1.140	1.116	1.085
m1/m3	1.318	1.296	1.299	1.246	1.243	1.205	1.155	1.153	1.128	1.096

$RMFE(\tau_i, h, m) > 1$ model β better than model α out of sample
 $RMFE(\tau_i, h, m) < 1$ model β worse than model α out of sample

The out of sample analysis with fixed parameters with $h = 1$ (one day ahead), shows that:

- The forecasting performance is poorer than in sample analysis because $RMSE$ (in sample) $<$ $RMSE$ (out of sample) ($RMFE$) in every period is analysed.
- Interest rates improve the forecasting accuracy of the default rates model in the period that finishes at the beginning of the European quantitative easing (i.e., in the evaluation sample from 02/08/2014 to 16/04/2015) with respect to the restricted model with no links with macro variables.
- The unspanned variables do not improve forecasts of future default rates in periods of negative rates. Model 2, always has better forecast accuracy than model 1.

This holds in every evaluation sample. The out of sample analysis confirms the poor performance of affine term structure models (Duffee, 2002). This

evidence holds after introducing unspanned macro variables in the term structure model.

4.3. Implication for the banking industry

From the models analysed we have that:

$$y^*(y, T) - y^r(y, T) = \left(\frac{A^r(\tau) - A^d(\tau)}{\tau} \right) + \left(\sum_{i=1}^3 \frac{B_i^d(\tau)}{\tau} - \sum_{i=1}^3 \frac{B_i^r(\tau)}{\tau} \right) x_i(t) + \sum_{i=1}^2 \frac{C_i^d(\tau)}{\tau} \lambda_i(t) \quad (41)$$

Risk-free rates affect banks' default rates. Does the risk-free rate affect more the long run default rates? Or does it affect only the bank default rates in the short run? Below it is shown how the term structure interest rates affect the term structure of default rates.

Table 7. Effect of the term structure of risk-free rates on the term structure of default rates over sample from November 3, 2008 till August 1, 2014

Mat	$C_1^d \bar{c}_1 \exp(-\bar{\alpha}_1 \Delta s)$	$C_2^d \bar{c}_2 \exp(-\bar{\alpha}_2 \Delta s)$	$B_1^d - B_1^r$	$B_2^d - B_2^r$	$B_3^d - B_3^r$
1	-0.0001	-0.0001	-0.061	-0.072	-0.060
2	-0.0003	-0.0003	-0.096	-0.131	-0.094
3	-0.0005	-0.0003	-0.119	-0.184	-0.114
4	-0.0007	-0.0004	-0.132	-0.231	-0.126
5	-0.0009	-0.0004	-0.140	-0.274	-0.132
6	-0.0011	-0.0004	-0.144	-0.314	-0.135
7	-0.0012	-0.0004	-0.146	-0.350	-0.136
8	-0.0014	-0.0004	-0.146	-0.385	-0.136
9	-0.0016	-0.0004	-0.146	-0.416	-0.135
10	-0.0017	-0.0004	-0.145	-0.446	-0.134

European interest rates span the EA banks default rates. As a consequence, EA banks default rate term structure variates as the European interest rates variates. This is confirmed by the default rates data and interest rates data. After the bursting of the sovereign debt crisis in July 2011, there were remarkable spillover effects among EA countries.

In every EA country, the debt/GDP ratio soared over 100%. This created trouble to EA bank balance sheets as the suffered from huge losses coming from the negative variation of sovereign bond prices. As a consequence, EA banks had liquidity shortages. This led to higher default probabilities.

As a result, the Zero Lower Bound policy put forward by the ECB had the aim to provide EA banks the liquidity to operate. This policy didn't allow EA banks to operate more efficiently leading them to lend fewer resources to the real economy.

The spanning relation between European interest rates and EA banks default rates shown here is stronger at longer maturities. As a result, when the European interest rates term structure flattens, the EA banks default rates term structure will be steeper. This is a consequence of what is shown in Table A.2 and Table A.3, where for every model specification (m1, m2) a reduction of short term European interest rates leads the EA banks short term default rates to increase more than two times.

This relation affects the allocation of EA banks assets and liabilities.

Below is shown the exposition of risk factors taken from the model estimated here. The results are compared with the factors computed with the PCA analysis.

For an illustrative example, consider a portfolio composed by a 1y risky bond, 5y risky bond and 10y risky bond. Consider a negative variation of risk free rate of 1 bp. At one year the risk bond price has a 1 milion, a variation of 5 milion at 5 year and a variation of 10 milion at 10 years.

Given that the risky bond yield is:

$$y^*(t, t+m) = -\frac{1}{m} \ln[Q(t, t+m)P(t, t+m)] \quad (42)$$

$$= y^d(t, t+m) + y(t, t+m)$$

whose dynamics is explained by five factors: two explain the variability of default rates and three factors explain the variability of risk free rates. Considering the factor loadings found by principal component analysis (Figure 4) and the factor loadings implied by the models (functions $B_i^d(\tau)$ and $C_i^d(\tau)$), the exposition of each yield factor is:

$$\text{Exposition first factor default rates PCA} = w^1_{d,pca(1)} 1 + w^1_{d,pca(5)} 5 + w^1_{d,pca(10)} 10 \quad (43)$$

$$\text{Exposition first factor risk - rates PCA} = w^1_{r,pca(1)} 1 + w^1_{r,pca(5)} 5 + w^1_{r,pca(10)} 10 \quad (44)$$

$$\text{Exposition second factor default rates PCA} = w^2_{d,pca(1)} 1 + w^2_{d,pca(5)} 5 + w^2_{d,pca(10)} 10 \quad (45)$$

$$\text{Exposition second factor risk - rates PCA} = w^2_{r,pca(1)} 1 + w^2_{r,pca(5)} 5 + w^2_{r,pca(10)} 10 \quad (46)$$

$$\text{Exposition third factor risk - rates PCA} = w^3_{r,pca(1)} 1 + w^3_{r,pca(5)} 5 + w^3_{r,pca(10)} 10 \quad (47)$$

$$\text{Exposition first factor default rates m1 and m3} = C^1_{d,(1)} 1 + C^1_{d,(5)} 5 + C^1_{d,(10)} 10 \quad (48)$$

$$\text{Exposition second factor default rates m1 and m3} = C^2_{d,(1)} 1 + C^2_{d,(5)} 5 + C^2_{d,(10)} 10 \quad (49)$$

$$\text{Exposition first factor risk - rates m1 and m3} = B^1_{d,(1)} 1 + B^1_{d,(5)} 5 + B^1_{d,(10)} 10 \quad (50)$$

$$\text{Exposition second factor risk - rates m1 and m3} = B^2_{d,(1)} 1 + B^2_{d,(5)} 5 + B^2_{d,(10)} 10 \quad (51)$$

$$\text{Exposition third factor risk - rates m1 and m3} = B^3_{d,(1)} 1 + B^3_{d,(5)} 5 + B^3_{d,(10)} 10 \quad (52)$$

where $w^i_{r,pca(\tau(j))}$ and $B^i_{d,\tau(j)}$ are the factor loadings of risk-free rates factor I at maturity $\tau(j)$ on risk-free yields, $C^i_{d,\tau(j)}$ is the factor loading of default rates factor I at maturity $\tau(j)$ and 1, 5 and 10 are bond's

prices variations of 1 year maturity bond, 5 year maturity bond and 10 year maturity bond respectively.

Figure 5. Exposition to default rate factors in million of euros given a negative variation of risk-free rate of 1 bp

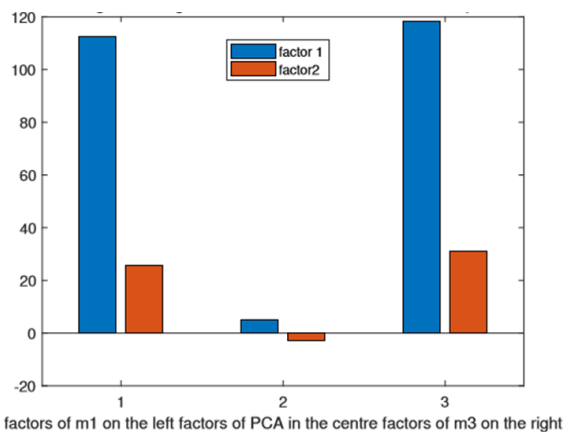
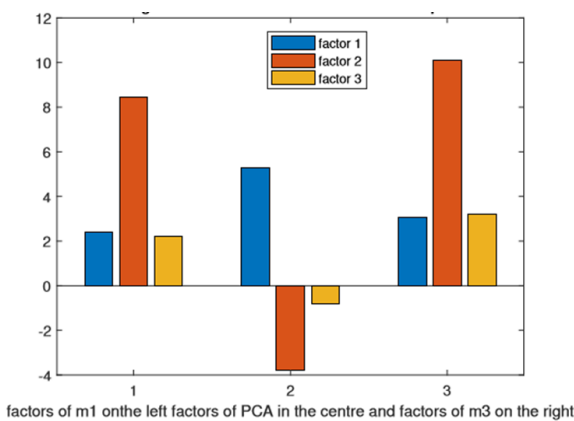


Figure 6. Exposition to interest rate risk factors in million of euros given a variation of risk-free rate of 1 bp



To a variation of 1bp of risk-free rates, we have a greater exposition to default rates than risk-free rates, more in the long run.

By decreasing the duration of assets to reduce the exposition to default risk, EA banks will further reduce the net interest margin as well. As a consequence, this leads the bank default risk to further increase.

Indeed EA banks will hold longer maturities and riskier assets in their portfolio to maximize the NIM at the cost of having a riskier portfolio of assets and lending fewer resources to the real economy.

This argument agrees with the recent literature on bank profitability and risk-taking behavior (Martynova, Ratnovski, & Vlahu, in press; Ferrero, Nobili, & Sene, 2019).

5. CONCLUSION

In this work, we estimate a bivariate Gaussian default rate model. We assume an unspanned relation between BEI rates and default rates. BEI rates affects only the expectation of future default rate in P through the default risk premium. We find that interest rates and inflation rates do not significantly affect the price of risk of default.

Furthermore:

1. Results on the correlation between interest rates and default rates in the Q -measure agree with the existing literature (Duffee, 1998, 1999).

2. Interest rates impact more the long term default rates. Negative interest rates make the slope of the default rate term structure steeper. This may create strains for banks.

3. The introduction of unspanned macroeconomic variables in the default rates model developed here does not improve the goodness of fit in sample with respect to reduced versions of the model. Model 2 outperforms both model 1 and model 3 in every samples taken into account. In particular, this holds in samples where interest rates are less negative (i.e., in the period before QE).

4. Macroeconomics variables improve the forecasting accuracy of the model when interest rates are higher and less negative. However, they don't improve the forecasting accuracy of the model after the implementation of QE (from the end of March 2015 onwards).

5. The poor forecasting ability of the term structure model developed here arises from the assumption of the Gaussian distribution for describing the evolution of both default rates and macroeconomic variables in periods on Zero Lower Bound and negative interest rates. The Gaussian distribution overestimates the price of risks asked by investors to bear any financial instrument in an economy of lower interest rates.

What are the consequences for EA banks? Taking for granted the term structure modelling issues, point 2 suggests that EA banks need to change their asset liability management in favour of longer maturity assets to maintain profitability. As a consequence, EA banks will be more risk taker (Bruno and Shin (2015) analyzed the role of financial intermediation and risk-taking behavior) by going through the reach of yield behavior. This may put further strains on the EA banking system as:

- Liquidity capital requirements, such as NSFR (net stable funding ratio), will be difficult to match as there will be fewer short term liquidity in EA banks balance sheets.
- Possibility of spillover effects as a consequence of the liquidity crisis of a G-SIB.
- Further increase of probability of default of EA banks putting further strains on the stability of the European banking system.

However, the model developed here, with an assumption of the Gaussian dynamics for describing both default rates and macroeconomic variables, is a serious source of bias for a risk manager for handling default risks and macro risks in the banking book and in the trading book that may put further strains to EA banking system.

Thus, in the ongoing macro-financial context we need to develop good financial models that consider the effect of macroeconomic determinants to price instruments to handle risks and make better forecasts to prevent the European banking system to collapse. This helps us to explain phenomena like the reach to yield behavior and to evaluate its effect on the stability of the banking system as a whole. We leave the further development of new and more complex financial models to further research objectives.

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APPENDIX

Table A.1. Interest rates and BEI rates over sample 03/11/2008 - 01/08/2014

Para	Interest rates	S.E.	BEI rates	S.E.
κ_1	0.569***	0.018	0.145***	0.002
γ_1	0.103***	0.001	0.072***	0.002
σ_1	0.152***	0.071	0.345***	0.006
λ_{01}	0.528***	0.263	0.158	0.122
λ_{11}	0.100***	0.026	-0.019	0.013
κ_2	0.027***	0.001	0.088***	0.001
σ_{21}	0.009***	0.001	-0.067***	0.002
σ_{22}	0.015***	0.001	0.045***	0.001
λ_{02}	-0.205***	0.105	-0.030	0.045
λ_{12}	-0.918***	0.163	0.972***	0.028
κ_3	0.479***	0.024	0.189***	0.002
σ_{31}	-0.159***	0.071	-0.290***	0.004
σ_{32}	-0.017***	0.001	-0.044***	0.002
σ_{33}	0.006***	0.001	0.009***	0.001
λ_{03}	-0.640**	0.270	-0.159	0.148
λ_{13}	-0.794*	0.463	-0.255	0.133
Loglike	-134249		-129229.12	

Notes: *significance at 10%; **significance at 5%; ***significance at 1%

Table A.2. Model parameters of default rates over sample 03/11/2008 - 01/08/2014

Para	m3	S.E.	m2	S.E.	m1	S.E.
α	0.0144***	0.0003	0.0256***	0.0007	0.0256***	0.0021
γ_λ	0.3265***	0.0090	0.3061***	0.0087	0.3061***	0.0170
σ_{λ_1}	0.0256***	0.0002	0.0263***	0.0003	0.0263***	0.0005
ω_{01}	-0.161	0.0235	-0.278	0.1248	-0.277	0.4401***
ω_{11}	-0.988***	0.104	-0.996	0.4368	-0.996	0.9155
ω_{b1}	-	-	-	-	-0.033	0.1814
ω_{r1}	-	-	-	-	0.1076	0.5650
c	-	-	-2.851***	0.0822	-2.851***	0.2987
c_1	-	-	-0.163***	0.0199	-0.163***	0.0719
α_1	0.4765***	0.0028	0.5910***	0.0056	0.5910***	0.0123
σ_{λ_2}	0.0237***	0.0006	0.0239***	0.0008	0.0239	0.0023
ω_{02}	-0.150	0.0175	-0.024***	0.0065	-0.011	0.0552
ω_{12}	-0.330	0.0752	-0.733***	0.1718	-0.933	1.0955
ω_{b2}	-	-	-	-	-0.977	2.2142
ω_{r2}	-	-	-	-	0.1057	0.4472
p_λ	0.0635***	0.0085	0.0216***	0.0088	0.0214	0.0548
σ_ϵ	0.0006***	0.0000	0.0006***	0.0000	0.0006***	0.0000
Loglike	-83851.8		-83911.3		-83911.4	

Notes: *significance at 10%; **significance at 5%; ***significance at 1%

Table A.3. Model parameters of default rates over sample 03/11/2008 - 17/04/2015

Para	m3	S.E.	m2	S.E.	m1	S.E.
α_1	0.0225***	0.0018	0.0171***	0.0007	0.0171***	0.0006
γ_λ	0.2419***	0.0258	0.3756***	0.0100	0.3757***	0.0128
σ_{λ_1}	0.0276***	0.0037	0.0253***	0.0007	0.0253***	0.0024
ω_{01}	-0.184	1.7356	-0.247***	0.0218	-0.248	0.1354
ω_{11}	-0.988	1.6364	-0.997**	0.2975	-0.997	1.0563
ω_{b1}	-	-	-	-	0.1090	0.3082
ω_{r1}	-	-	-	-	0.6488	1.9683
c	-	-	-3.218***	0.3079	-3.218***	0.1240
c_1	-	-	-0.040***	0.0015	-0.040	0.1314
α_2	0.4522***	0.0778	0.6256***	0.0140	0.6256***	0.0433
σ_{λ_2}	0.0225***	0.0068	0.0229***	0.0006	0.0229***	0.0010
ω_{02}	-0.023	0.2962	0.0305***	0.0063	0.0362	0.0911
ω_{12}	-0.226	1.8415	-0.734	0.1470	-0.593	0.8770
ω_{b2}	-	-	-	-	-0.983	2.1751
ω_{r2}	-	-	-	-	0.1073	0.6343
p_λ	-0.073	0.3462	0.0419***	0.0006	0.0421	0.0547
σ_ϵ	0.0007***	0.0001	0.0006***	0.0000	0.0006***	0.0000
Loglike	-93586.3		-94696.7		-94696.8	

Notes: *significance at 10%; **significance at 5%; ***significance at 1%

Table A.4. Model parameters of default rates over sample 03/11/2008 - 25/12/2015

Para	m3	S.E.	m2	S.E.	m1	S.E.
α_1	0.0282**	0.0151	0.0184***	0.0003	0.0184***	0.0033
γ_λ	0.2059**	0.0894	0.3447***	0.0117	0.3446***	0.0393
σ_{λ_1}	0.0285**	0.0027	0.0246***	0.0002	0.0246***	0.0017
ω_{01}	-0.174	0.7391	-0.239***	0.0301	-0.239***	0.1503
ω_{11}	-0.988	1.6763	-0.997***	0.1148	-0.997	2.3733
ω_{b1}	-	-	-	-	0.2440	1.7308
ω_{r1}	-	-	-	-	0.5275	2.0655
c	-	-	-3.396***	0.2027	-3.398***	0.1120
c_1	-	-	0.0007	0.0009	0.0013	0.0681
α_2	0.4296***	0.0817	0.6266***	0.0032	0.6261***	0.0103
σ_{λ_2}	0.0218***	0.0019	0.0221***	0.0007	0.0221***	0.0007
ω_{02}	-0.072	0.1112	-0.078**	0.0070	-0.080	1.1723
ω_{12}	0.0088	0.0461	-0.838**	0.3729	-0.551	10.549
ω_{b2}	-	-	-	-	-0.984	11.354
ω_{r2}	-	-	-	-	-0.077	1.9830
p_λ	-0.177	0.1462	0.0022**	0.0010	0.0034	0.0372
σ_ϵ	0.0007***	0.0000	0.0006***	0.0000	0.0006***	0.0000
Loglike	-103899		-105679		-105679	

Notes: *significance at 10%; **significance at 5%; ***significance at 1%

Table A.5. Model parameters of default rates over sample 03/11/2008 - 03/11/2016

Para	m3	S.E.	m2	S.E.	m1	S.E.
α_1	0.0303***	0.0021	0.0178***	0.0003	0.0178***	0.0026
γ_λ	0.1984***	0.0136	0.3465***	0.0087	0.3465***	0.0193
σ_{λ_1}	0.0294***	0.0022	0.0250***	0.0003	0.0250***	0.0015
ω_{01}	-0.160	0.0473	-0.222	0.0341	-0.222	0.1661
ω_{11}	-0.999	3.0796	-0.999	0.1475	-0.999	1.0437
ω_{b1}	-	-	-	-	0.7601	1.4762
ω_{r1}	-	-	-	-	0.0609	0.3817
c	-	-	-3.158***	0.0674	-3.157***	0.2176
c_1	-	-	0.0418***	0.0010	0.0417***	0.0113
α_2	0.1259*	0.0817	0.6191***	0.0034	0.6191***	0.0124
σ_{λ_2}	0.0012	0.0019	0.0252***	0.0007	0.0252***	0.0020
ω_{02}	0.0166	0.1112	-0.090	0.0280	-0.087	0.4398
ω_{12}	0.3283**	0.0461	-0.391**	0.0802	-0.988	0.9094
ω_{b2}	-	-	-	-	-0.995	0.9570
ω_{r2}	-	-	-	-	-0.065	0.4065
p_λ	-0.267**	0.0402	0.0496**	0.0074	0.0496	0.2131
σ_ϵ	0.0007***	0.0001	0.0006***	0.0000	0.0006***	0.0000
Loglike	-116901		-118876		-118876	

Notes: *significance at 10%; **significance at 5%; ***significance at 1%

Figure A.1. Model parameters of default rates over sample 03/11/2008 - 03/11/2016

